Math 127A, Fall 2019.

Discussion problems 7

1. In each case below, determine the accumulation points of A. Also determine the closure $\overline{A}$, the boundary ∂A, and interior(A).
(a) $A = (0, 2)$.
(b) $A = (0, 2) \cap \mathbb{Q}$.
(c) $A = (0, 2) \cup \{3\}$.
(d) $A = \{n + 1/n : n \in \mathbb{N}\}$.

2. Let $A \subseteq \mathbb{R}$, and $a \in A$. Prove that a is either an interior point of A or an accumulation point of A^c, but that it cannot be both.

3. Assume that $A \subseteq \mathbb{R}$ is bounded above. Prove that if $\sup A \notin A$, then $\sup A$ is an accumulation point of A.

4. Determine, with proof, whether each of the following statements is true or false.
(a) If $A \subseteq \mathbb{R}$ and $x \in \mathbb{R}$ is an upper bound of A, then x is not in the interior of A.
(b) If $A, B \subseteq \mathbb{R}$, and x is both an interior point of A and an interior point of B, then x is an interior point of $A \cap B$.
(c) If $A, B \subseteq \mathbb{R}$, and x is both a boundary point of A and a boundary point of B, then x is a boundary point of $A \cap B$.
(d) If $A \subseteq \mathbb{R}$ is open, then $A \cap (0, 1)$ is open.
(e) If $A \subseteq \mathbb{R}$ is closed, then $A \cap (0, 1)$ is not closed.
(f) If $A \subseteq \mathbb{R}$, then $\overline{A} = \overline{A \cap \mathbb{Q}}$.
(g) If $A \subseteq \mathbb{R}$, then $\overline{A}^c = \overline{A^c}$.

5. Assume that $a_n > 0$ for all n, that the sequence (a_n) is decreasing, and that $\sum_n a_n$ converges. Let $b_n = 2a_{n-1}$ if n is even and $b_n = a_n$ otherwise. (The sequence b_n then is: $a_1, 2a_1, a_3, 2a_3, \ldots$) Show that $\sum_n b_n$ converges, but $\limsup b_{n+1}/b_n = 2$. (Therefore, $\limsup b_{n+1}/b_n > 1$ does not imply divergence of $\sum_n b_n$.)