Math 135A, Fall 2025.

Homework 2

The homework will not be collected, but do it by Tue., Oct 14. The solutions are provided.

1. Three tours, A, B, and C, are offered to a group of 100 tourists. It turns out that 28 tourists sign for A, 26 for B, 16 for C, 12 for both A and B, 4 for both A and C, 6 for both B and C, and 2 for all three tours. (a) What is the probability that a randomly chosen tourist is taking none of these tours? (b) What is the probability that a randomly chosen tourist is taking exactly one of these tours? (c) What is the probability that two randomly chosen tourists are both taking at least one of these tours?

2. The *Poker Dice* is a game in which a player rolls 5 dice, and the hand is observed as in poker, except there are no flushes (because there are no suits), 1 may only be the lowest number in a straight, and five of a kind is an additional possibility. Compute the probabilities of all seven hands: one pair, two pairs, three of a kind, straight, full house, four of a kind, and five of a kind.

3. An urn contains 5 red, 6 green, and 8 blue balls. Take three balls out at random one by one (a) without and (b) with replacement. In each case compute the probability that the balls are of (1) the same color and (2) three different colors.

4. Assume a birthday is equally likely to be in any month of the year. In a group of 20 people, what is the probability that 4 months contain *exactly* 2 birthdays each and 4 months contain *exactly* 3 birthdays each?

5. You are dealt 13 cards from a shuffled deck of 52 cards. Compute the probability that (a) your hand lacks at least one suit, (b) you get the both Ace and King of at least one suit, (c) you get all four cards of at least one denomination (all Aces, or all Kings, or all Queens, . . . , or all Twos).

6. Three dice, one red, one blue, and one green, have the following numbers on their sides, two sides per number:

- red: 1, 6, 8;
- blue: 3, 5, 7;
- green: 2, 4, 9.

Going first, Player 1 is free to choose any of the three dice, then rolls the selected die. Then Player 2 is free to select any one of the remaining two dice, and rolls it. (The players see the colors of the dies and thus know the numbers on their faces, but the rolls are of course random.) The player who rolls the highest number wins. When you play the game, which of the two players would you prefer to be?

You should also do the six Problems in Section 3 of the book. Note. There is an inaccuracy in the solution of the last part of Problem 6 in the book: on the penultimate line, the last probability should be $P(1, 2, 4, 5 \text{ all hit, but not } 3)$. The answer is correct.

Solutions

1. Let A, B, C be the events that the random tourist takes tour A, B, C.
(a) $P((A\cup B\cup C)^c)=1-P(A\cup B\cup C)=1-P(A)-P(B)-P(C)+P(A\cap B)+P(A\cap C)+P(B\cap C)-P(A\cap B\cap C)=1-0.28-0.26-0.16+0.12+0.04+0.06-0.02=0.5$.
(b) This probability equals $P(A)+P(B)+P(C)-2P(A\cap B)-2P(A\cap C)-2P(B\cap C)+3P(A\cap B\cap C)$.
(c) $\binom{50}{2}/\binom{100}{2}$.

2. Number of outcomes is 6^5, and the number of good outcomes is given as follows. One pair (choose a pair, then 3 others, then place the pair, then place others): $6\cdot\binom{5}{3}\cdot\binom{5}{2}\cdot 3!$; two pairs: $\binom{6}{2}\cdot 4\cdot\binom{5}{2}\cdot\binom{3}{2}$; three of a kind: $6\cdot\binom{5}{3}\cdot\binom{5}{2}\cdot 2!$; straight: $2\cdot 5!$; full house: $6\cdot 5\cdot\binom{5}{3}$; four of a kind: $6\cdot 5\cdot 5$; five of a kind: 6.

3. (a1): $(\binom{5}{3}+\binom{6}{3}+\binom{8}{3})/\binom{19}{3}$; (a2): $5\cdot 6\cdot 8/\binom{19}{3}$; (b1): $(5^3+6^3+8^3)/19^3$; (b2): $(5\cdot 6\cdot 8\cdot 3!)/19^3$.

4. $\binom{12}{4}\binom{8}{4}\binom{20}{2}\binom{18}{2}\binom{16}{2}\binom{14}{2}\binom{12}{3}\binom{9}{3}\binom{6}{3}/12^{20}$.

5. (a) Let $A_i=\{\text{suit } i \text{ is missing}\}$. By inclusion-exclusion, $P(A_1\cup A_2\cup A_3\cup A_4)=\binom{4}{1}\cdot\binom{39}{13}/\binom{52}{13}-\binom{4}{2}\cdot\binom{26}{13}/\binom{52}{13}+\binom{4}{3}\cdot 1/\binom{52}{13}$.
(b) Let $A_i=\{\text{Ace and King of suit } i \text{ are present}\}$. By inclusion-exclusion, $P(A_1\cup A_2\cup A_3\cup A_4)=\binom{4}{1}\cdot\binom{50}{11}/\binom{52}{13}-\binom{4}{2}\cdot\binom{48}{9}/\binom{52}{13}+\binom{4}{3}\cdot\binom{46}{7}/\binom{52}{13}-\binom{4}{4}\cdot\binom{44}{5}/\binom{52}{13}$.
(c) Let $A_i=\{\text{ all four cards of denomination } i \text{ are present}\}$. By inclusion-exclusion, $P(A_1\cup\cdots\cup A_{13})=\binom{13}{1}\cdot\binom{48}{9}/\binom{52}{13}-\binom{13}{2}\cdot\binom{44}{5}/\binom{52}{13}+\binom{13}{3}\cdot\binom{40}{1}/\binom{52}{13}$.

6. This is an example of *non-transitivity* of winning advantage: P(red beats blue) = 5/9, P(blue beats green) = 5/9, P(green beats red) = 5/9. It is better to be Player 2 because no matter what Player 1 chooses, Player 2 can choose a die with a winning advantage.