Math 108, Fall 2013

HW7 SOLUTIONS

4.4

1(a) Fix a $y \in (-\infty, -1)$. Show that $f(x) = y$ has a unique solution: $-\frac{x}{x+2} = y$, $-x = yx - 2y$, $x = \frac{2y}{y+1} = 2 \cdot \frac{-y}{-y-1}$
$x \in (2, \infty)$

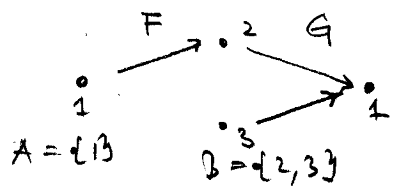
As $-y \in (1, \infty)$, $-y-1 > 0$ and $-y-1 < -y$, $\frac{-y}{-y-1} > 1$, and $x > 2$.

2(b) $f(x) = x+1$. 2(c) $f(x) = \frac{5}{3}x$

3(a) $f^{-1} = f$ 3(b) omitted

3(d) $\frac{5(x-1)}{x-3} = y$, solve for x, $x = \frac{3y-5}{y-5}$, $f^{-1}(y) = \frac{3y-5}{y-5}$.

5(b) F must be one-to-one and G must be onto, so we try

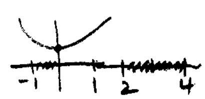


$G \circ F = I_A$, but F^{-1} is not a function!

4.5

1(b) Omitted.

2(b) $f([-1,0] \cup [2,4]) = [0,1] \cup [4,16]$



2(c) $f^{-1}([2,5]) = [-2,-1] \cup [1,2]$

5(a) $\{2^m \cdot 3^n : m = 1,2,3;\ n = 3,4\} = \{2 \cdot 27, 4 \cdot 27, 8 \cdot 27, 2 \cdot 81, 4 \cdot 81, 8 \cdot 81\}$.

5(b) $f(m,n)$ is always even and divisible by 3, so
$f^{-1}(\{5,6,7,8,9,10\}) = f^{-1}(\{6\}) = \{(2,3)\}$.

10(a) Assume $y \in f(f^{-1}(E))$. Then $\exists x \in f^{-1}(E)$ so that $y = f(x)$. But then $f(x) \in E$ and so $y \in E$.

10(b) Assume $x \in A - f^{-1}(E)$. Then $x \notin f^{-1}(E)$ and so $f(x) \notin E$. Therefore $f(x) \in B - E$ and $x \in f^{-1}(B-E)$.

13 $(\subseteq)$ Assume $y \in f(A-X)$. Then $y = f(x)$ for some $x \in A - X$. As $x \in A$, $y \in f(A)$. To show $y \in f(A) - f(X)$, we need to show that $y \notin f(X)$. Assume $y \in f(X)$, then there exists an $x_1 \in X$ so that $f(x_1) = y$. But $x_1 \neq x$ and $f(x_1) = f(x)$, a contradiction with the assumption that f is one-to-one.

HW7 SOLUTIONS, CONT'D

13, cont'd. ($\supseteq$) Assume $y \in f(A) - f(X)$. Then there exist an $x \in A$ so that $y = f(x)$. We need to show that $x \notin X$. But if $x \in X$, then $y = f(x) \in f(X)$, a contradiction. Thus $x \in A - X$ and $y \in f(A - X)$. $\square$

18 (a) F. It is possible that $f(x) \in f(X)$ but $x \notin X$.
In fact the statement is not true, e.g. $f: \{1,2\} \to \{1,2\}$, $f(1) = 1$, $f(2) = 1$. Then $f(\{1\}) = \{1\}$, $f^{-1}(f(\{1\})) = \{1,2\} \not\subseteq \{1\}$.

18 (b) A. Correct proof.

Counterexample to equality in 10(a)
Same function as in 18(a), $E = \{1,2\}$, $f^{-1}(E) = \{1,2\}$,
$f(f^{-1}(E)) = \{2\} \neq \{1,2\}$.