Math 108, Fall 2013

HW6 SOLUTIONS

$\boxed{4.2}$ 1(b) Omitted.

12 $f \cap g = \{(x,y) : (x,y) \in f \text{ and } (x,y) \in g\}$
$= \{(x,y) : y = f(x) = g(x)\}$
$= \{(x,y) : x \in A \text{ and } y = g(x)\} = g|A = f|A.$

13 ($\Rightarrow$) Take $x \in E$. We need to show $h(x) = g(x)$.
As $h \cup g$ is a function, and $(x, h(x)) \in h \cup g$, $(x, g(x)) \in h \cup g$, $h(x) = g(x)$ by the single value property.
($\Leftarrow$) The domain of $h \cup g$ is $A \cup C$: if $x \in A \cup C$, then either $x \in A = \mathrm{Dom}(h) \subseteq \mathrm{Dom}(h \cup g)$ or $x \in C = \mathrm{Dom}(g) \subseteq \mathrm{Dom}(h \cup g)$.
Single value: Assume $(x,y) \in h \cup g$ and $(x,z) \in h \cup g$.
If both are in h (or both in g), then $y = z$ by single value property of h (or g). Otherwise, without loss of generality, $(x,y) \in h$ and $(x,z) \in g$; then $x \in A \cap C = E$ and $y = f(x) = g(x) = z$. □

14 (c) We need to check whether $h|[0,1] = g|[0,1]$.
For $x \in [0,1]$, $h(x) = x$ and $g(x) = 3-(3-x) = x$. Yes.

14 (d) Is $h|\{2\} = g|\{2\}$; i.e., is $\cos 2 = 2^2$? No.

$\boxed{4.3}$ 1(b) Yes. For any y, $-x+1000 = y$ can be solved for x:
$x = 1000 - y$.

1(d) Yes. $x^3 = y$ has solution $x = y^{1/3}$ for any $y \in \mathbb{R}$.

1(f). No. $(\forall x)(2^x > 0)$, so $-1 \notin \mathrm{Rng}(f)$.

1(h). Yes. Take a $z \in \mathbb{R}$. Let $x = z$, $y = 0$. Then $f(x,y) = z$.

1(l). Yes. Take a $y \in (1,\infty)$. Solve $\frac{x}{x-1} = y$, $x = yx - y$, $x = \frac{y}{y-1} = 1 + \frac{1}{1-y}$, so $x \in (1,\infty)$.

2(b) Yes. $-x_1 + 1000 = -x_2 + 1000 \Rightarrow x_1 = x_2$.

2(d) Yes. $x_1^3 = x_2^3 \Rightarrow (x_1 - x_2)\underbrace{(x_1^2 + x_1x_2 + x_2^2)}_{\text{never } 0} = 0 \Rightarrow x_1 = x_2$.
(Or use Rolle: $(x_1 < x_2) \wedge (f(x_1) = f(x_2)) \Rightarrow f'(c) = 0$ for c between x_1 and x_2, but then $c = 0$ and $x_1 < 0$, $x_2 > 0$ and so $x_1^3 < 0$, $x_2^3 > 0$, a contradiction.

2(f) Yes. $\frac{d}{dx}(2^x) = 2^x \ln 2 > 0$

2(h) No. $f(1,1) = f(0,0)$.

2(l) Yes. The solution x in 1(l) is unique for any y.

HW6 SOLUTIONS, cont'd.

<u>3(c)</u> $f(x) = \lfloor x \rfloor$ = largest integer $\leq x$. It is <u>onto</u> as, for every $y \in \mathbb{Z}$, $f(y) = y$. It is <u>not 1-1</u> as $f(2.5) = f(2)$.

<u>6</u> Assume $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$. Then $g(f(x_1)) = g(f(x_2))$ and so $(g \circ f)(x_1) = (g \circ f)(x_2)$; as $g \circ f$ is 1-1, $x_1 = x_2$. $\square$

<u>8(c)</u> $A = \{1, 2\}$, $B = \{1, 2\}$, $C = \{1\}$. $f(1) = f(2) = 1$, $g(1) = g(2) = 1$.

<u>8(d)</u> —//— $f(1) = 1$, $f(2) = 2$, —//—

<u>14(c)</u> F. The argument only proves that $g \circ f$ is a function with codomain C (i.e., that it maps <u>onto</u> C).