Math 108, Fall 2013

HW 2 Solutions

$\boxed{1.5}$ **3g.** Proof. We prove that x odd implies $8 \mid (x^2-1)$. If x is odd, there is a $k \in \mathbb{Z}$ so that $x = 2k+1$. Then $x^2 - 1 = 4k^2 + 4k + 1 - 1 = 4k(k+1)$. Either k or $k+1$ is even, so $4k(k+1)$ is divisible by 8. $\square$

4c Proof. Assume $x \le 0$. Then $x^3 + x = x \cdot (x^2+1) \le 0$, as x^2+1 is positive. $\square$

5c Proof. The distance $d((2,4),(0,3)) = \sqrt{4+1} = \sqrt{5}$, while $d((2,4),(3,1)) = \sqrt{1+9} = \sqrt{10} > \sqrt{5}$. $\square$

6e Proof. If $a \ge 2$, $a < b$ and $ab < 3$, then $b \ge 3$ (as $b > a$), and then $ab \ge 6$. $\square$

7b Proof. $(\Leftarrow)$ Easy check.

$(\Rightarrow)$ If $b \mid (b+3)$, then $b + 3 = \ell b$ for some $\ell \ge 1$, but then $3 = (\ell - 1)b$, and $b \mid 3$; as 3 is a prime, $b = 1$ or $b = 3$. But $b = 1$ is impossible, as $(a+1) \mid 1$ is impossible because $a + 1 \ge 2$. Therefore $b = 3$; $a+1 \mid 3$ and $a+1 \ge 2$ then together imply $a + 1 = 3$, $a = 2$. $\square$

10. Omitted.

12(d) The proof is correct, but unnecessarily convoluted. Erase the first two sentences, replace next "However" by "By the assumption" and shorten the last sentence to "Thus a divides b+c".

$\boxed{1.6}$ **4(b)** True. Take $y = -x$.

4(c) False. Counterexample: $x = 2$, $y = 1$.

4(g) False. Counterexample: $x = 1$.

5(a) Proof. ($\Rightarrow$) Contrapositive.

($\Leftarrow$) Also contrapositive. Assume x is not a prime. Then $x = m \cdot n$ for some $m, n \in \mathbb{N}$. At least one of m, n must be less or equal $\sqrt{x}$: if both $m > \sqrt{x}$ and $n > \sqrt{x}$, then $m \cdot n > \sqrt{x} \cdot \sqrt{x} = x$, a contradiction with $m \cdot n = x$. Assume that $m \le \sqrt{x}$, without loss of generality. Then m is a positive integer less of equal to $\sqrt{x}$ that divides x. $\square$

6(i) Pf. $\frac{1}{r^2} < \frac{1}{100}$ is equivalent to $r > 10$.

So we can take $K = 10$. $\square$

7(h). F. What is proved is the converse

$(x=0) \vee (y=0) \Rightarrow xy = 0$.