Math 108, Fall 2013

HW4 SOLUTIONS

2.5

1(b). Check for 34, 35, 36, 37, 38. Now take $n \geq 39$.
By I.H., $n-5$ can be written as stated: $n-5 = 4s+5t$.
Then $n = 4s + 5(t+1)$. □

2. ($n=1, n=2$) True.
($n \geq 3$)($n-1 \to n$) $a_n = 5a_{n-1} - 6a_{n-2}$, so, using IH,
$a_n = 5 \cdot 2^{n-1} - 6 \cdot 2^{n-2} = 10 \cdot 2^{n-2} - 6 \cdot 2^{n-2} = 4 \cdot 2^{n-2} = 2^n$. □

5(b). ($n=1, n=2$) True, as $f_1 = f_2 = 1$.
($n \geq 3$)($n-1 \to n$) We will show that $\gcd(f_n, f_{n-1}) = 1$ assuming that $\gcd(f_{n-1}, f_{n-2}) = 1$. If $d | f_n$ and $d | f_{n-1}$, then $d | (f_n - f_{n-1})$, but $f_n - f_{n-1} = f_{n-2}$, so $d | f_{n-2}$ and $d = 1$ by IH.

5(c). If $d | f_{n+2}$, $d | f_n$, then $d | f_{n+2} - f_n$, $d | f_{n+1}$. So $d = 1$ by 5(b).

13(c). (The statement is true, but the proof is badly written. The base case is not checked, and the use of the induction hypothesis is unclear.

3.1

1 Omitted.

2b Domain: all x, Range: $y \geq 3$.

4d $\{(x,y) \in \mathbb{R} \times \mathbb{R} : x \geq 2,\ y = +\sqrt{x-2}$ or $y = -\sqrt{x-2}\}$

5a, 5h Omitted.

7c, 7d, 7f. Omitted.

15(c) F. Cannot divide by a set.