Math 108, Fall 2013

HW1 solutions.

1.1 3(h)

P	Q	$\sim P \wedge \sim Q$
F	F	T
F	T	F
T	F	F
T	T	F

4(d) $\sim S$ is T, so the disjunction is T.

6(h) Not equivalent. If P is T, Q and R are F, then $(P \wedge Q) \vee R$ is F, while $P \vee (Q \wedge R)$ is T.

9(a) Neither. If both P and Q are T, then the value is T, while if P is T and Q is F, the value is F.

10(b) Tautology.

1.2 3(e) Q is False. 5(d) True, as $7 < 2$ is False.

6(b). True, as both sides are True.

7(a). This is equivalent to $P \Rightarrow Q$, so it is F iff P is T and Q is F.

12(f). If both P and Q are T, or both are F, both statements are T. If P is F and Q is T (or vice versa), both statements are F.

16(b). Tautology. If P is T, $P \wedge (P \vee Q)$ is T regardless of Q, and if P is F, $P \wedge (P \vee Q)$ is F regardless of Q.

1.3 6(c) In T (as $(x > 8)$ is always True), in U (as (x odd) is always False), in V (as $(x>8)$ is always True), but not in W (take $x = 3$).

8(c) False (the only solution $x=-1$ is not in $\mathbb{N}$)

10(c) False. The negation $(\forall x)(\forall y)(x^2+y^2 \neq -1)$ is True as $x^2+y^2 \geq 0$.

10(e) True. Take $x=0$. (So even $(\exists x)(\forall y)(\forall z)(xy=xy)$ is True!)

$\boxed{1.4}$

4 (b) By (iv), the weapon was candlestick. By (iii), Scarlett is guilty.

5 (f) Proof. There exist integers k, l, so that $x = 2k+1$, $y = 2l+1$. Then

$$3x - 5y = 3(2k+1) - 5(2l+1) = 6k - 10l - 2$$
$$= 2(3k - 5l - 1),$$

an even number. □

6(f). Proof. ~~Case 1:~~ $a \geq 0$. Then $|a| \leq b$ iff $a \leq b$, which holds by assumption.
Case 2: $a < 0$. Then $|a| \leq b$ iff $-a \leq b$, which also holds by assumption. □

8a. Proof Case 1: n even. Then $n = 2k$ for some $k \in \mathbb{Z}$, and $n^2 + n + 3 = 4k^2 + 2k + 3 = 2(2k^2 + k + 1) + 1$, an odd number.
Case 2: n odd. Then $n = 2k+1$ for some $k \in \mathbb{Z}$, and $n^2 + n + 3 = 4k^2 + 4k + 1 + 2k + 1 + 3$
$= 4k^2 + 6k + 5 = 2(2k^2 + 3k + 2) + 1$,
an odd number. □

9d. Proof. Note: $2x + 5 < 11$ iff $x < (11-5)/2 = 3$.
Further $x^3 + 2x^2 = x^2(x+2) < 0$ iff $x < -2$. But $x < -2$ implies $x < 3$. □

11(b). C. The proof needs this correction: ..., and for some integer r, $c = ar$. Then $b + c = a(q + r)$, so $a \mid (b+c)$. □