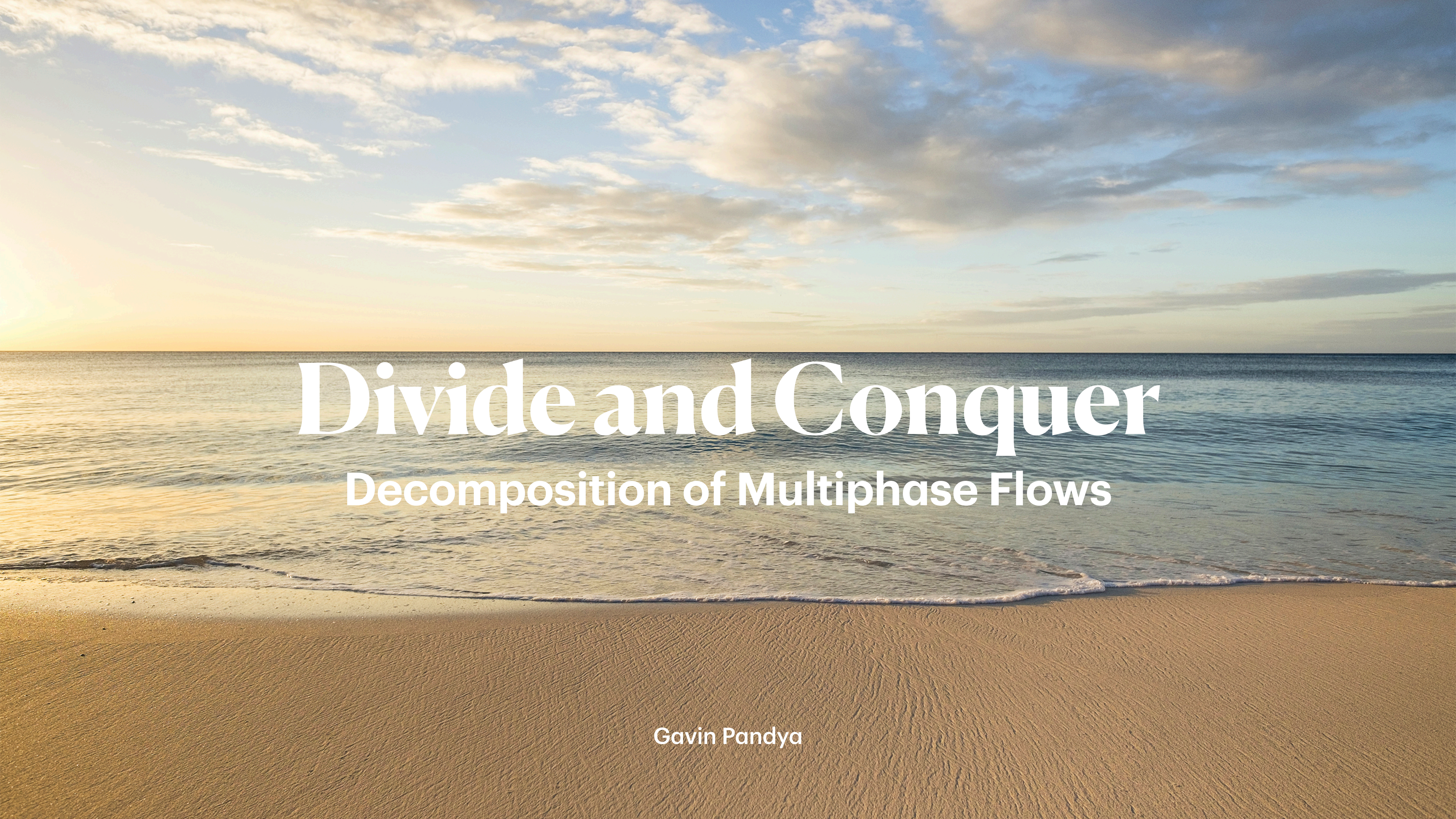




Divide and Conquer

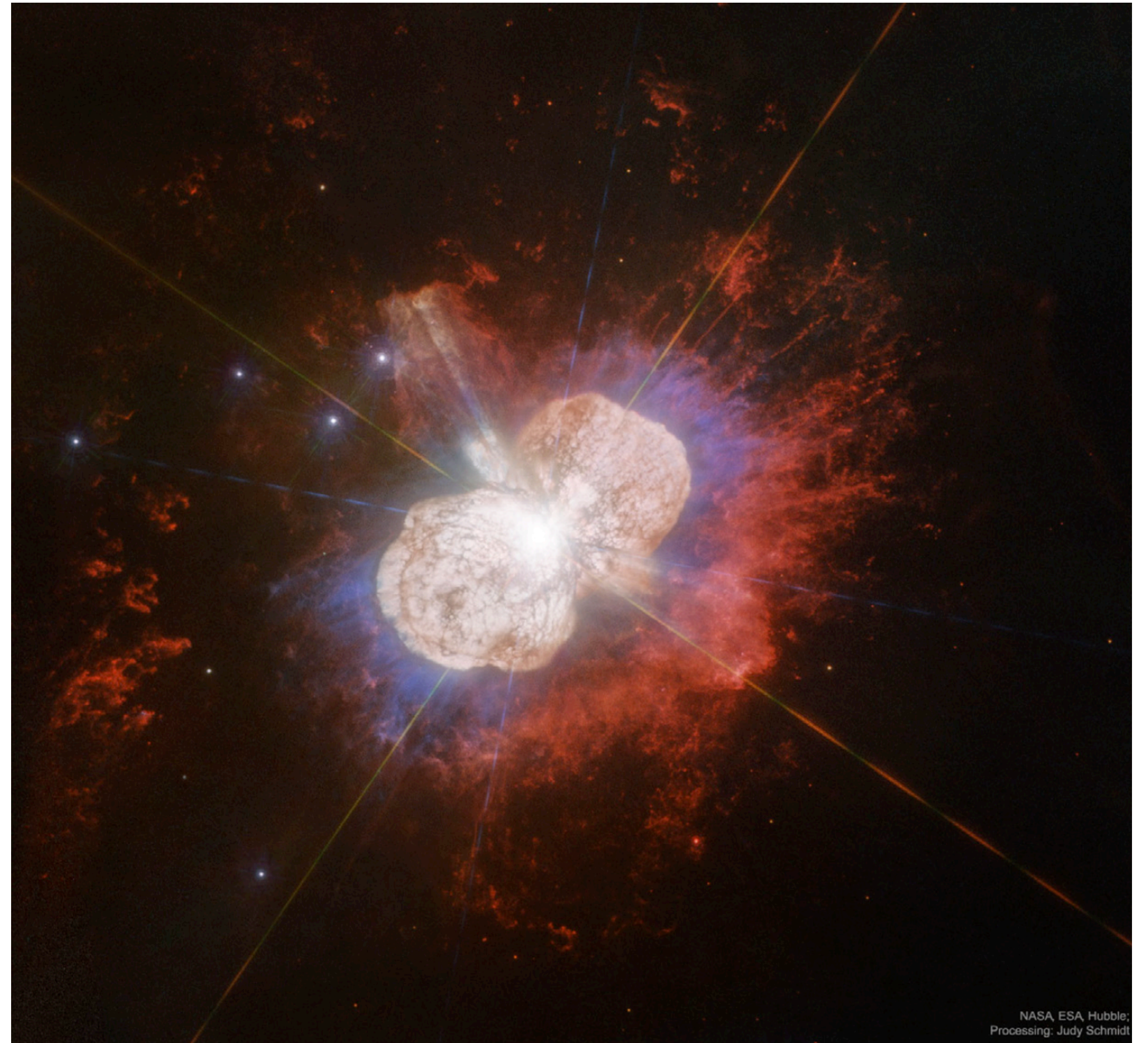
Decomposition of Multiphase Flows

Gavin Pandya

Methodology

Fluid Equations are HARD 😞

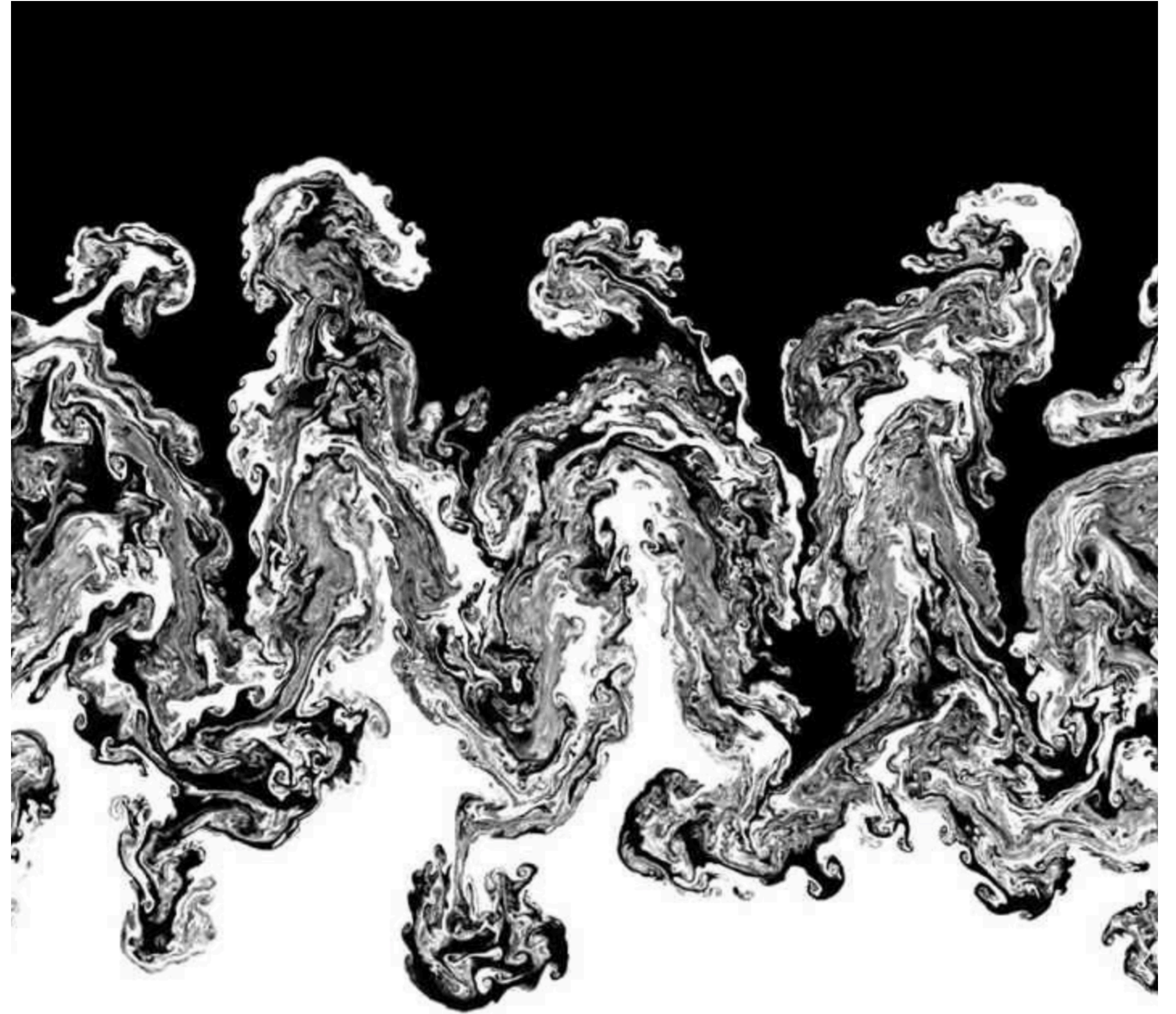
- Action at many scales
- Direct numerical simulation is computationally unfeasible
- **Idea:** decompose flow into simple constituent parts
 - e.g. transport, diffusion, shock waves, contact discontinuities, reaction, combustion, etc.



Fluid Equations

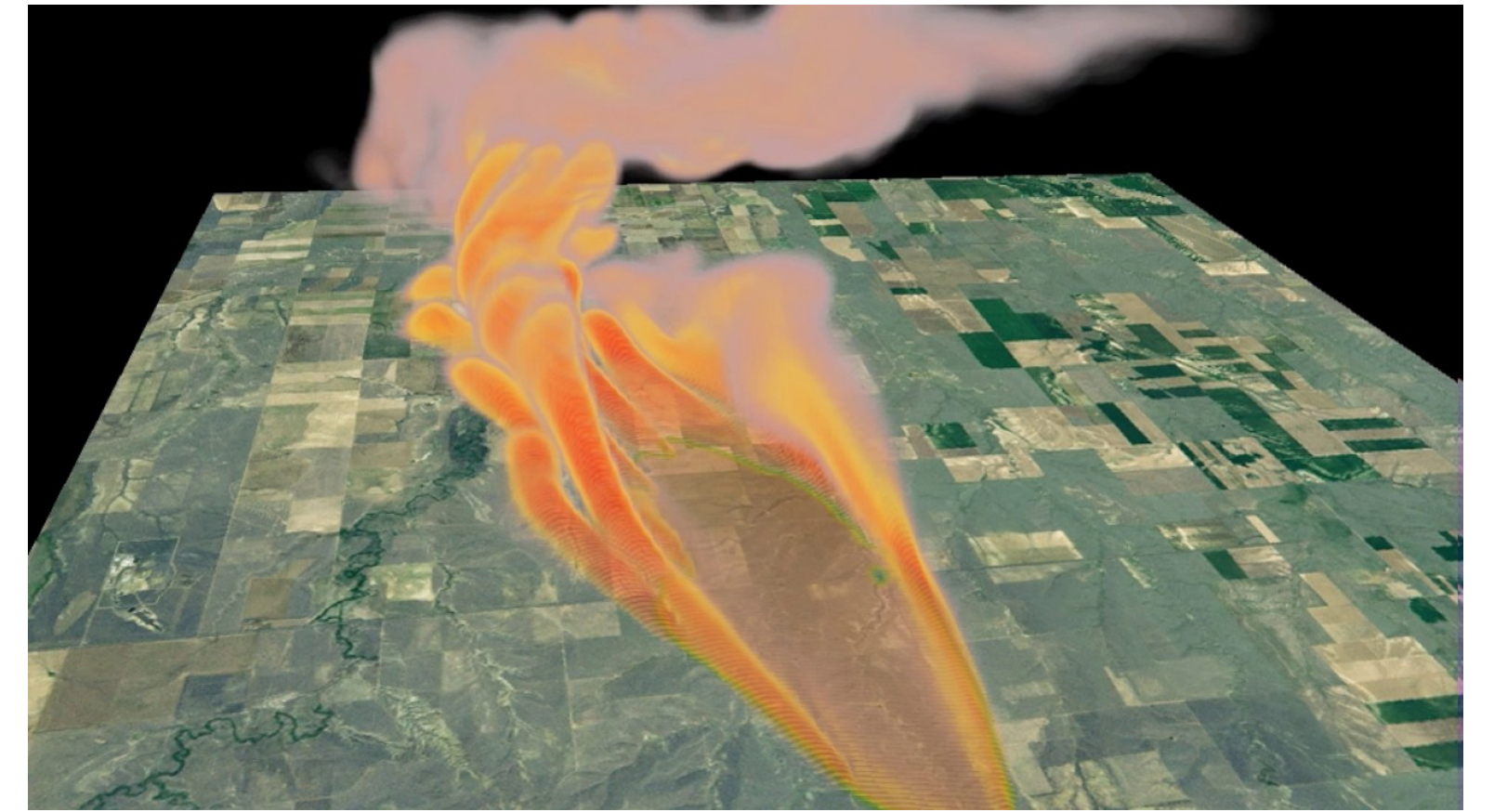
But WHY are they hard? 🤔

- Euler/Navier-Stokes equations come from conservation of mass, momentum, and energy.
- Turbulence activates small scales. Discontinuities form in finite time from smooth initial data.
- Avoiding smearing discontinuities or introducing spurious oscillation (Gibbs) is very difficult numerically.

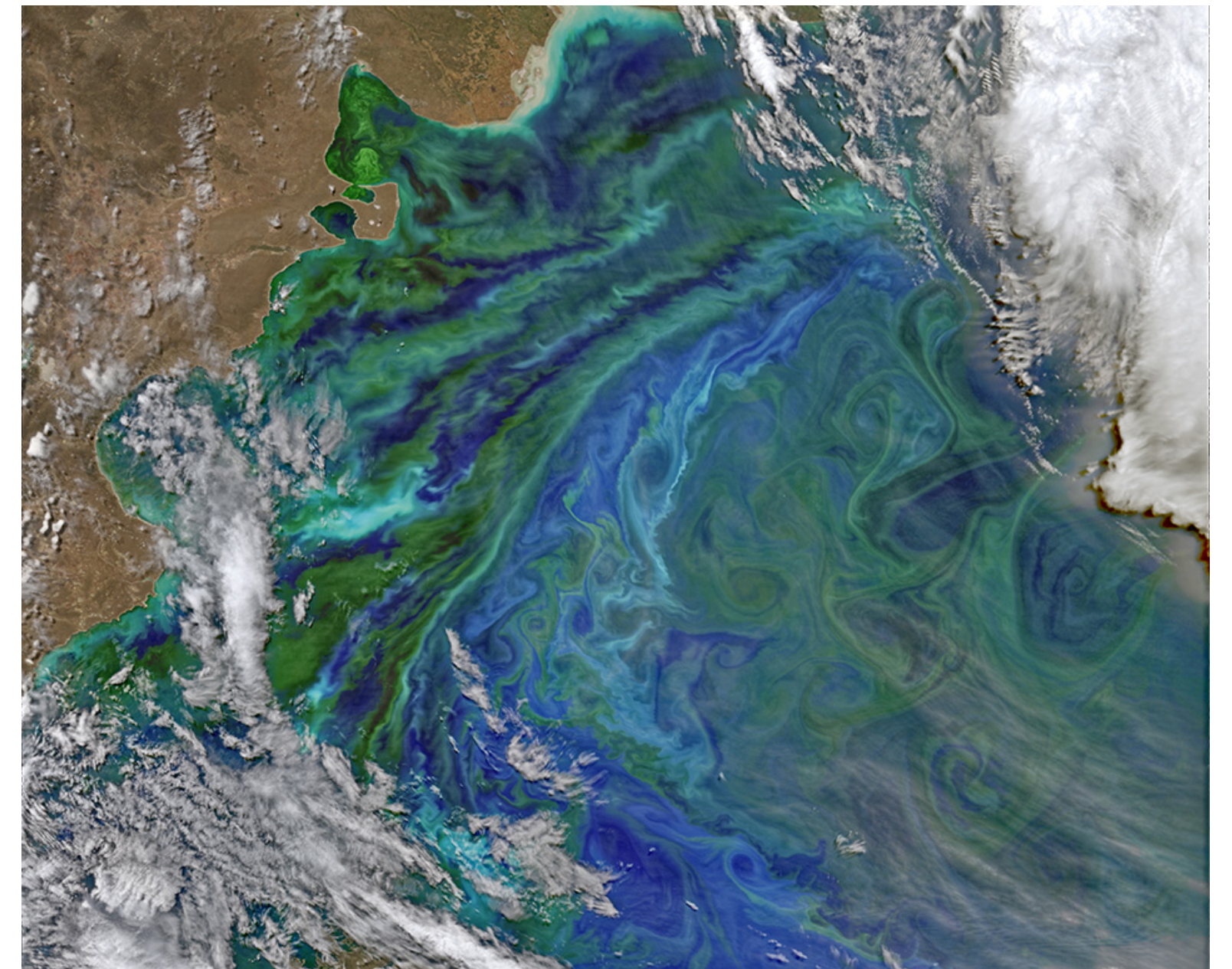




Smoke plume from the eruption of Mt. Saint Helens (USGS)



Multiscale wildfire simulation (NCAR)

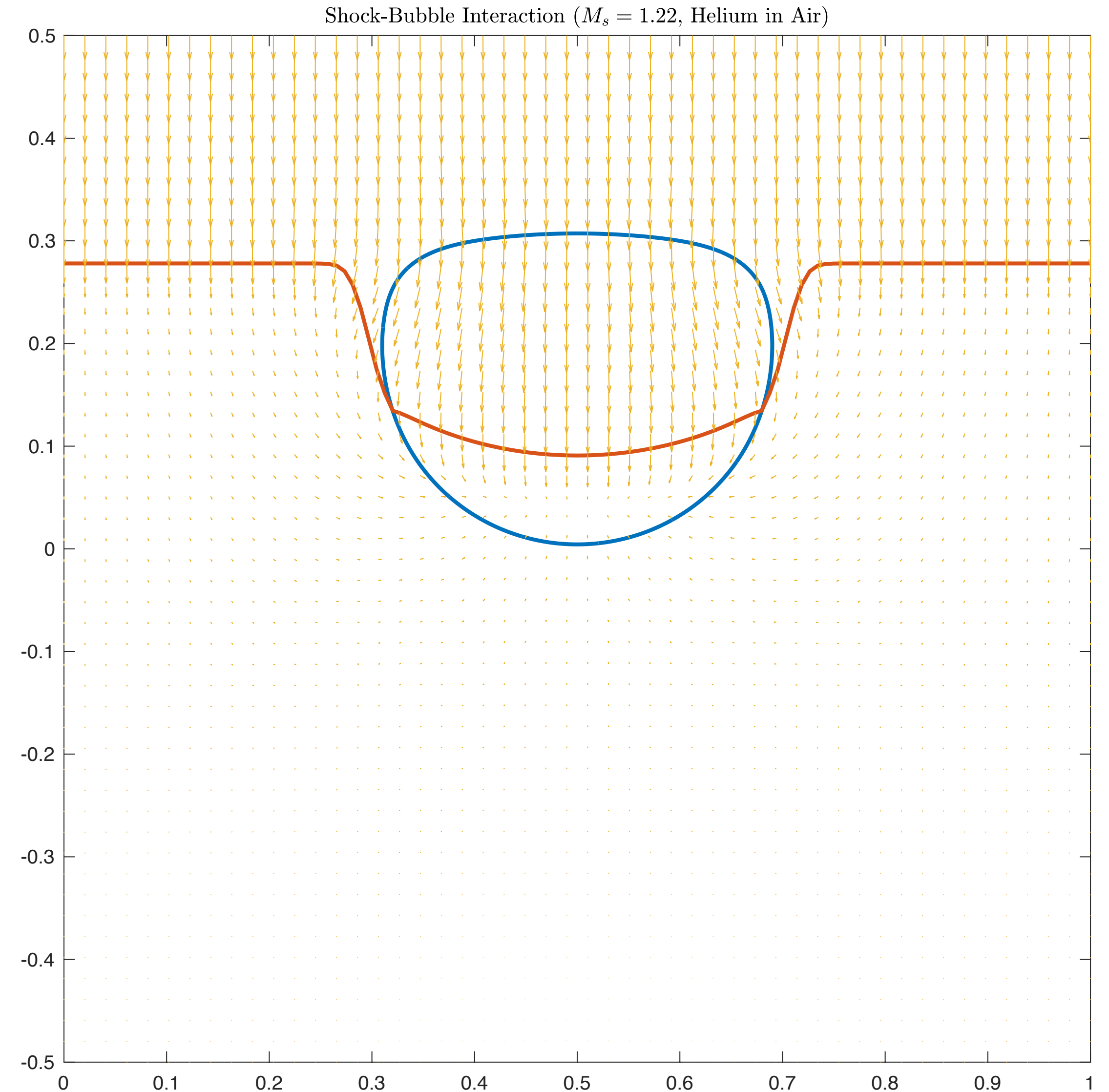


Phytoplankton, as seen from space (NASA)

Our Contribution

Shock/Contact Models

- Interface model for contact discontinuities (Granero & Shkoller '17 in 2d, P. and Shkoller '22 in 3d)
- Coupling contact model to compressible flow field (Ramani and Shkoller '20, '22)
- Interface model for shock fronts and contact discontinuities, with full velocity reconstruction.



Terminology

- **Surface of discontinuity:** curve (in 2d) or surface (in 3d) along which one or more of the fluid variables (density, velocity, pressure) are discontinuous.
- **Shock:** surface of discontinuity where the **normal velocity** is discontinuous.
- **Contact:** surface of discontinuity where the **tangential velocity** is discontinuous.
- **Incompressible flow:** flow where the **divergence** of the velocity vanishes.
- **Irrotational flow:** flow where the **curl** of the velocity vanishes.
- **Interface model:** model where the dependent variables are defined only on a curve (in 2d) or a surface (in 3d).

Flow Reconstruction

- An arbitrary velocity field u can be reconstructed from its **vorticity** $\omega = \text{curl } u$ and **compression** $\chi = \text{div } u$:

$$u(x) = \frac{1}{4\pi} \iiint_{\mathbb{R}^3} \frac{\omega(x') \times (x - x')}{|x - x'|^3} + \frac{\chi(x')(x - x')}{|x - x'|} dx'$$

- This continues to work when the vorticity and compression are distributions supported on a surface of discontinuity, $\omega = \bar{\omega}\delta_\Gamma, \chi = \bar{\chi}\delta_\Gamma$:

$$u(x) = \frac{1}{4\pi} \iint_{\Gamma} \frac{\bar{\omega}(x') \times (x - x')}{|x - x'|^3} + \frac{\bar{\chi}(x')(x - x')}{|x - x'|^3} dS(x')$$

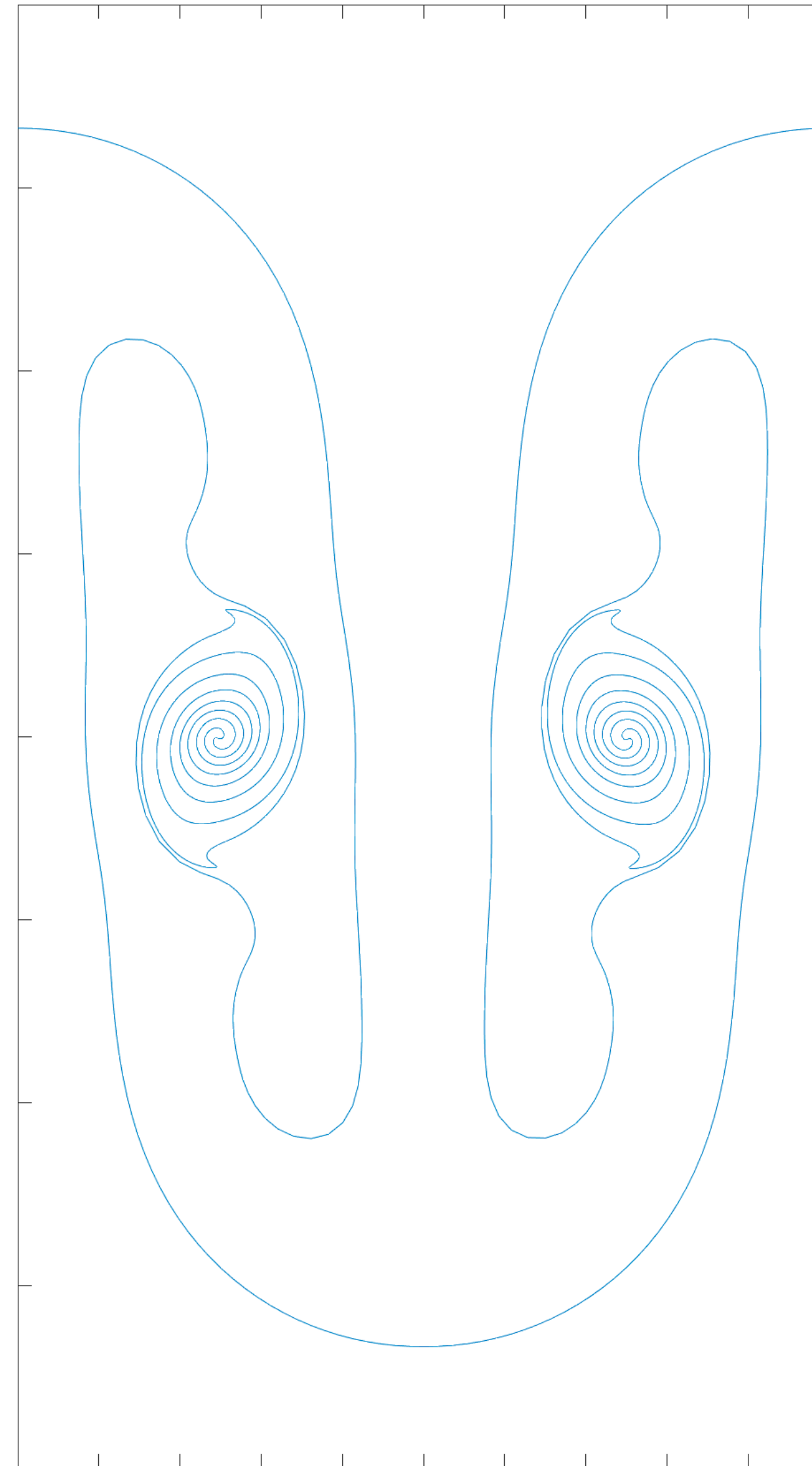
Contacts

The z-model

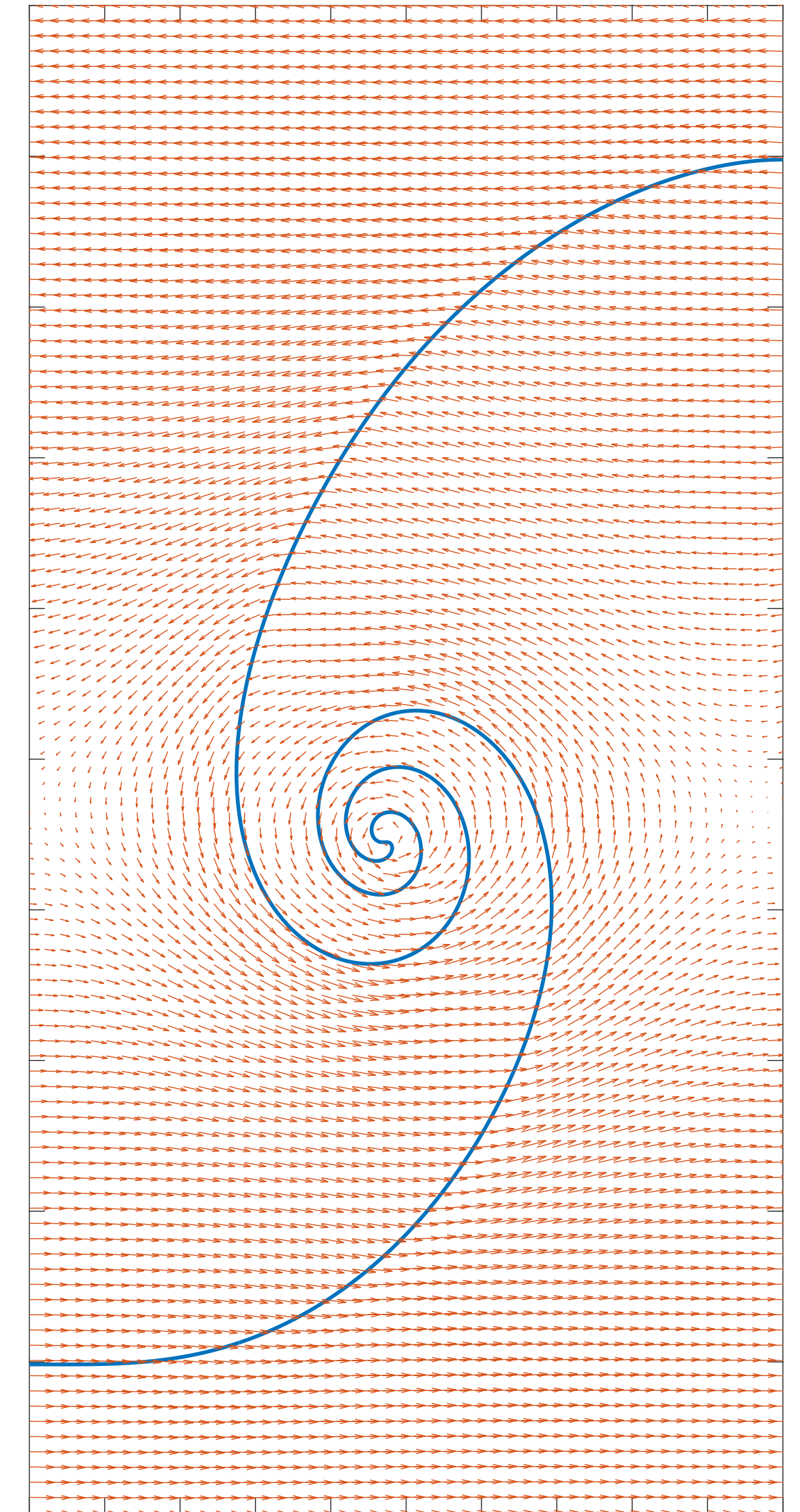
- Interface model for the interface position and amplitude of vorticity.
- Models the deposition of vorticity on density discontinuities:

$$\frac{D\omega}{Dt} = \frac{\nabla \rho \times \nabla p}{\rho^2}.$$

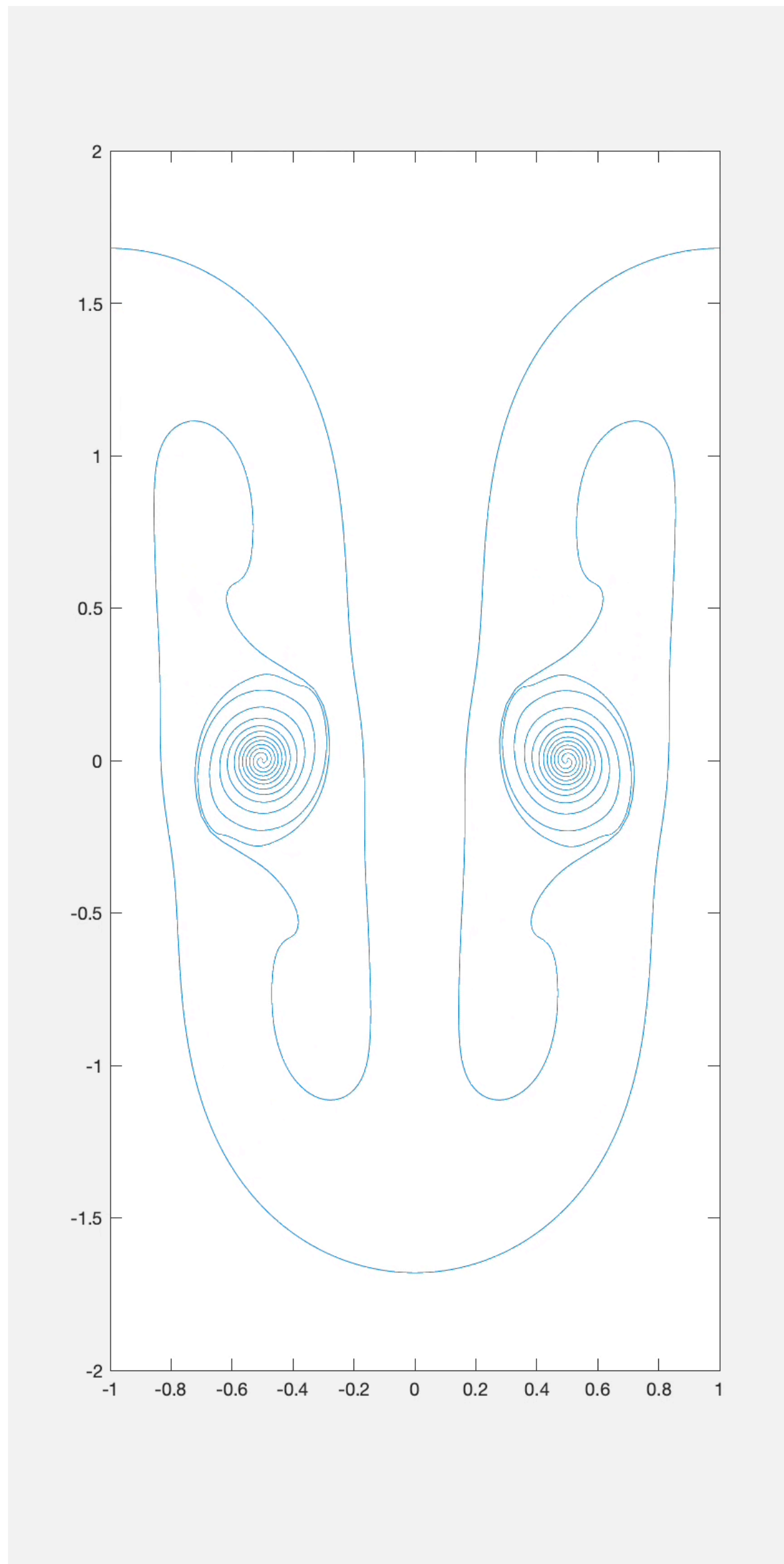
- In everyday terms, this means the interface between oil/water, hot air/cold air, etc.



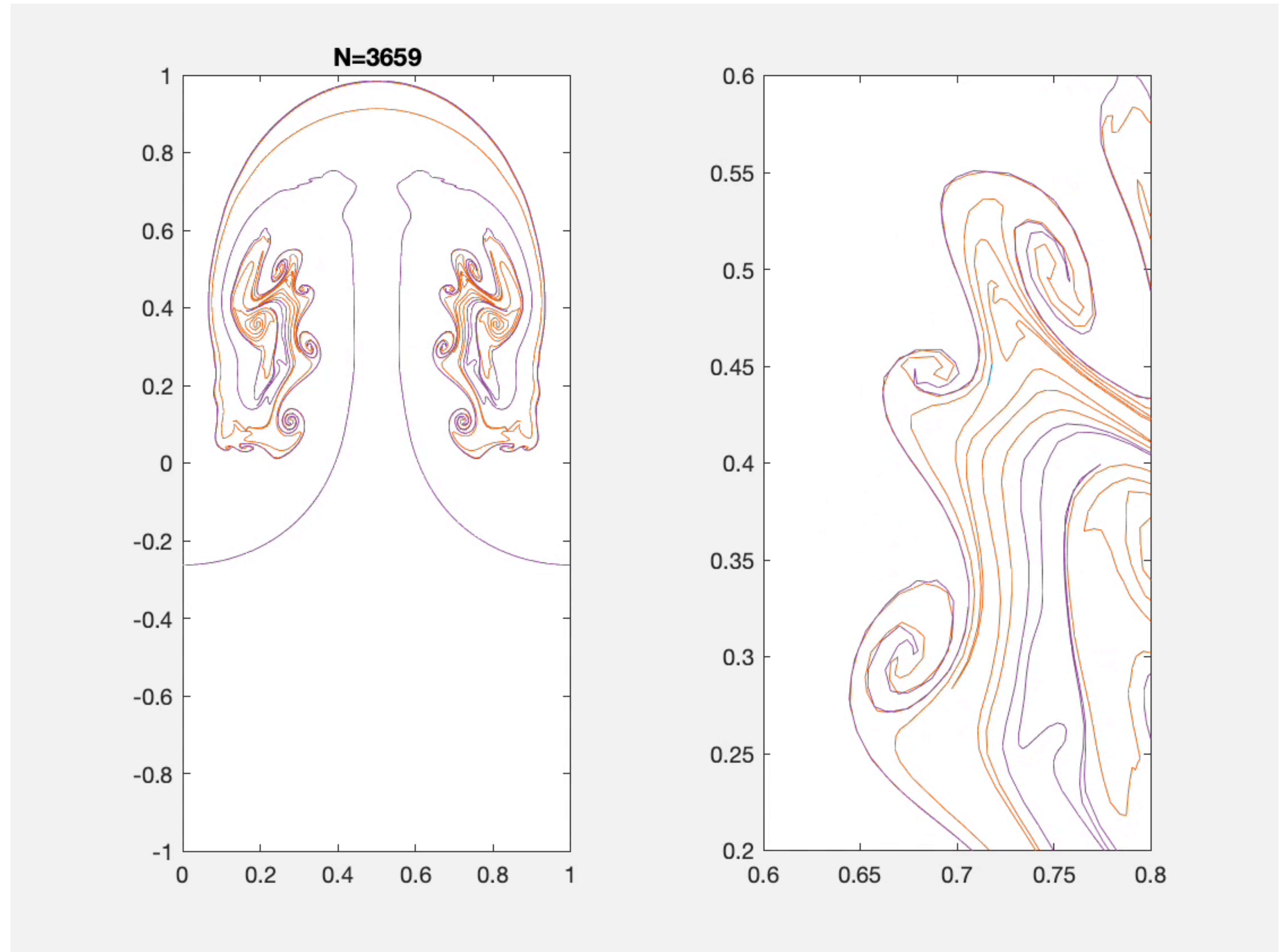
z-model interface



with reconstructed velocity



Single-mode Rayleigh-Taylor instability



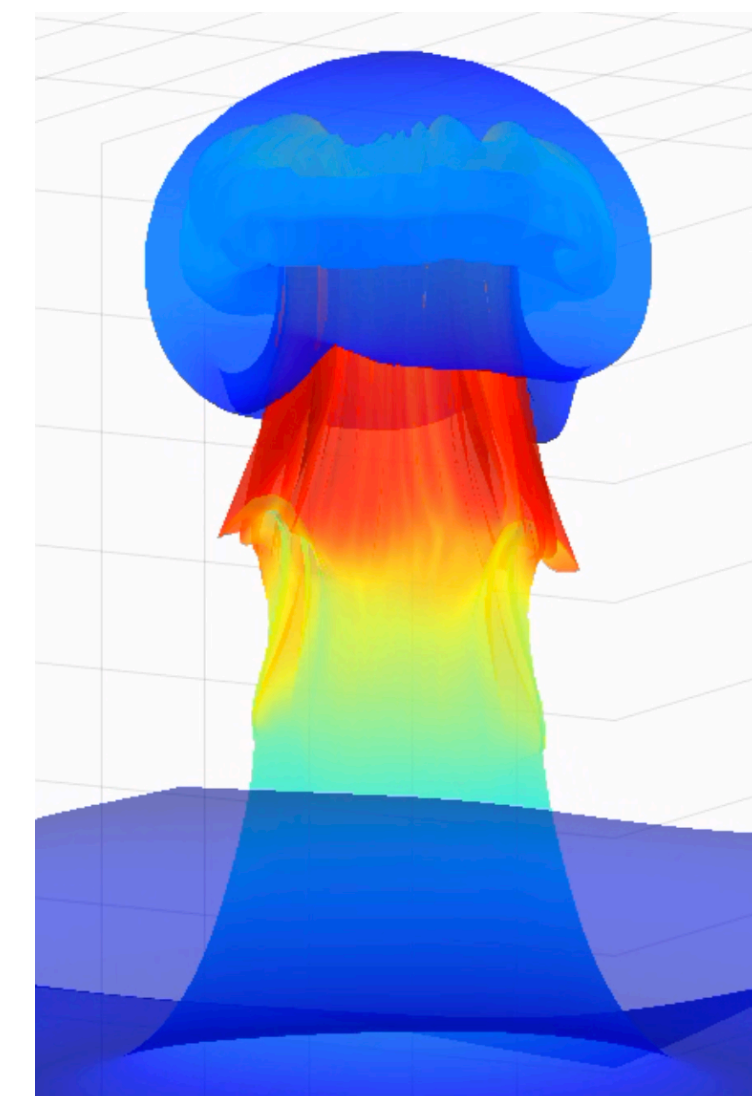
Rising bubble interacting with unstable density interface

Diffuse
Interface



Sharp
Interface

Rayleigh-Taylor instability in clouds vs. z-model



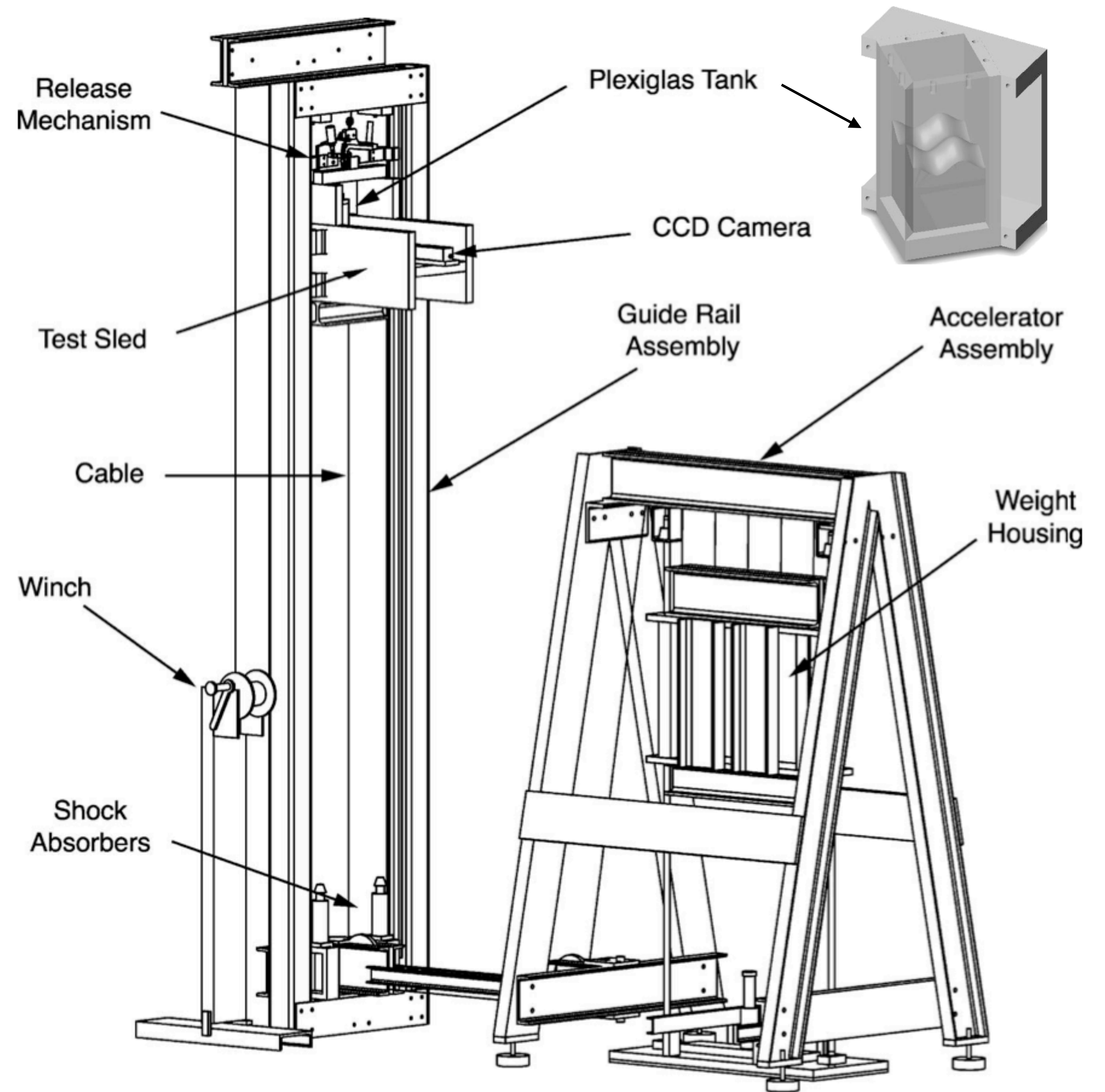
Rayleigh-Taylor Instability

- 1) water droplet impact
- 2) plume of superheated gas from atomic bomb
- 3) z-model simulation

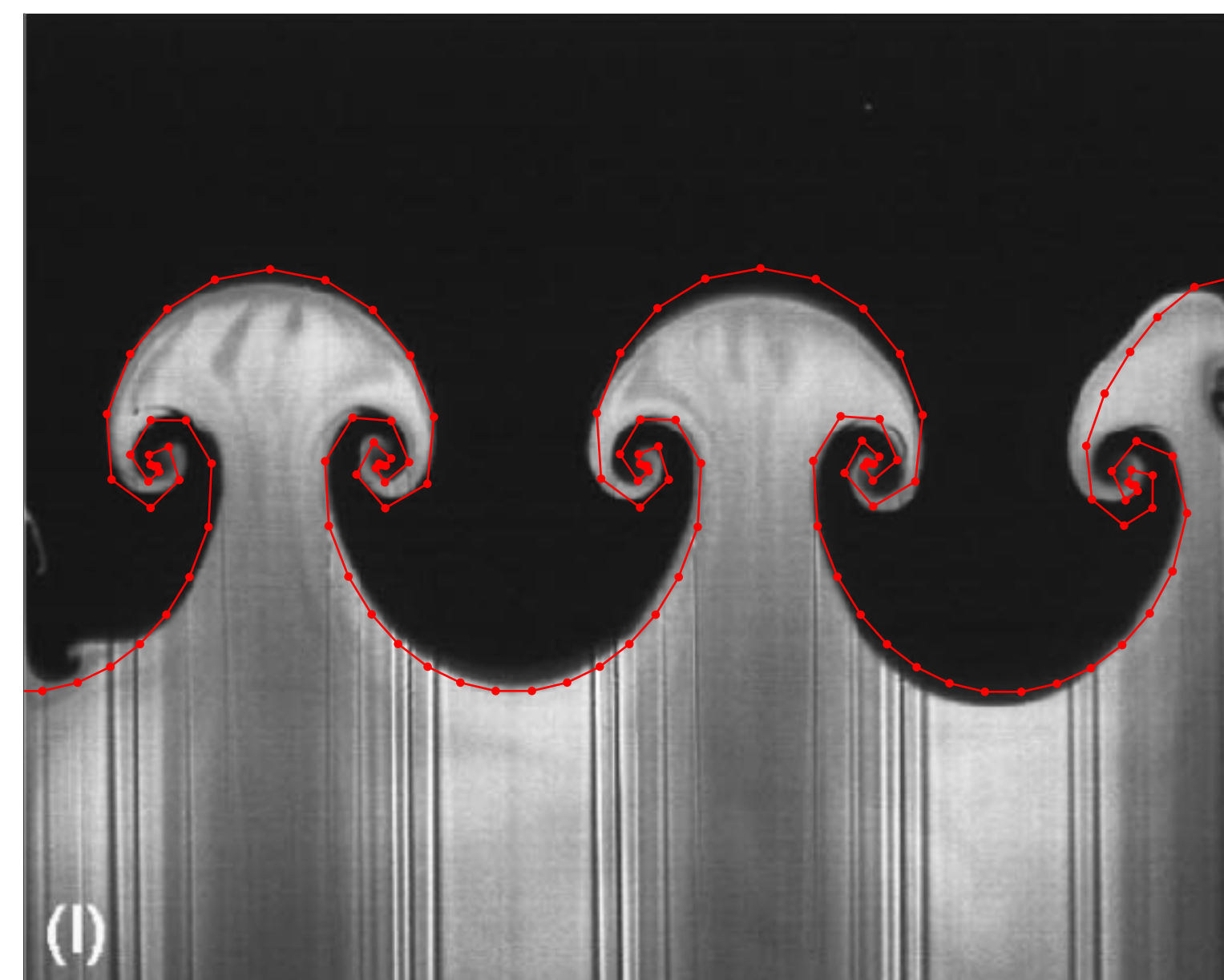
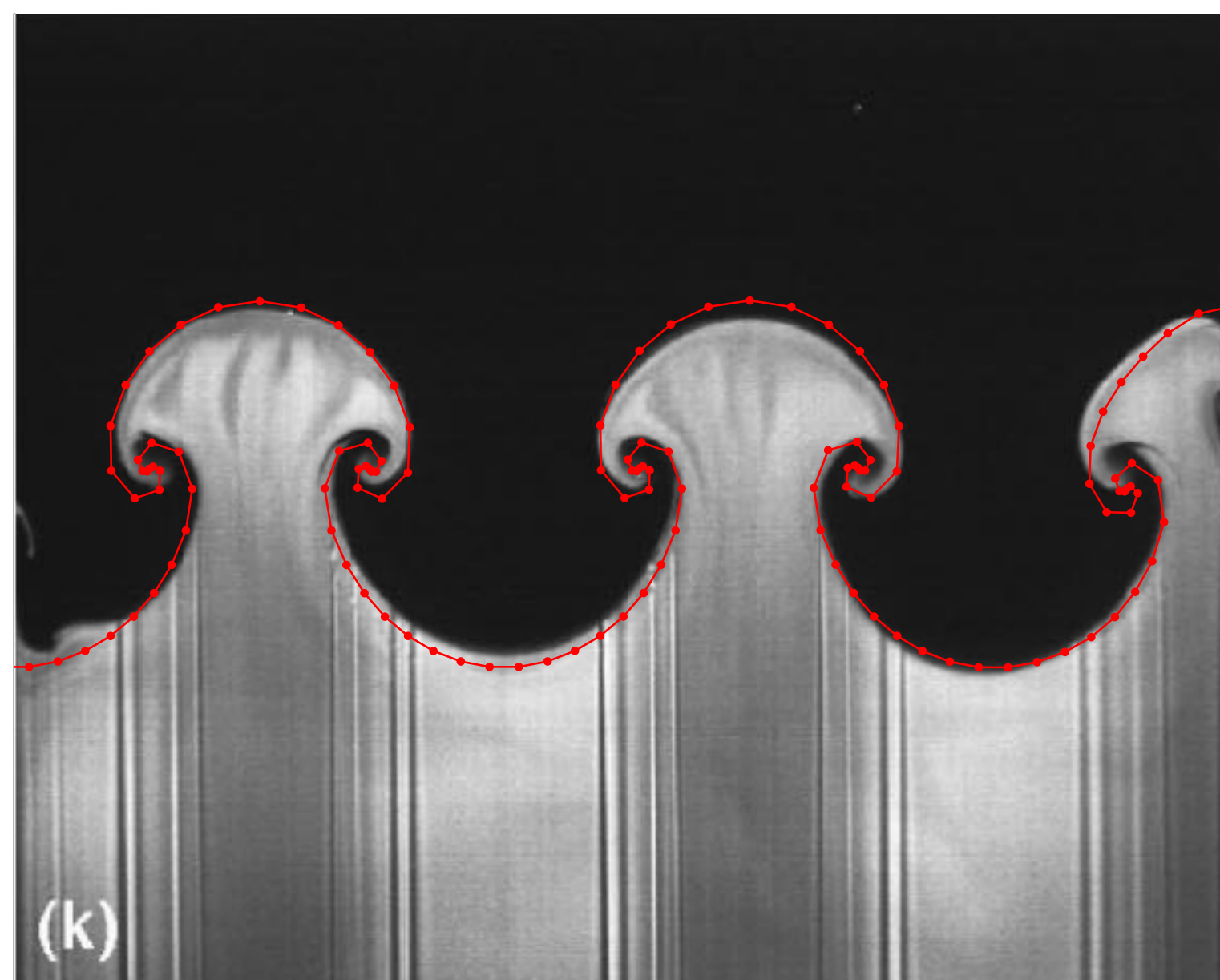
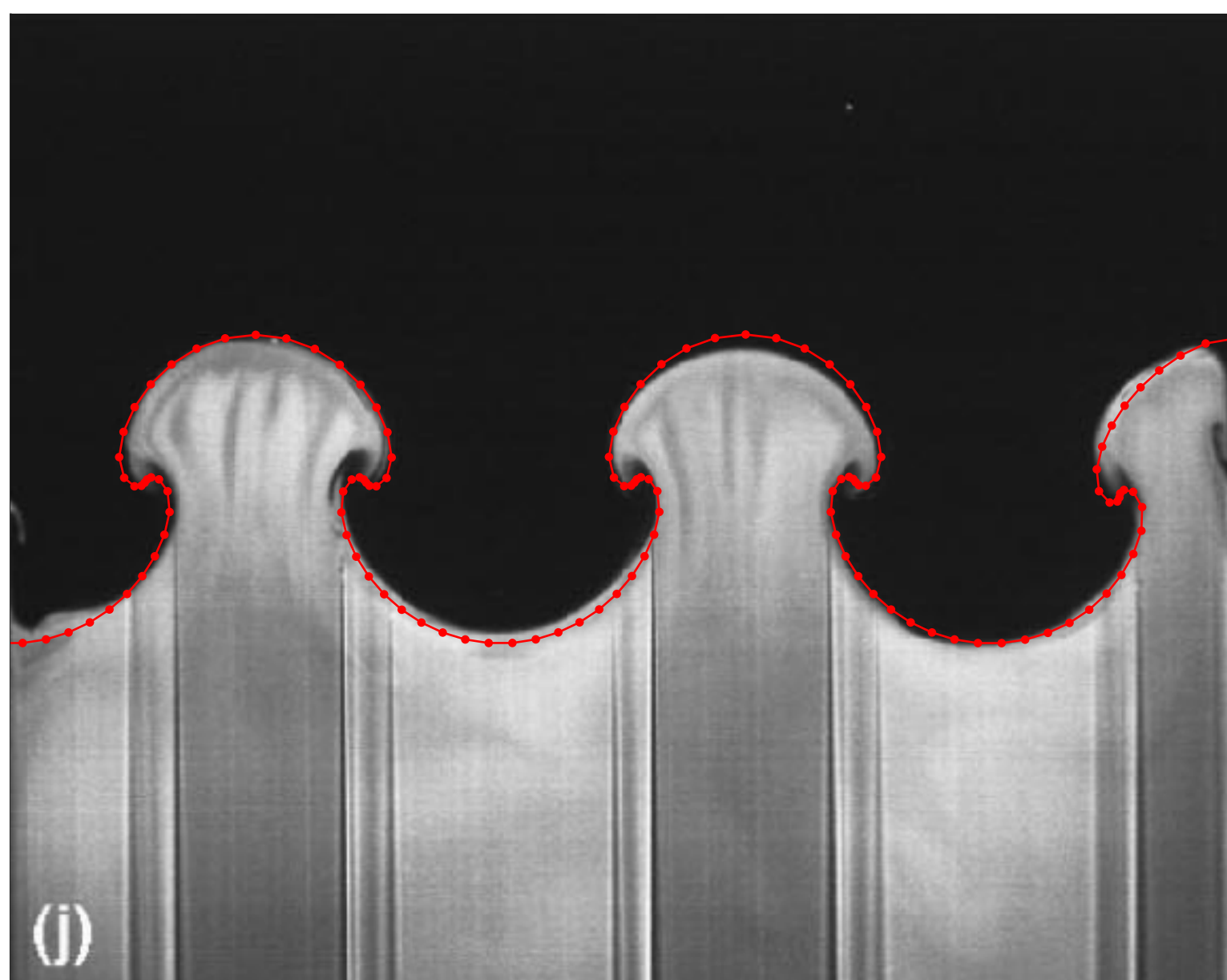
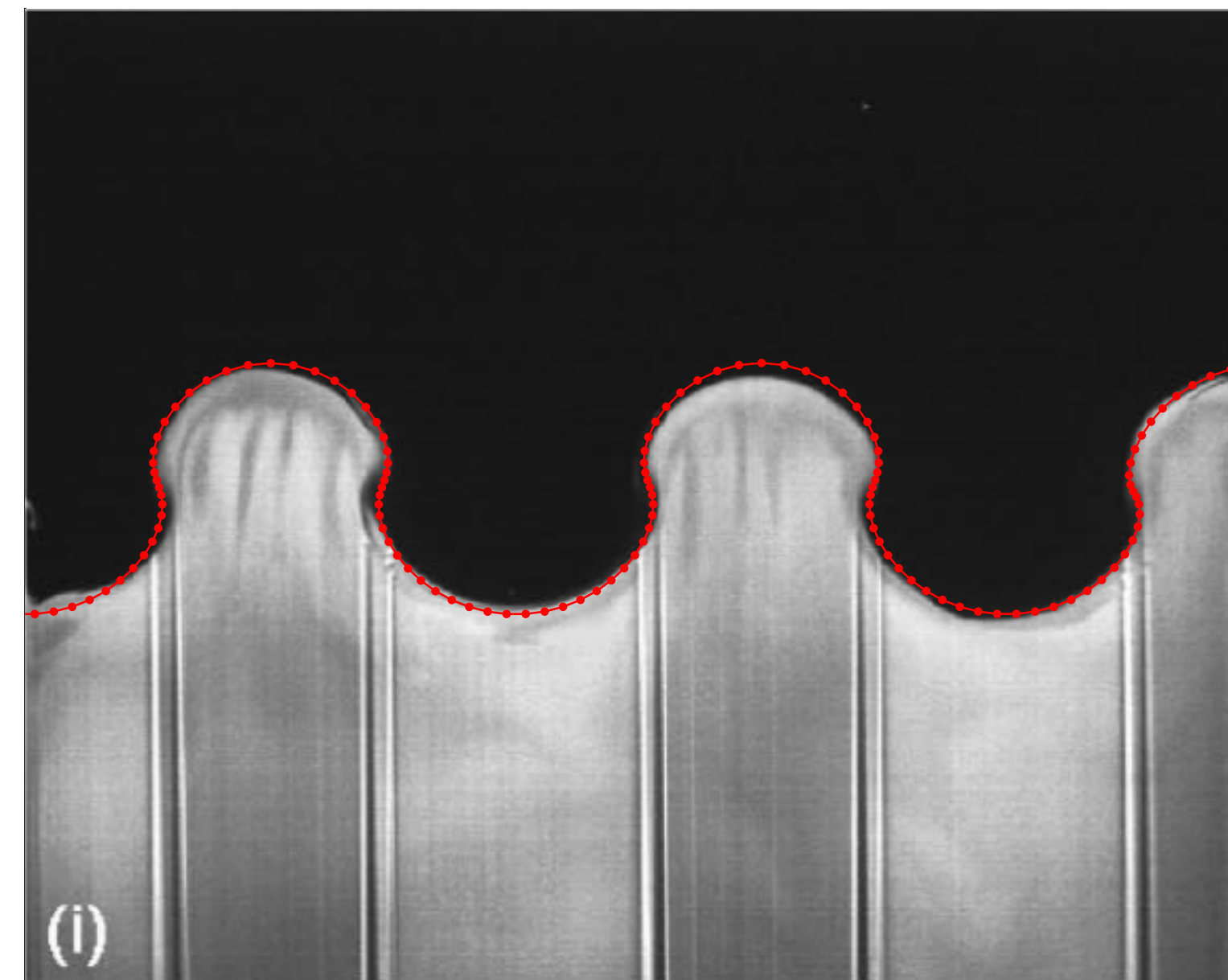
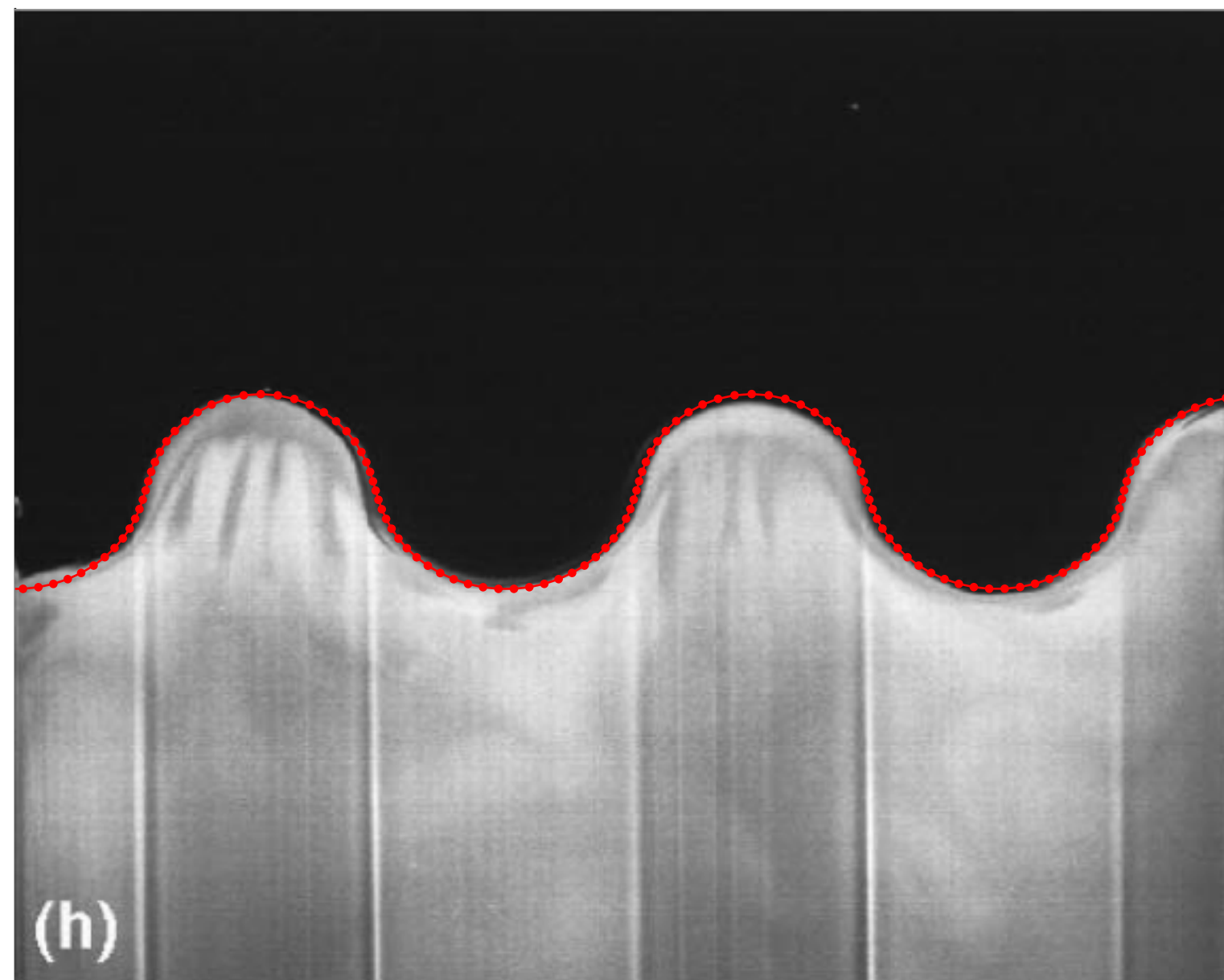
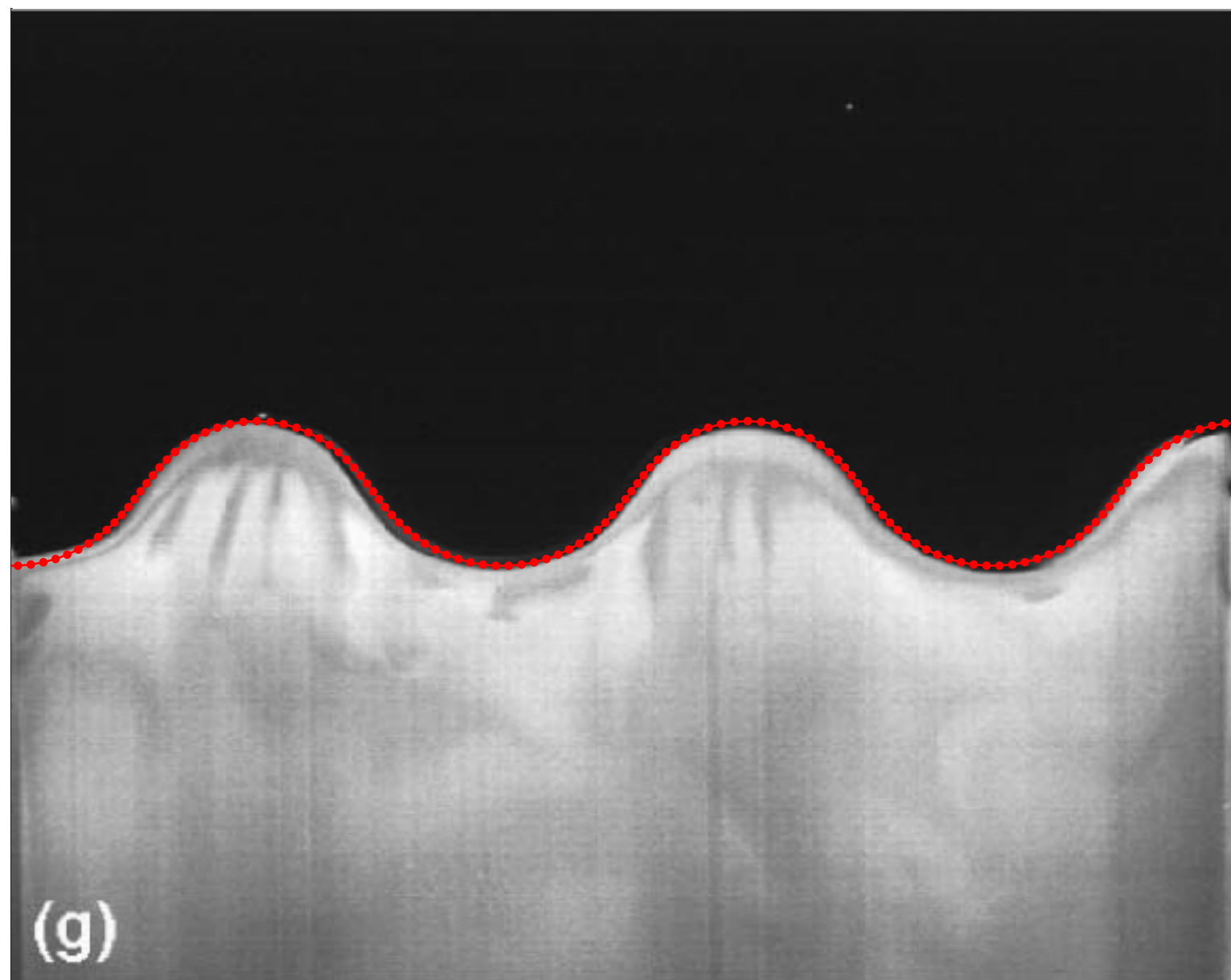
Comparison With Experiments

Single-Mode RT Instability

- A standing wave is excited in a tank containing two fluids, the heavier fluid on the bottom.
- The tank is then accelerated downward, so that the acceleration points from the heavier fluid into the lighter fluid.
- The resulting Rayleigh-Taylor instability is photographed along the tank's diagonal using PLIF (planar laser induced fluorescence).



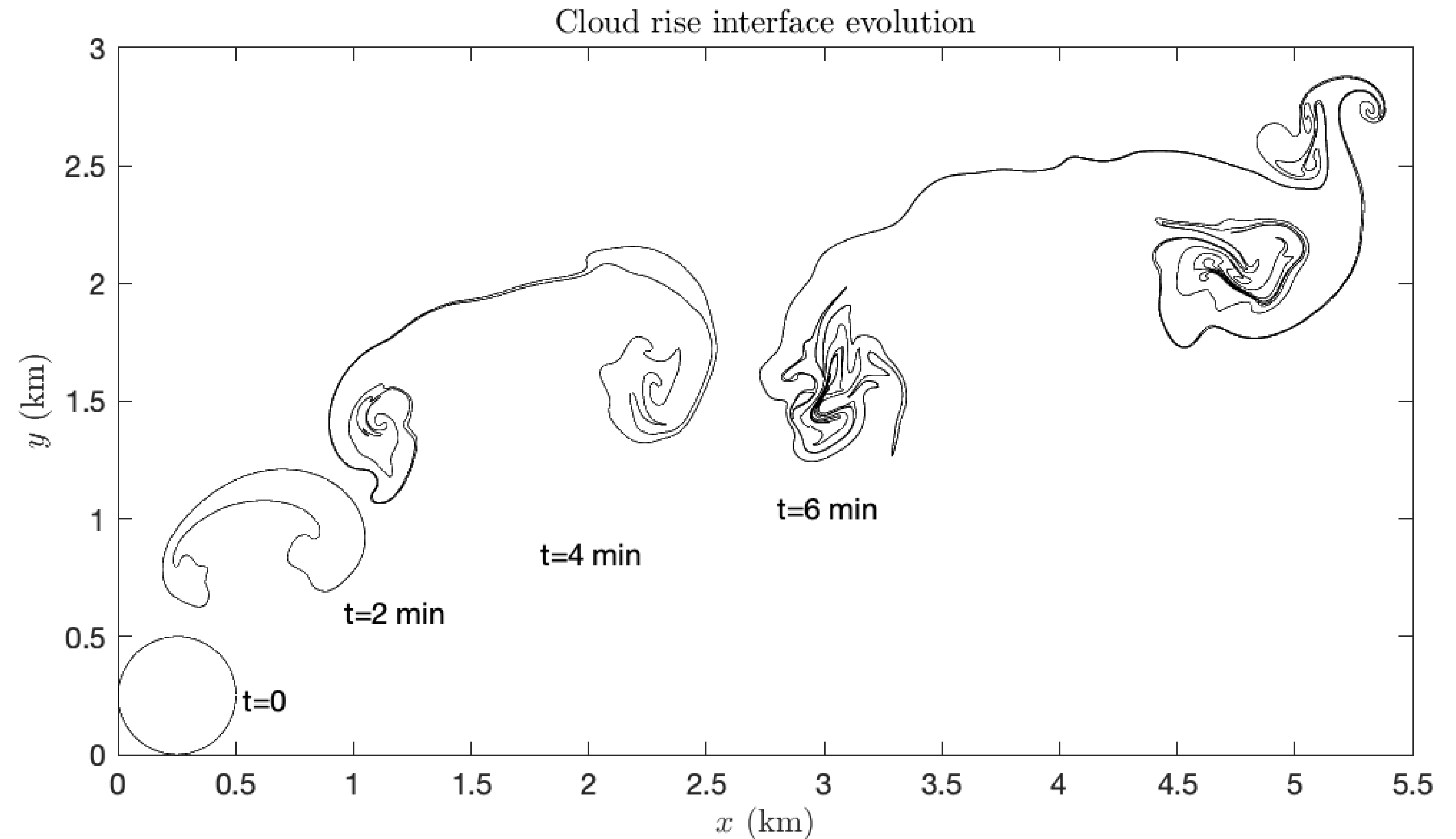
Experimental setup of Waddell/Jacobs and Wilkinson/Jacobs



Cloud Rise

z-model + shear flow

- A sphere of warm air (density ρ_0) rises into air of density $\rho(y)$, $d\rho/dy < 0$.
- Wind blows horizontally with speed $U(y)$, $dU/dy > 0$.
- Cool rollup happens!



Comparison With Experiments

Rocket Rig Experiment

- Theory predicts that the RT mixing layer grows quadratically with time:

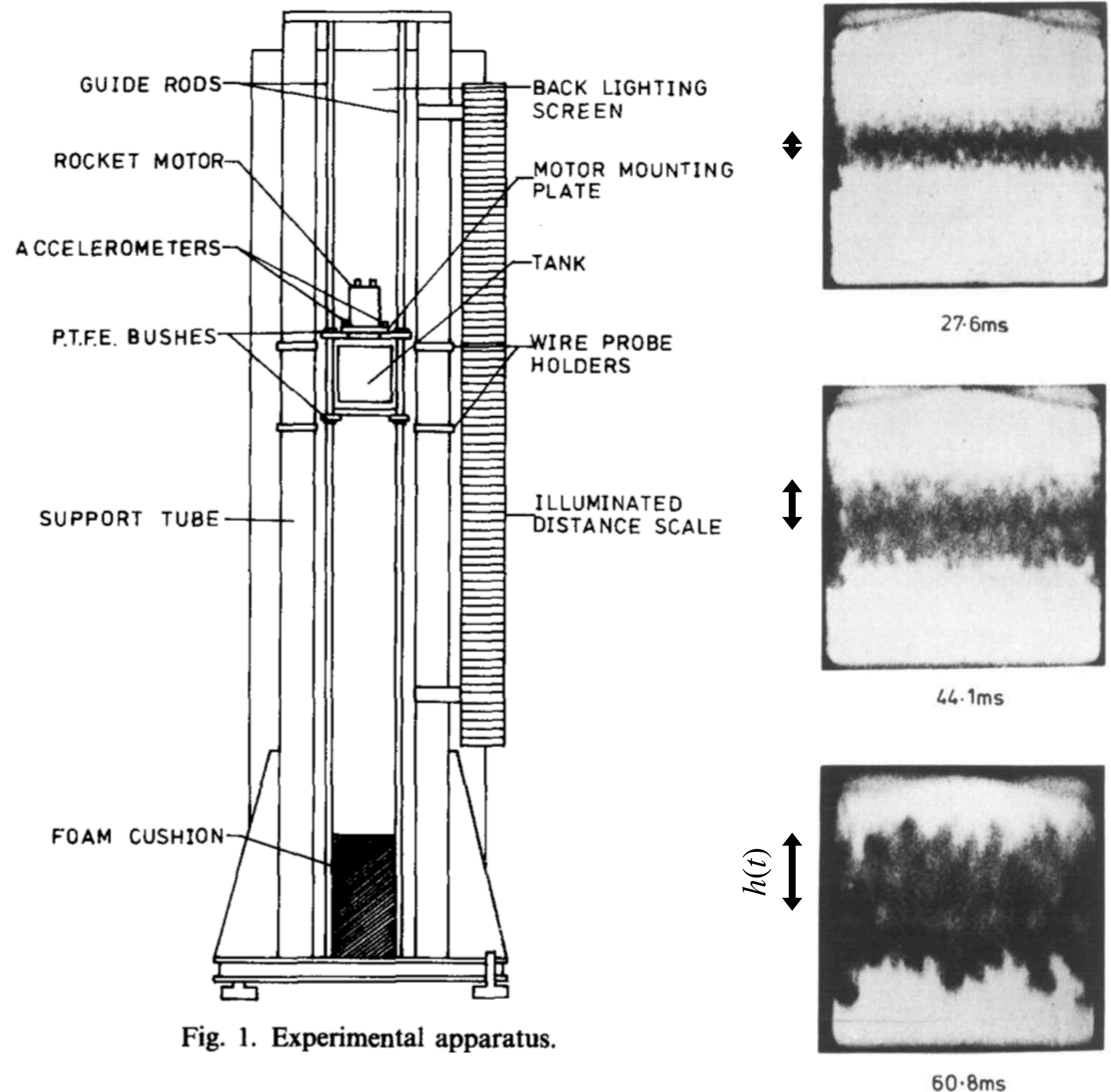
$$h - h_0 = \alpha A g t^2$$

- Experiments of Read and Youngs found constant of proportionality

$$\alpha \approx 0.06 \text{ or } 0.07$$

- Model of Cabot/Cook & Ristorcelli/
Clark measures α dynamically:

$$\alpha = \dot{h}^2 / (4Agh)$$



Comparison With Experiments

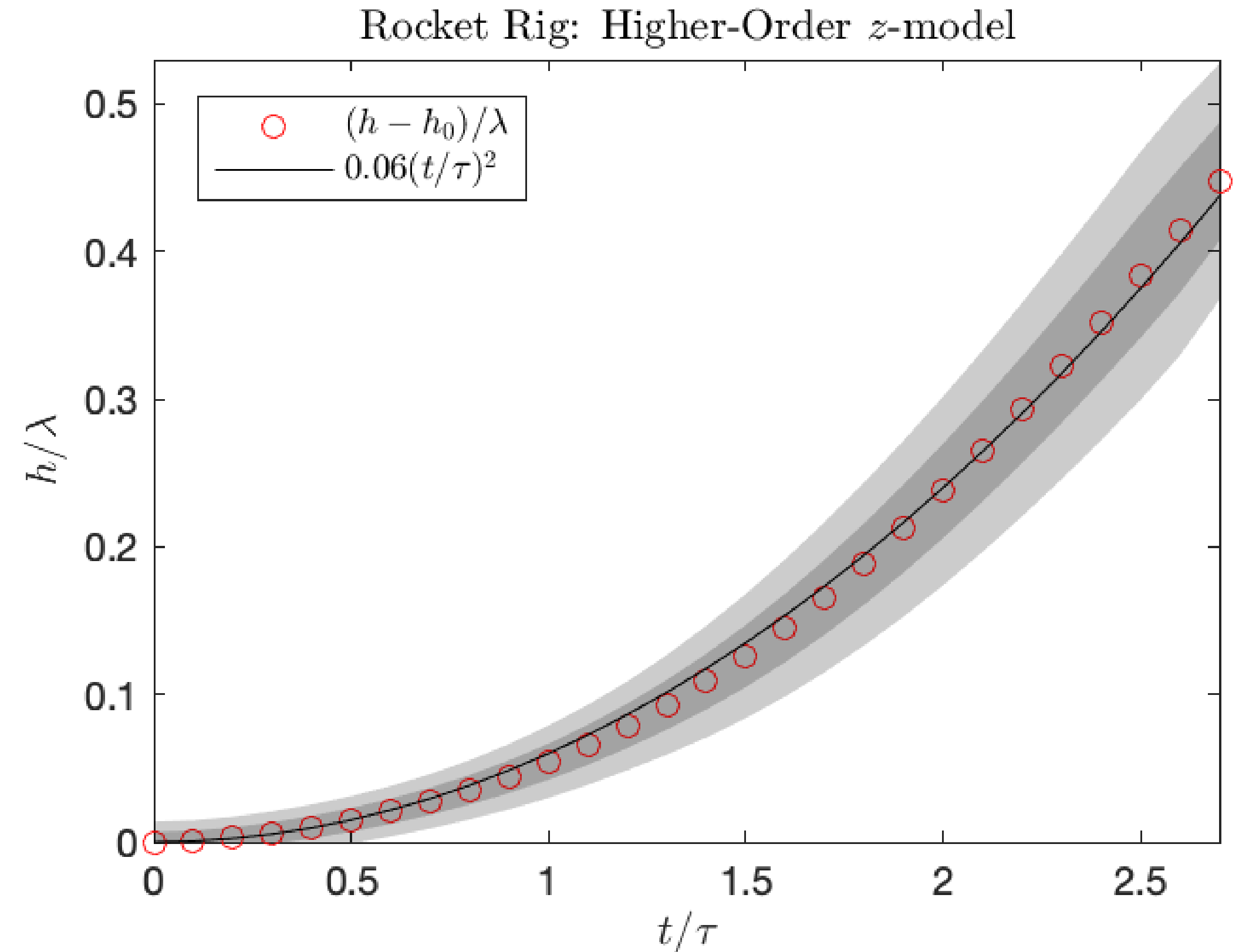
Rocket Rig

We begin with randomized initial data

$$z_3 = \text{Re} \sum_{k_1, k_2 = -N}^N c_{k_1, k_2} e^{2\pi i(k_1 s_1 + k_2 s_2)}$$

where the coefficients c_{k_1, k_2} are sampled from a normal distribution.

We averaged the result of 100 runs, using parameters that match Read's NaI/Pentane experiment.

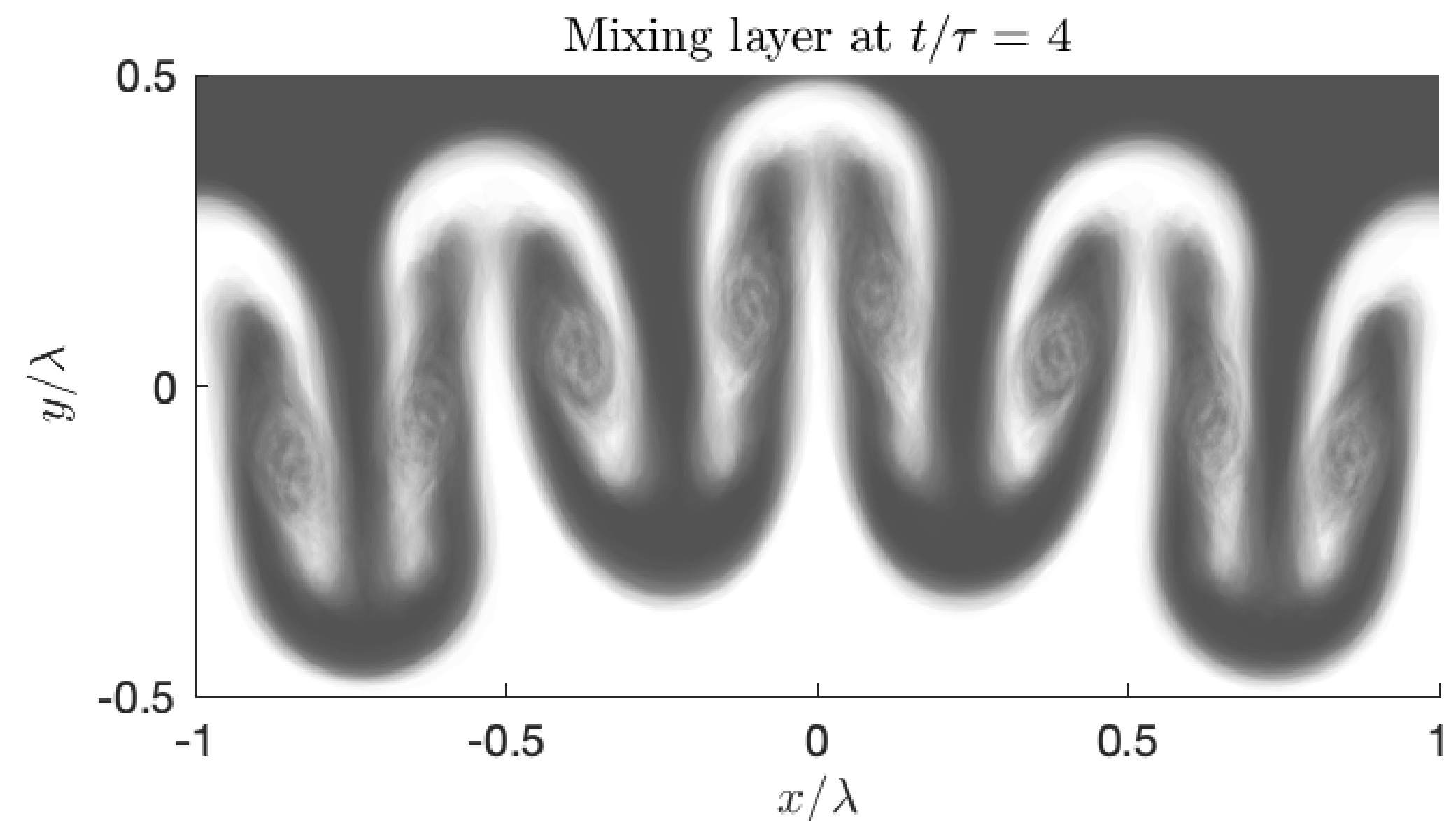
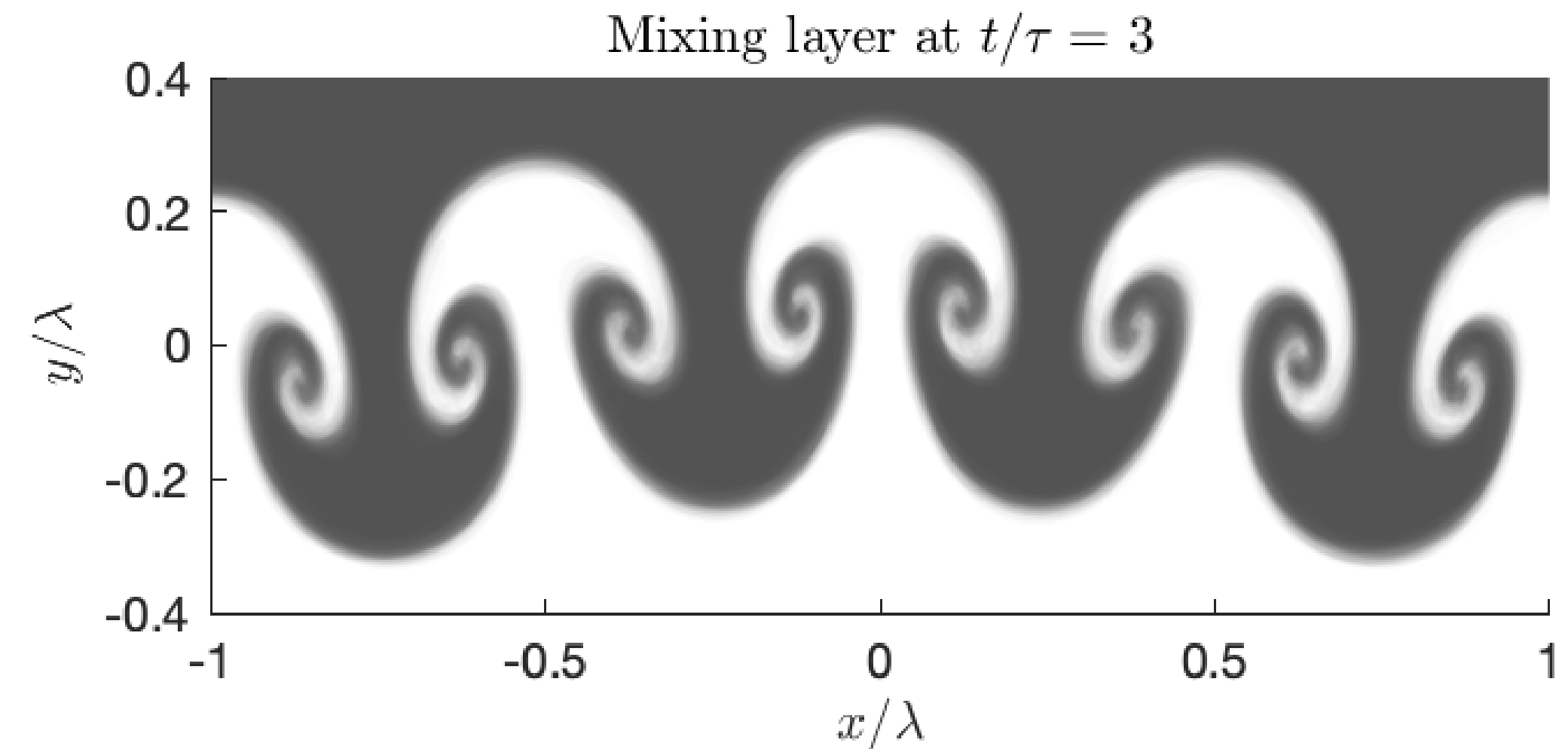


Width of mixing layer, averaged over 100 runs. The mean is shown in red, and the grey bands show one and two standard deviations. Here λ is a characteristic length scale and $\tau = (\lambda/Ag)^{1/2}$ is the characteristic time scale. The simulations and the experiment were both run for 60ms, or approximately 2.7τ .

Side Note

Ensemble Averaging

- Brownian motion $\leftrightarrow$ diffusion.
- **Conjecture:** ensemble average of inviscid flows with random initial data gives viscous flow.
- We have observed this qualitatively from our z-model runs. Can we make it quantitative?



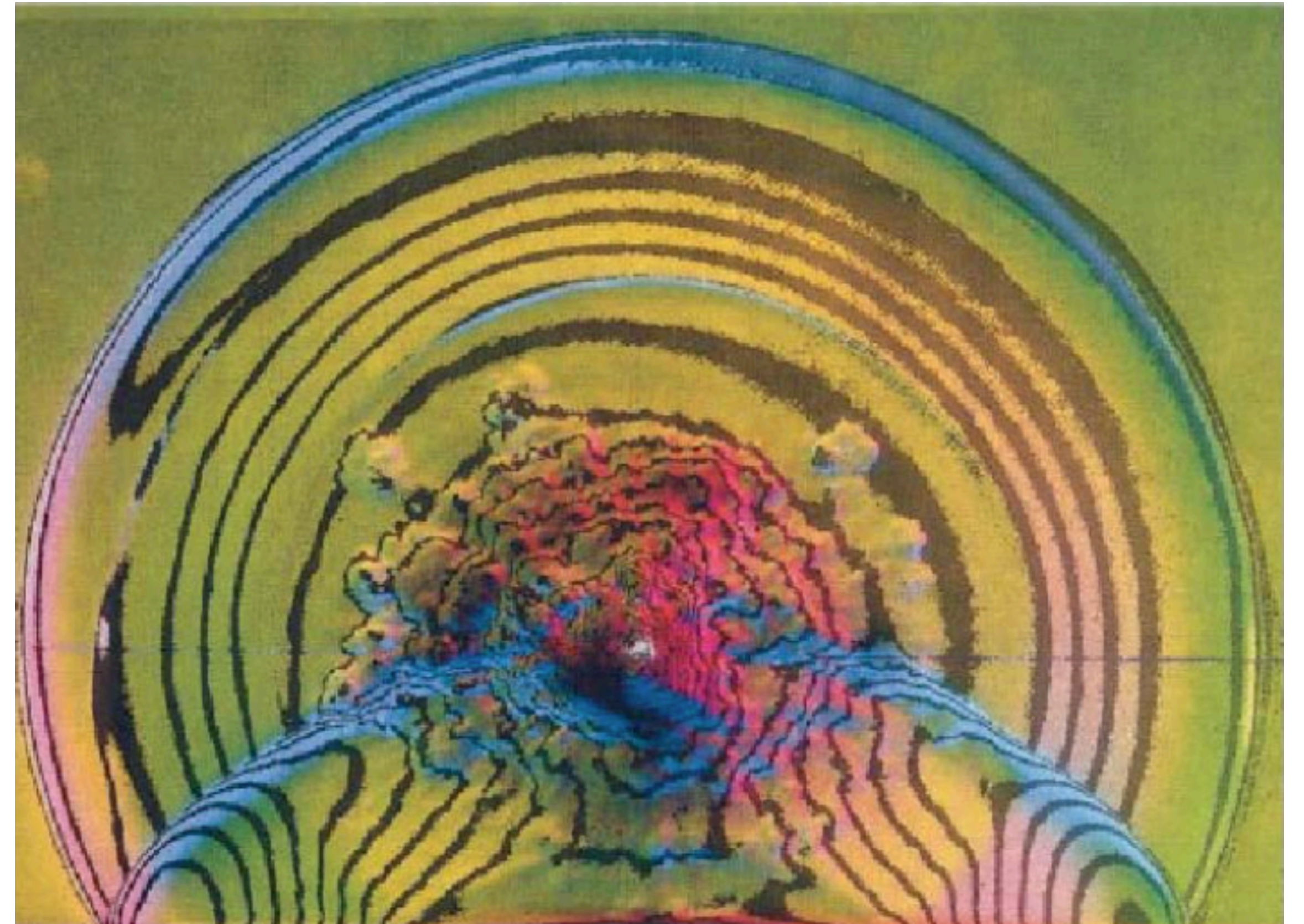
Shocks

Geometrical Shock Dynamics (Whitham, Schwendeman)

- Interface model for the shock position and shock Mach number

$$M = \frac{\text{shock speed}}{\text{sound speed}}.$$

- Models shock wave propagating into quiescent gas (e.g. blast wave from explosion, shock wave from supersonic airplane).



Schlieren/Interferometer image of blast wave (Kleine/Takayama)

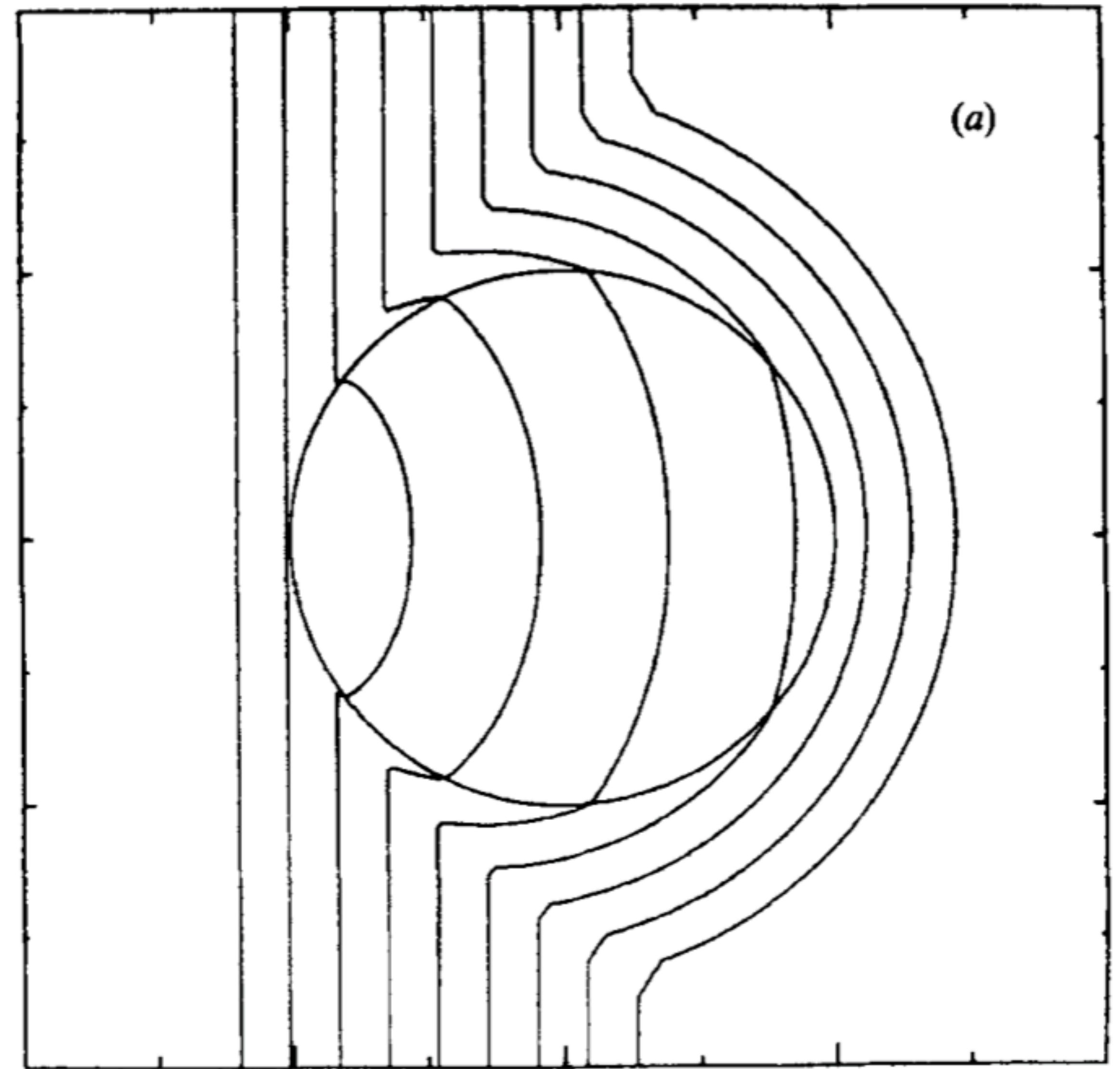
Shocks

Nonuniform Media

- When the sound speed has a jump, the Mach number jumps too:

$$\Delta M = \frac{M \cos \theta}{\beta(M, \gamma)} \frac{\Delta c}{c}.$$

- In particular, where the shock crosses a contact, we have a corner in the shock surface.



Shock propagating through sphere of lighter gas (Schwendeman)

Shocks and Contacts

Coupling the z-model and the kinematic shock model

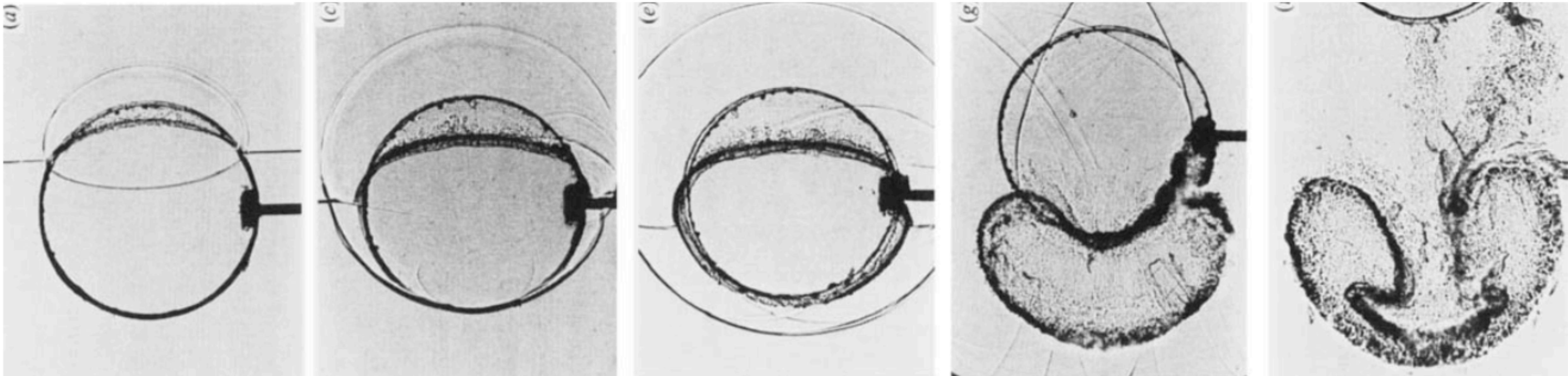
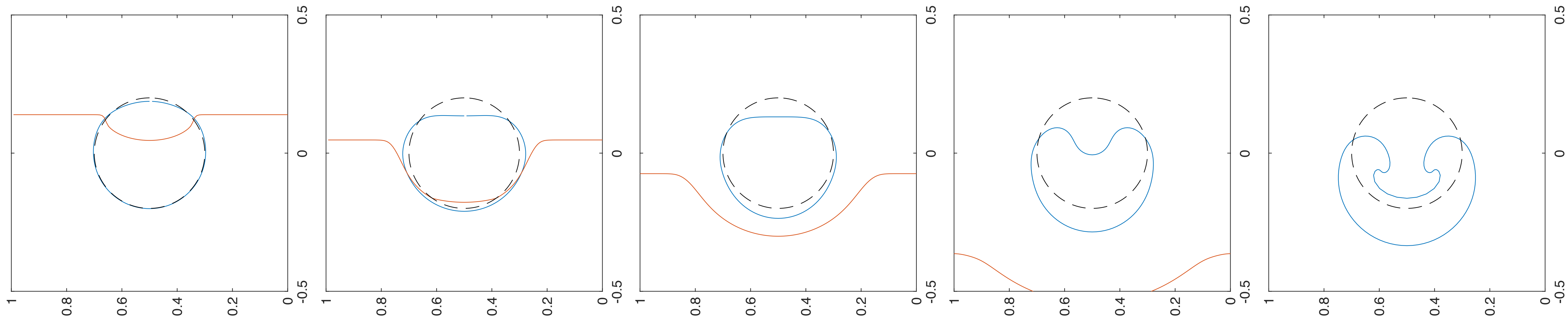
- The contact is affected by the shock via baroclinic vorticity production:

$$\Delta\omega = \left(\frac{1}{\rho^+} - \frac{1}{\rho^-} \right) \Delta p \sin \theta.$$

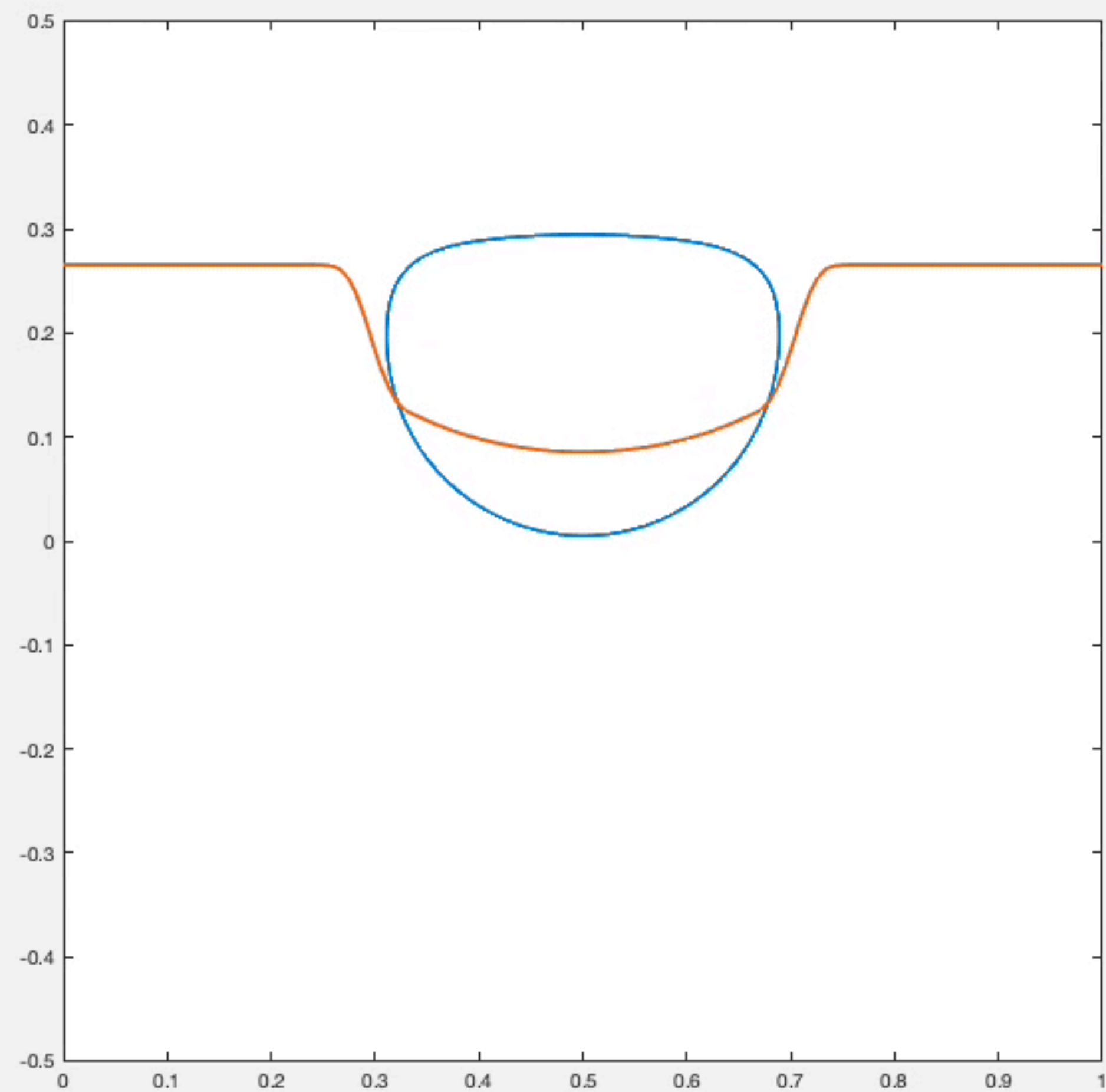
- The shock is affected by the contact via the change in sound speed:

$$\Delta M = \frac{M \cos \theta}{\beta(M, \gamma)} \frac{\Delta c}{c} \qquad c^2 = \frac{\gamma p}{\rho}.$$

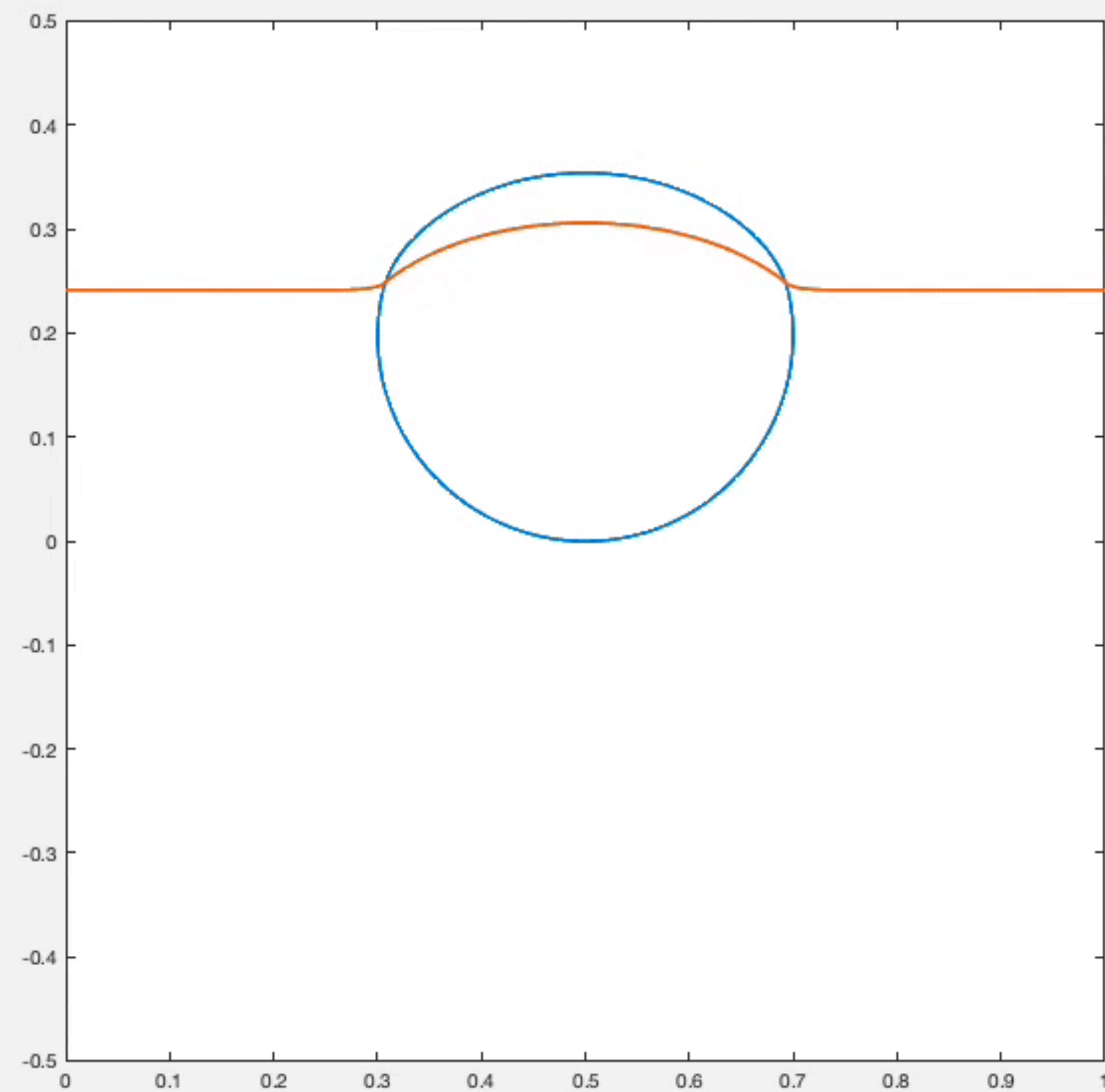
- The full velocity is reconstructed from the amplitudes of vorticity and compression, using our boundary integral formulation.



Mach 1.22 shock propagating through helium bubble in air;
our simulation vs. experiments of Haas/Sturtevant.



Air-Helium



Air-Freon

Future Directions

- Species transport: $\partial_t X + (u \cdot \nabla)X = 0$.
- Reaction/Combustion: $\partial_t G + (u \cdot \nabla)G = U |\nabla G|$.
- Applications to wildfire modeling, cloud formation, inertial confinement fusion, blast wave modeling, and more!

A black and white artistic photograph. In the background, a person's face is shown in profile, looking towards the left. The face is softly lit, with the eyes closed or looking down. In the foreground, a liquid, possibly water or a light-colored oil, is captured in a dynamic splash or ripple, creating a series of peaks and valleys. The liquid is positioned horizontally across the middle of the frame, partially obscuring the lower part of the face. The overall mood is serene yet dynamic, with a focus on texture and light.

Thank You!