# MAT 22B Problem Set 5 (Due 8/29)

1. Solve the following initial value problems, and determine the behavior of the solution as $t \to \infty$.
   (a) $y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 2$
   (b) $9y'' - 12y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = -1$
   (c) $y'' + 4y' + 4y = 0, \quad y(-1) = 2, \quad y'(-1) = 1$

2. Solve the following differential equations given one of the solutions to the differential equaiton.
   (a) $t^2y'' - 4ty' + 6y = 0, \quad t > 0, \quad y_1(t) = t^2$
   (b) $t^2y'' + 3ty' + y = 0, \quad t > 0, \quad y_1(t) = t^{-1}$
   (c) $x^2y'' + xy' + (x^2 - 0.25)y = 0, \quad x > 0, \quad y_1(x) = x^{-\frac{1}{2}}\sin(x)$

3. Solve the following initial value problems.
   (a) $y'' + 4y = t^2 + 3e^t, \quad y(0) = 0, \quad y'(0) = 2$
   (b) $y'' - 2y' + y = te^t + 4, \quad y(0) = 2, \quad y'(0) = -1$
   (c) $y'' + 3y' + 2y = e^t(t^2 + 1)\sin(2t) + 3e^{-t}\cos(t) + 4e^t, \quad y(0) = 0, \quad y'(0) = 0$

4. Solve the following differential equation
$$y'' + y = \begin{cases} t & 0 \le t \le \pi \\ \pi e^{\pi - t} & t > \pi \end{cases}$$
   with initial conditions $y(0) = 0$ and $y'(0) = 1$, and assume that $y$ and $y'$ are continuous at $t = \pi$.

5. Find the general solution of the following differential equations and determine the interval of validity.
   (a) $y'' + y = \tan(t)$
   (b) $y'' + 4y' + 4y = t^{-2}e^{-2t}$
   (c) $y'' - 2y' + y = \frac{e^t}{1+t^2}$

6. Show that the solution to the initial value problem
$$y'' + by' + cy = g(t), \quad y(t_0) = 0, \quad y'(t_0) = 0$$
   may be written in the form
$$y(t) = \int_{t_0}^{t} K(t - s)g(s)\, ds.$$

7. A spring is streched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damped that exerts a force of 3 $N$ when the velocity of the mass is 5 meters per second. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm per second, determine its position $u$ at any time $t$. Find the quasi-frequency $\mu$ and the ratio of $\mu$ to the natural frequency of the corresponding undamped motion.

8. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin\left(\frac{t}{2}\right)$ N and moves in a madium that imparts a viscous force of 2 N when the speed of the mass is 4 cm per second. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm per second, formulate the initial value problem describing the motion of the mass.