

Discussion problems 1

1. Is the statement $(\forall x \in \mathbb{Z})((x > 0) \Leftrightarrow (x^2 > 0))$ true or false? Give a proof. Then do the same for the statement $(\exists x \in \mathbb{Z})((x > 0) \Leftrightarrow (x^2 > 0))$
2. For all natural numbers a and b , if 10 divides ab , then either 10 divides a or 10 divides b . Is this statement true or false? Justify.
3. Write out the negation of the statement

$$(\forall x \in \mathbb{N})(\forall y \in \mathbb{N}) [(x < y) \Rightarrow ((\exists z \in \mathbb{N})(x < z < y))].$$

Then prove that the negation is true.

4. Let A, B, C be sets. Prove that $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$.
5. Find infinitely many nonempty sets $A_1, A_2, \dots$ so that

$$\mathbb{N} \supseteq A_1 \supseteq A_2 \supseteq A_3 \dots$$

and $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Prove your claim.

6. Prove that $(1+x)^n \geq 1+nx$ for every $n \in \mathbb{N}$ and $x \in (-1, \infty)$.
7. Prove that $\sum_{i=1}^n i(i+1) = n(n+1)(n+2)/3$ for all $n \in \mathbb{N}$.