Math 125B, Winter 2015.

Homework 8 Solutions

11.4.2(b). By the Chain Rule, $D(f \circ g)(a) = Df(g(a)) \cdot Dg(a)$, and the determinant of the product of two square matrices is the product of determinants, thus $\det D(f \circ g)(a) = \det Df(g(a)) \cdot \det Dg(a)$

11.4.4. By the Chain Rule, $u_x = f' \cdot y$ and $u_y = f' \cdot x$ (where f' is evaluated at xy), thus $xu_x - yu_y = f' \cdot (yx - xy) = 0$. Similarly $v_x = f' + g'$, then $v_{xx} = f'' + g''$ (where derivatives of f are evaluated at $x + y$ and derivatives of g are evaluated at $x + y$), and $v_y = f' - g'$, then $v_{yy} = f'' + g''$. Thus $v_{xx} = v_{yy}$.

11.5.6. (a) By the Chain Rule,

$$g'(t) = f_x(tx + (1-t)a, y)(x - a) + f_y(a, ty + (1-t)b)(y - b).$$

11.5.6. (b) By the Mean Value Theorem for the function g, there exists a $t_0 \in [0, 1]$ so that $g'(t_0) = g(1) - g(0)$. However, $g(1) - g(0) = f(x, y) - f(a, b)$ and so we can take $c = t_0 x + (1 - t_0)a$ and $d = t_0 y + (1 - t_0)b$.

11.6.1 (c). If we assume $0 < u < v$, there is a unique point (u, v) for which $f(u, v) = (2, 5)$: $uv = 2$, $v = 2/u$, and then $5 = u^2 + 4/u^2$, $u^4 - 5u^2 + 4 = 0$, $(u^2 - 4)(u^2 - 1) = 0$, so $(u, v) = (2, 1)$ or $(1, 2)$, and we keep $(u, v) = (1, 2)$. Now

$$Df = \begin{bmatrix} v & u \\ 2u & 2v \end{bmatrix}$$

and so at $(1, 2)$,

$$Df(1, 2) = \begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}$$

Thus $\det Df(1, 2) = 6 \neq 0$ and

$$Df^{-1}(2, 5) = Df(1, 2)^{-1} = \begin{bmatrix} 2/3 & -1/6 \\ -1/3 & 1/3 \end{bmatrix}$$