

## Homework 1

*The homework will not be collected, but do it by Tue., Oct 7. The solutions are provided.*

1. Fifteen married couples are at a dance lesson. Assume that husband-wife dance pairs are assigned at random. (a) What is the number of possible assignments? (b) What is the probability that each husband ends up dancing with his wife?
2. Three Finns and three Danes sit in a row at random. Compute: (a) the probability that the three Finns sit in three adjacent seats, and the same is true for the Danes; (b) the probability that the three Finns sit in three adjacent seats; (c) no two adjacent seats are occupied by citizens of the same nation.
3. Fifteen married couples are at a dance lesson, but now only five husbands and five wives are selected at random, then randomly paired. (a) What is the number of possible dancing arrangements? (b) What is the probability that John Smith, one of the husbands, dances with Jane Smith, his wife?
4. You are walking on points in the plane with integer coordinates. Each time you can move either one unit up or one unit right; for example, from  $(2, 4)$  you can move either to  $(3, 4)$  or  $(2, 5)$ . You start at the origin  $(0, 0)$  want to reach  $(4, 3)$ . (a) How many possible routes do you have? (b) You choose one of the routes at random. What is the probability that you visit  $(2, 2)$  on your route?
5. Twelve people are divided at random into three committees: A (3 people) B (4 people) and C (5 people). What is the probability that A consists of the 3 youngest people and C of the 5 oldest people?

*You should also do the five Problems in Section 2 of the book.*

### Solutions

1. (a)  $15!$ . (b)  $\frac{1}{15!}$ .
2. (a)  $\frac{2 \cdot 3! \cdot 3!}{6!}$ . (b) Order the Finns within their block, and the Finnish block and three Danes:  $\frac{3! \cdot 4!}{6!}$ . (c) Finns and Danes must alternate, same answer as (a).
3. (a)  $\binom{15}{5}^2 \cdot 5!$ . (b)  $\frac{\binom{14}{4}^2 \cdot 4!}{\binom{15}{5}^2 \cdot 5!}$  or  $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{5}$  (each of the two has to be chosen, then paired).
4. (a)  $\binom{7}{3}$ , as you need to choose when to make the 3 upward steps. (b) You have to make two upward steps within the first four steps, then the last upward step within the last three steps:  $\frac{\binom{4}{2} \binom{3}{1}}{\binom{7}{3}}$ .
5.  $\frac{1}{\binom{12}{3} \binom{9}{4} \binom{5}{5}} = \frac{3!4!5!}{12!}$ .