Mat 108, Fall 2013

HW5 SOLUTIONS

$\boxed{8.2}$ 1(b) reflexive and transitive, 1(c) transitive, 1(f) symmetric

5(b) Proof omitted. $106|R = 1|R = 2001|R = 20001|R$.
$635|R = 301|R = 230|R = 2030|R$.

5(e) Proof omitted. Equivalence classes are circles with center $(0,0)$; in the case of $(1,2)$ with radius $\sqrt{5}$, and in the case of $(4,0)$ with radius 4.

5(f). Proof omitted. $\{m\}|R = \{\{m\}, \{n\}, \{p\}, \{q\}, \{r\}, \{s\}\}$,
$X|R = \{X\}$.
$\mathcal{P}(X)|R$ has 7 elements, because there are 7 possible numbers of elements for subsets of X.

5(h). Proof omitted. $x^2|R = x^2+1|R = x^2+2|R$.
In general $f|R = g|R$ iff f and g differ by a constant.

7(b). Equivalence relation, 7(d) Neither symmetric, nor transitive, but reflexive.

9(a). $5, -5$.

11. Not reflexive: $3 \nmid (1+1)$, so $1 \not S 1$. Also not transitive:
$3 | 1+2$, $3 | 2+1$ but $3 \nmid (1+1)$, so $1 S 2$, $2 S 1$, but $1 \not S 1$.

12. Reflexivity: $\forall x \in A$: $(x,x) \in R$ and $(x,x) \in S$ so $(x,x) \in R \cap S$.
Symmetry: Assume $(x,y) \in R \cap S$. Then $(x,y) \in R$ and $(x,y) \in S$, and then $(y,x) \in R$ and $(y,x) \in S$, and so $(y,x) \in R \cap S$.
Transitivity. Assume $(x,y) \in R \cap S$, $(y,z) \in R \cap S$. Then $(x,y) \in R$ and $(y,z) \in R$, so $(x,z) \in R$. Similarly, $(x,z) \in S$ and so $(x,z) \in R \cap S$.

13(c). $(\Rightarrow)$ Assume $(x,z) \in R \circ R$. Then there is a $y \in A$ so that $(x,y) \in R$, $(y,z) \in R$. Then $(x,z) \in R$ by transitivity.
$(\Leftarrow)$ Assume $(x,y) \in R$ and $(y,z) \in R$. Then $(x,z) \in R \circ R \subseteq R$ and therefore $(x,z) \in R$. □

19(d) F. The proof is about ~~the~~ particular relations, not about the general ones. The statement is correct, see problem 12.

HW 5 sol., cont'd

$\boxed{3.3}$ 2(a) No. 2(c) Yes.

3 (a) $\{\{\ldots, a-2, a-1, a, a+1, a+2, \ldots\} : a \in [0,1)\}$

3 (d) $\{\{x, -x\} : x \in [0, \infty)\}$

4 $\{\{i, -i\}, \{1, -1\}\}$

7 (b) Disjoint: if $(x,y) \in A_a \cap A_b$, then $y = a - x^2$ and $y = b - x^2$, so $y + x^2 = a = b$. Cover: Take $(x_0, y_0) \in \mathbb{R}^2$. Then let $a = y_0 + x_0^2$. Clearly, $(x_0, y_0) \in A_a$. Nonempty: $(0, a) \in A_a$.

(c) This is a relation between points in $\mathbb{R}^2$:

$(x_1, y_1) \; R \; (x_2, y_2)$ iff $x_1^2 + y_1 = x_2^2 + y_2$.

15 (c). C. Claim is not true. The proof forgets the possibility that $A \cap B \neq \emptyset$. For example, if $A \subseteq B$, $A \neq \emptyset$, $B \neq A$, $\mathcal{A} = \{A\}$ and $\mathcal{B} = \{B\}$, then $\mathcal{A} \cup \mathcal{B} = \{A, B\}$ and $A \cap B = A \neq \emptyset$. The proof is correct under the additional assumption that $A \cap B = \emptyset$. (Precise location of the mistake in the proof is in (ii), when $X \in \mathcal{B}$ and $Y \in \mathcal{A}$; it does not necessarily hold that $X \cap Y = \emptyset$.)

$\boxed{4.1}$ 1(b) No. 1(c) Yes. Domain and codomain are both $\{1,2\}$.

1(i) Yes. Domain $\{a, b, c, d\}$ and codomain $\{1,2,3,4\}$.

4(d) Domain $(-\infty, 5]$, range $[0, \infty)$.

6(d). Two reasons: domain not $\mathbb{R}$ (if $x<0$, no $(x,y) \in R$) and no uniqueness of value ($(1,-1) \in R$ and $(1,1) \in R$).

11(e). We need to show that $\overline{x} = \overline{y}$ implies $[x] = [y]$, that is, that $3 \mid y-x$ implies $4 \mid y-x$. This is clearly not true, e.g. $y = 6$, $x = 3$.

19(e) A. Correct proof.