Math 135A, Fall 2011.
Oct. 21, 2011.

MIDTERM EXAM 1

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________

ID#: ______________________

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. Unless directed to do so, do *not* evaluate complicated expressions to give the result as a decimal number.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

1. A group of 40 people consists of 18 Swedes, 10 Norwegians and 12 Finns. In a *selection step*, choose a committee of 8 of them at random.

(a) Assume the selection step is performed once. Compute the probability that the committee consists of 3 Swedes, 3 Norwegians and 2 Finns.

$$\frac{\binom{18}{3}\binom{10}{3}\binom{12}{2}}{\binom{40}{8}}$$

(b) A committee is *acceptable* if it consists of representatives of a single nation (i.e., consists entirely of Swedes, or entirely of Norwegians, or entirely of Finns). Assume the selection step is performed once and compute the probability that the committee is acceptable.

$$\frac{\binom{18}{8}+\binom{10}{8}+\binom{12}{8}}{\binom{40}{8}}$$

(c) Assume the selection step is repeated (independently) until the committee is acceptable. Compute the probability that the resulting committee consists entirely of Swedes.

P(Swedes only | committee acceptable)

$$= \frac{\binom{18}{8}}{\binom{18}{8}+\binom{10}{8}+\binom{12}{8}}$$

2. Roll a fair die ten times.

(a) Compute the probability that the ten numbers you roll are the same.

$$\frac{6}{6^{10}} = \underline{\underline{\frac{1}{6^9}}}$$

(b) Compute the probability that you roll exactly five 1's and exactly five 2's.

$$\frac{\binom{10}{5}}{6^{10}}$$

(d) Compute the probability that you roll exactly two 1's and exactly five 2's.

position of 1's   pos. of 2's   rest

$$\frac{\binom{10}{2}\binom{8}{5} \cdot 4^3}{6^{10}}$$

(c) Compute the probability that you the numbers 1, 2, and 3 are all represented among the numbers your roll.

$$A_i = \{i \text{ missing}\}, \quad i = 1, 2, 3$$

$$1 - P(A_1 \cup A_2 \cup A_3)$$

$$= 1 - \big(P(A_1) + P(A_2) + P(A_3)\big)$$
$$+ \big(P(A_1 \cap A_2) + P(A_1 \cap A_3) + P(A_2 \cap A_3)\big)$$
$$- P(A_1 \cap A_2 \cap A_3)$$

$$= 1 - 3 \cdot \frac{5^{10}}{6^{10}} + 3\,\frac{4^{10}}{6^{10}} + \frac{3^{10}}{6^{10}}$$

3. A bag contains 60 golf balls: 15 yellow, 25 red, and 20 green golf balls. Select four balls from the bag one by one *without* replacement.

(a) Compute the probability that first ball selected is yellow, the second is green, the third is red and the fourth is again yellow.

$$\frac{15}{60} \cdot \frac{20}{59} \cdot \frac{25}{58} \cdot \frac{14}{57}$$

(b) Compute the probability that all four balls selected are green.

$$\frac{20}{60} \cdot \frac{19}{59} \cdot \frac{18}{58} \cdot \frac{17}{57} \quad \text{or} \quad \frac{\binom{20}{4}}{\binom{60}{4}}$$

(c) Let A be the event that the second ball has a different color than the first ball, the third ball has a different color than the first ball, and the fourth ball has a different color than the first ball. Compute the probability of A.

$$\underset{\substack{\uparrow \\ \text{1st yellow}}}{\frac{15}{60}} \cdot \frac{45}{59} \cdot \frac{44}{58} \cdot \frac{43}{57} + \underset{\substack{\uparrow \\ \text{1st red}}}{\frac{25}{60}} \cdot \frac{35}{59} \cdot \frac{34}{58} \cdot \frac{33}{57}$$

$$+ \underset{\substack{\uparrow \\ \text{1st green}}}{\frac{20}{60}} \cdot \frac{40}{59} \cdot \frac{39}{58} \cdot \frac{38}{57} \quad \text{or} \quad \frac{\frac{15}{60}\binom{45}{3} + \frac{25}{60}\binom{35}{3} + \frac{20}{60}\binom{40}{3}}{\binom{59}{3}}$$

4. You have a deck of 13 cards labeled with values 1, 2, ..., 13. Shuffle this deck. Let A be the event that the cards 1, 2, and 3 are together, i.e., next to each other in any order, in the deck. Let B be the event that the six cards 1, 2, 3, 4, 5, and 6 are together in the deck.

(a) Compute $P(A)$. Give the answer as a simple fraction.

$$P(A) = \frac{3! \cdot 11!}{13!} = \frac{6}{12 \cdot 13} = \underline{\underline{\frac{1}{26}}}$$

(b) Compute $P(B)$.

$$P(B) = \frac{6! \cdot 8!}{13!}$$

(c) Compute the conditional probability $P(A|B)$. Give the answer as a simple fraction.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3!\,4!\,8!}{13!}}{\frac{6!\,8!}{13!}} = \frac{3!}{5 \cdot 6} = \underline{\underline{\frac{1}{5}}}$$

(d) Are A and B independent? Explain.

<u>No</u> : $P(A) \neq P(A|B)$.