Math 127A, Fall 2019.

Discussion problems 3

1. (a) Assume (a_n) is a sequence of real numbers, and $a \in \mathbb{R}$. Define precisely what it means that $\lim_{n\to\infty} a_n = a$.
Prove, using this definition (and *not* the algebraic limit theorem), that
(b) $\lim_{n\to\infty} \frac{4n+3}{n+2} = 4$,
(c) $\lim_{n\to\infty} \frac{4n^2+3}{n^2+2} = 4$,
(d) $\lim_{n\to\infty} \frac{4\sqrt{n}+3}{\sqrt{n}+2} = 4$,

2. Now use limit theorems (and not the definition) to compute the limits
(a) $\lim_{n\to\infty} \frac{4n^7+3}{n\cdot(2n+17)^6+n^5}$,
(b) $\lim_{n\to\infty} \frac{4n^2+n^{(-1)^n}}{n^2+n\cdot(-1)^n}$.

3. Assume that a sequence (a_n) satisfies $a_n > 0$ and $\lim_{n\to\infty} a_n = 0$. Find

$$\bigcap_{n=1}^{\infty} (-a_n, 2a_n)\,.$$

Carefully prove your assertion.

4. Assume a sequence (a_n) satisfies $|a_n| \le n$ for all $n \in \mathbb{N}$, and form a new sequence $b_n = \frac{a_n+5}{n^2+a_n}$. Prove that b_n must have a limit, and find it. Carefully prove your assertions. You may use any theorem from the lecture.

5. Define, for every $n \in \mathbb{N}$,

$$a_n = \sum_{i=1}^{n} \frac{1}{\sqrt{i}}.$$

(a) Show, by induction, that the following inequality holds for every $n \in \mathbb{N}$:

$$a_n \le 2\sqrt{n} - 1.$$

(*Hint*. I think it is easier to do the $n-1 \to n$ induction step in this case.)
(b) Compute the limit of the sequence $b_n = a_n/n$.