Math 108, Fall 2013

HW 3 SOLUTIONS

$\boxed{2.1}$ 5(d) True, 5(j) False.

6(d) $A = B = \{1\}$, $C = \{2\}$.

14(d) $\mathcal{P}(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$, then substitute $a = 1$, $b = \{\emptyset\}$, $c = \{2, \{3\}\}$.

15(c) True. If $x \in A$, then $\{x\} \subseteq A$, so $\{x\} \in \mathcal{P}(A)$.

15(h) True. If $B \in \mathcal{P}(A)$, then $B \subseteq A$; as $C \in B$, $C \in A$, so $C \in \mathcal{P}(A)$.

17(d) True. $\emptyset$ is a subset of any set.

19(g) F. Completely wrong. The statement is not true, e.g. $\mathcal{P}(\{1\}) = \{\emptyset, \{1\}\}$, but $\{1\} \not\subseteq \{\emptyset, \{1\}\}$ as $1 \notin \{\emptyset, \{1\}\}$. What is true is that $A \in \mathcal{P}(A)$. The mistake in the proof is confusion between "$\in$" and "$\subseteq$".

$\boxed{2.2}$ 1(j) $A \cup B = \{1,2,3,4,5,6,7,8,9\}$, $C \cap D = \{1,2,5,7,8\}$, so $(A\cup B) - (C \cap D) = \{3,4,6,9\}$.

2(f) $[2,5]$.

9(b) Assume $x \in A$. Then $x \in B \cup C$; so either $x \in B$ or $x \in C$. But $x \in B$ implies $x \notin A$ as $A \cap B = \emptyset$. So we must have $x \in C$.

9(c) $(A - B) - C = (A \cap B^c) \cap C^c$

$$(A - C) - (B - C) = (A \cap C^c) \cap (B \cap C^c)^c = A \cap (C^c \cap (B^c \cup C))$$

$$= A \cap ((C^c \cap B^c) \cup \underbrace{(C^c \cap C)}_{=\emptyset}) = A \cap (C^c \cap B^c)$$

11(b) $A = \{1\}$, $B = C = \emptyset$.

12(b) If $X \in \mathcal{P}(A) \cup \mathcal{P}(B)$, then $X \subseteq A$ or $X \subseteq B$. In either case, $X \subseteq A \cup B$, so $X \in \mathcal{P}(A \cup B)$.

12(c) $A = \{1\}$, $B = \{2\}$, $A \cup B = \{1,2\}$; $A \cup B \notin \mathcal{P}(A)$, $A \cup B \notin \mathcal{P}(B)$, so $A \cup B \notin \mathcal{P}(A) \cup \mathcal{P}(B)$. The equality holds if and only if $A \subseteq B$ or $B \subseteq A$.

HW 3 Solutions, cont'd

19(f) F. The claim is not true, e.g., take $A=\{1,2\}$, $B=\{2,3\}$, $C=\{3,4\}$. The mistake in the proof is that "the two x's are not the same, necessarily," i.e. there exist $x_1 \in A\cap B$ and $x_2 \in B\cap C$.

$\boxed{2.3}$ **1(n)** $\bigcap_{n\in\mathbb{N}} D_n = (-1, 0]$, $\bigcup_{n\in\mathbb{N}} D_n = (-\infty, 1)$.

$\boxed{2.4}$ **6g** $(n=1)$ $\frac{1}{1\cdot 2} = \frac{1}{1+1}$.

$(n\to n+1)$ $\frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} \underset{\uparrow}{=} \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)}$

by the I.H. $= \frac{(n+1)^2}{\cancel{(n+1)}(n+2)} = \frac{n+1}{n+2}$

7a $(n=1)$ $1+5+6 = 12 = 4\cdot 3$,

$(n\to n+1)$ $(n+1)^3 + 5(n+1) + 6 = n^3 + 3n^2 + 3n + 1 + 5n + 5 + 6$

$= n^3 + 5n + 6 + 3(n^2+n+2) \underset{\substack{\uparrow\\ \text{I.H.}}}{=} 3k + 3(n^2+n+2)$,

for some $k\in\mathbb{Z}$.

7h $(n=1)$ $3 \geq 1+2$

$(n\to n+1)$ $3^{n+1} \geq 3\cdot 3^n \underset{\substack{\uparrow\\ \text{I.H.}}}{\geq} 3(1+2^n) = 3 + 3\cdot 2^n \geq 1 + 2\cdot 2^n = 1 + 2^{n+1}$

8h $(n=2)$ $\sqrt{2} < 1 + \frac{1}{\sqrt{2}}$ as $2 < \sqrt{2}+1$.

$(n\to n+1)$ $\sqrt{n+1} = \sqrt{n} + \sqrt{n+1} - \sqrt{n} = \sqrt{n} + \frac{1}{\sqrt{n+1}+\sqrt{n}}$

$< \sqrt{n} + \frac{1}{\sqrt{n+1}} \underset{\substack{\uparrow\\ \text{I.H.}}}{<} \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}}$

13(c) F. $n^2+n = n(n+1)$ is always even. Verification for $n=1$ was wrong, although the inductive step is correct.