Math 127A, Fall 2019.

**Discussion problems 8**

1. In each case below, determine whether the set $A$ is compact. Whenever $A$ is not compact, find a sequence of elements of $A$ that has no convergent subsequence with limit in $A$, and an open cover of $A$ without a finite subcover.
(a) $A = [0,2] \cap \mathbb{Q}$.
(b) $A = [0,2] \cup \{3\}$.
(c) $A = \{\frac{6n+7}{2n+5} : n \in \mathbb{N}\} \cup \{3\}$.
(d) $A = [0,\infty) \setminus (\bigcup_{n=1}^{\infty}(n^2+2, n^2+3n))$.
(e) $A = [0,1] \cup (2,3]$.

2. Show that a set $A \subseteq \mathbb{R}$ is bounded if and only if its closure $\overline{A}$ is compact.

3. Assume that $A \subseteq \mathbb{R}$ is compact. Prove that $\sup A \in A$, and $\inf A \in A$; that is, the set $A$ has a maximum and a minimum.

4. For each of the following statements determine, with proof, whether it is true or false.
(a) If If $A \subseteq \mathbb{R}$ is finite, then it is compact.
(b) If $A \subseteq \mathbb{R}$ is compact then $A^c$ is open.
(c) If $A \subseteq \mathbb{R}$ is compact then $A \cap (0,1)$ is compact.
(d) If $A \subseteq \mathbb{R}$ is compact, and $x_n \in A$, then $(x_n)$ is a convergent sequence.
(e) If $A \subseteq \mathbb{R}$, and there exist an open cover of $A$ with a finite subcover, then $A$ is compact.
(f) If $A \subseteq \mathbb{R}$ is compact, and $(x_n)$ is a convergent sequence with limit in $A$, then $A \cup \{x_n : n \in \mathbb{N}\}$ is compact.