3 CHAPTER REVIEW EXERCISES

In Exercises 1–4, find the critical numbers of the function.

1. $f(x) = -x^2 + 2x + 4$

2. $g(x) = (x - 1)^2(x - 3)$

3. $h(x) = \sqrt{x}(x - 3)$

4. $f(x) = (x + 1)^3$

In Exercises 5–8, determine the open intervals on which the function is increasing or decreasing. Solve the problem analytically and graphically.

5. $f(x) = x^2 + x - 2$

6. $g(x) = -x^2 + 7x - 12$

7. $h(x) = \dfrac{x^2 - 3x - 4}{x - 3}$

8. $f(x) = -x^3 + 6x^2 - 2$

9. ***Meteorology*** The monthly normal temperature T (in degrees Fahrenheit) for New York City can be modeled by

$$T = 0.0385t^4 - 1.122t^3 + 9.67t^2 - 21.8t + 47$$

where $1 \le t \le 12$ and $t = 1$ corresponds to January. *(Source: National Climatic Data Center)*

(a) Find the interval(s) on which the model is increasing.

(b) Find the interval(s) on which the model is decreasing.

(c) Interpret the results of parts (a) and (b).

(d) Use a graphing utility to graph the model.

10. ***CD Shipments*** The number S of manufacturer unit shipments (in millions) of CDs in the United States from 1998 through 2002 can be modeled by

$$S = 5.8583t^3 - 28.943t^2 - 34.36t + 940.6$$

where $-2 \le t \le 2$ and $t = 0$ corresponds to 2000. *(Source: Recording Industry Association of America)*

(a) Find the interval(s) on which the model is increasing.

(b) Find the interval(s) on which the model is decreasing.

(c) Interpret the results of parts (a) and (b).

(d) Use a graphing utility to graph the model.

In Exercises 11–20, use the First-Derivative Test to find the relative extrema of the function. Then use a graphing utility to verify your result.

11. $f(x) = 4x^3 - 6x^2 - 2$

12. $f(x) = \frac{1}{4}x^4 - 8x$

13. $g(x) = x^2 - 16x + 12$

14. $h(x) = 4 + 10x - x^2$

15. $h(x) = 2x^2 - x^4$

16. $s(x) = x^4 - 8x^2 + 3$

17. $f(x) = \dfrac{6}{x^2 + 1}$

18. $f(x) = \dfrac{2}{x^2 - 1}$

19. $h(x) = \dfrac{x^2}{x - 2}$

20. $g(x) = x - 6\sqrt{x}, \quad x > 0$

In Exercises 21–30, find the absolute extrema of the function on the closed interval. Then use a graphing utility to confirm your result.

21. $f(x) = x^2 + 5x + 6; \quad [-3, 0]$

22. $f(x) = x^4 - 2x^3; \quad [0, 2]$

23. $f(x) = x^3 - 12x + 1; \quad [-4, 4]$

24. $f(x) = x^3 + 2x^2 - 3x + 4; \quad [-3, 2]$

25. $f(x) = 4\sqrt{x} - x^2; \quad [0, 3]$

26. $f(x) = 2\sqrt{x} - x; \quad [0, 9]$

27. $f(x) = 3x^4 - 6x^2 + 2; \quad [0, 2]$

28. $f(x) = -x^4 + x^2 + 2; \quad [0, 2]$

29. $f(x) = \dfrac{2x}{x^2 + 1}; \quad [-1, 2]$

30. $f(x) = \dfrac{8}{x} + x; \quad [1, 4]$

31. ***Surface Area*** A right circular cylinder of radius r and height h has a volume of 25 cubic inches. The total surface area of the cylinder in terms of r is given by

$$S = 2\pi r\left(r + \frac{25}{\pi r^2}\right).$$

Use a graphing utility to graph S and S' and find the value of r that yields the minimum surface area.

32. ***Environment*** When organic waste is dumped into a pond, the decomposition of the waste consumes oxygen. A model for the oxygen level O (where 1 is the normal level) of a pond as waste material oxidizes is

$$O = \frac{t^2 - t + 1}{t^2 + 1}, \quad 0 \le t$$

where t is the time in weeks.

(a) When is the oxygen level lowest? What is this level?

(b) When is the oxygen level highest? What is this level?

(c) Describe the oxygen level as t increases.

In Exercises 33–36, det
graph of the function i:
Then use a graphing ut

33. $f(x) = (x - 2)^3$

34. $h(x) = x^5 - 10x^2$

35. $g(x) = \frac{1}{4}(-x^4 + 8$

36. $h(x) = x^3 - 6x$

In Exercises 37–40, finc
the function.

37. $f(x) = \frac{1}{2}x^4 - 4x^3$

38. $f(x) = \frac{1}{4}x^4 - 2x^2$

39. $f(x) = x^3(x - 3)^2$

40. $f(x) = (x + 2)^2(x$

In Exercises 41–44, use
relative extrema of the

41. $f(x) = x^5 - 5x^3$

42. $f(x) = x(x^2 - 3x$

43. $f(x) = (x - 1)^3(x$

44. $f(x) = (x - 2)^2(x$

Point of Diminishi
identify the point of c
function. For each fun
dollars) and x is the am
of dollars).

45. $R = \frac{1}{1500}(150x^2 -$

46. $R = -\frac{2}{3}(x^3 - 12x$

47. ***Minimum Sum***
product is 169 an
problem analytical
problem graphical

48. ***Length*** The wa
that must pass ove
building and 4 feet
shortest beam that

49. ***Newspaper Circ***
newspapers in circ
from 1970 through

$N = 0.020t^3 -$

where $0 \le t \le 3$
(Source: Editor &

(a) Find the absc
time period.

(b) Find the year
greatest rate.

(c) Briefly explai

In Exercises 33–36, determine the open intervals on which the graph of the function is concave upward or concave downward. Then use a graphing utility to confirm your result.

33. $f(x) = (x - 2)^3$

34. $h(x) = x^5 - 10x^2$

35. $g(x) = \frac{1}{4}(-x^4 + 8x^2 - 12)$

36. $h(x) = x^3 - 6x$

In Exercises 37–40, find the points of inflection of the graph of the function.

37. $f(x) = \frac{1}{2}x^4 - 4x^3$

38. $f(x) = \frac{1}{4}x^4 - 2x^2 - x$

39. $f(x) = x^3(x - 3)^2$

40. $f(x) = (x + 2)^2(x - 4)$

In Exercises 41–44, use the Second-Derivative Test to find the relative extrema of the function.

41. $f(x) = x^5 - 5x^3$

42. $f(x) = x(x^2 - 3x - 9)$

43. $f(x) = (x - 1)^3(x + 4)^2$

44. $f(x) = (x - 2)^2(x + 2)^2$

Point of Diminishing Returns In Exercises 45 and 46, identify the point of diminishing returns for the input-output function. For each function, R is the revenue (in thousands of dollars) and x is the amount spent on advertising (in thousands of dollars).

45. $R = \frac{1}{1500}(150x^2 - x^3), \quad 0 \le x \le 100$

46. $R = -\frac{2}{3}(x^3 - 12x^2 - 6), \quad 0 \le x \le 8$

47. ***Minimum Sum*** Find two positive numbers whose product is 169 and whose sum is a minimum. Solve the problem analytically, and use a graphing utility to solve the problem graphically.

48. ***Length*** The wall of a building is to be braced by a beam that must pass over a five-foot fence that is parallel to the building and 4 feet from the building. Find the length of the shortest beam that can be used.

49. ***Newspaper Circulation*** The total number N of daily newspapers in circulation (in millions) in the United States from 1970 through 2000 can be modeled by

$$N = 0.020t^3 - 1.19t^2 + 9.0t + 1746$$

where $0 \le t \le 30$ and $t = 0$ corresponds to 1970. *(Source: Editor and Publisher Company)*

(a) Find the absolute maximum and minimum over the time period.

(b) Find the year when the circulation was changing at the greatest rate.

(c) Briefly explain your results for parts (a) and (b).

50. ***Minimum Cost*** A fence is to be built to enclose a rectangular region of 4800 square feet. The fencing material along three sides costs \$3 per foot. The fencing material along the fourth side costs \$4 per foot.

(a) Find the most economical dimensions of the region.

(b) How would the result of part (a) change if the fencing material costs for all sides increased by \$1 per foot?

51. ***Biology*** The growth of a red oak tree is approximated by the model

$$y = -0.003x^3 + 0.137x^2 + 0.458x - 0.839, \quad 2 \le x \le 34$$

where y is the height of the tree in feet and x is its age in years. Find the age of the tree when it is growing most rapidly. Then use a graphing utility to graph the function to verify your result. (*Hint:* Use the viewing window $2 \le x \le 34$ and $-10 \le y \le 60$.)

52. ***Consumer Trends*** The average number of hours N (per person per year) of TV usage in the United States from 1996 through 2001 can be modeled by

$$N = -2.870t^3 + 79.62t^2 - 639.1t + 3473$$

where $t = 6$ corresponds to 1996. *(Source: Veronis Suhler Stevenson)*

(a) Find the intervals on which dN/dt is increasing and decreasing.

(b) Find the limit of N as $t \to 0$.

(c) Briefly explain your results for parts (a) and (b).

53. ***Medicine: Poiseuille's Law*** The speed of blood that is r centimeters from the center of an artery is modeled by

$$s(r) = c(R^2 - r^2)$$

where c is a constant, R is the radius of the artery, and s is measured in centimeters per second. Show that the speed is a maximum at the center of an artery.

54. ***Profit*** The demand and cost functions for a product are $p = 36 - 4x$ and $C = 2x^2 + 6$.

(a) What level of production will produce a maximum profit?

(b) What level of production will produce a minimum average cost per unit?

55. ***Revenue*** For groups of 20 or more, a theater determines the ticket price p according to the formula

$$p = 15 - 0.1(n - 20), \quad 20 \le n \le N$$

where n is the number in the group. What should the value of N be? Explain your reasoning.

56. ***Minimum Cost*** The cost of fuel to run a locomotive is proportional to the $\frac{3}{2}$ power of the speed. At a speed of 25 miles per hour, the cost of fuel is \$50 per hour. Other costs amount to \$100 per hour. Find the speed that will minimize the cost per mile.

57. ***Inventory Cost*** The cost C of inventory modeled by

$$C = \left(\frac{Q}{x}\right)s + \left(\frac{x}{2}\right)r$$

depends on ordering and storage costs, where Q is the number of units sold per year, r is the cost of storing one unit for 1 year, s is the cost of placing an order, and x is the number of units in the order. Determine the order size that will minimize the cost when $Q = 10{,}000$, $s = 4.5$, and $r = 5.76$.

58. ***Profit*** The demand and cost functions for a product are given by

$$p = 600 - 3x$$

and

$$C = 0.3x^2 + 6x + 600$$

where p is the price per unit, x is the number of units, and C is the total cost. The profit for producing x units is given by

$$P = xp - C - xt$$

where t is the excise tax per unit. Find the maximum profits for excise taxes of $t = \$5$, $t = \$10$, and $t = \$20$.

In Exercises 59–62, find the intervals on which the demand is elastic, inelastic, and of unit elasticity.

59. $p = 30 - 0.2x, \quad 0 \le x \le 150$

60. $p = 60 - 0.04x, \quad 0 \le x \le 1500$

61. $p = \sqrt{300 - x}, \quad 0 \le x \le 300$

62. $p = \sqrt{960 - x}, \quad 0 \le x \le 960$

In Exercises 63–68, find the vertical and horizontal asymptotes of the graph. Then use a graphing utility to graph the function.

63. $h(x) = \dfrac{2x + 3}{x - 4}$

64. $g(x) = \dfrac{5x^2}{x^2 + 2}$

65. $f(x) = \dfrac{\sqrt{9x^2 + 1}}{x}$

66. $h(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

67. $f(x) = \dfrac{3}{x^2 - 5x + 4}$

68. $h(x) = \dfrac{2x^2 + 3x - 5}{x - 1}$

In Exercises 69–76, find the limit, if it exists.

69. $\displaystyle\lim_{x \to 0^+} \left(x - \frac{1}{x^3}\right)$

70. $\displaystyle\lim_{x \to 0^-} \left(3 + \frac{1}{x}\right)$

71. $\displaystyle\lim_{x \to -1^+} \frac{x^2 - 2x + 1}{x + 1}$

72. $\displaystyle\lim_{x \to 3^-} \frac{3x^2 + 1}{x^2 - 9}$

73. $\displaystyle\lim_{x \to \infty} \frac{5x^2 + 3}{2x^2 - x + 1}$

74. $\displaystyle\lim_{x \to \infty} \frac{3x^2 - 2x + 3}{x + 1}$

75. $\displaystyle\lim_{x \to -\infty} \frac{3x^2}{x + 2}$

76. $\displaystyle\lim_{x \to -\infty} \left(\frac{x}{x - 2} + \frac{2x}{x + 2}\right)$

77. ***Health*** For a person with sensitive skin, the maximum amount T (in hours) of exposure to the sun that can be tolerated before skin damage occurs can be modeled by

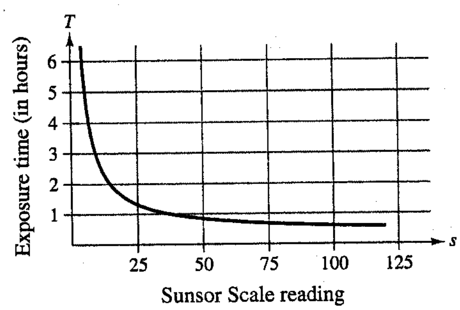
$$T = \frac{0.37s + 23.8}{s}, \quad 0 < s \le 120$$

where s is the Sunsor Scale reading. *(Source: Sunsor, Inc.)*

(a) Use a graphing utility to graph the model. Compare your result with the graph below.

(b) Describe the value of T as s increases.

Sensitive Skin



78. ***Average Cost and Profit*** The cost and revenue functions for a product are given by

$$C = 10{,}000 + 48.9x$$

and

$$R = 68.5x.$$

(a) Find the average cost function.

(b) What is the limit of the average cost as x approaches infinity?

(c) Find the average profits when x is 1 million, 2 million, and 10 million.

(d) What is the limit of the average profit as x increases without bound?

In Exercises 79–86, ske
intercepts, relative extr
State the domain of the

79. $f(x) = 4x - x^2$

81. $f(x) = x\sqrt{16 - x}$

83. $f(x) = \dfrac{x + 1}{x - 1}$

85. $f(x) = x^2 + \dfrac{2}{x}$

In Exercises 87–90, find

87. $y = 6x^2 - 5$

88. $y = (3x^2 - 2)^3$

89. $y = -\dfrac{5}{\sqrt[3]{x}}$

90. $y = \dfrac{2 - x}{x + 5}$

In Exercises 91–94, use
cost, revenue, or profit
one unit.

91. $C = 40x^2 + 122$

92. $C = 1.5\sqrt[3]{x} + 5$

93. $R = 6.25x + 0.4$

94. $P = 0.003x^2 + 0.$

95. ***Revenue Per***
dollars) for the W
through 2003 are
Disney Company,

Year, t	
Revenue per share, R	

Year, t	8
Revenue per share, R	1

(a) Use a graphi
data, where t
sponding to 1

(b) Describe any

(c) A model for t

$R = \frac{}{1 -}$

$2 \le t \le$

Graph the m
window.

In Exercises 79–86, sketch the graph of the function. Label the intercepts, relative extrema, points of inflection, and asymptotes. State the domain of the function.

79. $f(x) = 4x - x^2$

80. $f(x) = 4x^3 - x^4$

81. $f(x) = x\sqrt{16 - x^2}$

82. $f(x) = x^2\sqrt{9 - x^2}$

83. $f(x) = \dfrac{x+1}{x-1}$

84. $f(x) = \dfrac{2x}{1 + x^2}$

85. $f(x) = x^2 + \dfrac{2}{x}$

86. $f(x) = x^{4/5}$

In Exercises 87–90, find the differential dy.

87. $y = 6x^2 - 5$

88. $y = (3x^2 - 2)^3$

89. $y = -\dfrac{5}{\sqrt[3]{x}}$

90. $y = \dfrac{2 - x}{x + 5}$

In Exercises 91–94, use differentials to approximate the change in cost, revenue, or profit corresponding to an increase in sales of one unit.

91. $C = 40x^2 + 1225, \quad x = 10$

92. $C = 1.5\sqrt[3]{x} + 500, \quad x = 125$

93. $R = 6.25x + 0.4x^{3/2}, \quad x = 225$

94. $P = 0.003x^2 + 0.019x - 1200, \quad x = 750$

95. ***Revenue Per Share*** The revenues per share R (in dollars) for the Walt Disney Company for the years 1992 through 2003 are shown in the table. *(Source: The Walt Disney Company)*

Year, t	2	3	4	5	6	7
Revenue per share, R	4.77	5.31	6.40	7.70	10.50	11.10

Year, t	8	9	10	11	12	13
Revenue per share, R	11.21	11.34	12.09	12.52	12.40	13.23

(a) Use a graphing utility to create a scatter plot of the data, where t is the time in years, with $t = 2$ corresponding to 1992.

(b) Describe any trends and/or patterns of the data.

(c) A model for the data is

$$R = \frac{4.72 - 1.605t + 0.1741t^2}{1 - 0.356t + 0.0420t^2 - 0.00112t^3},$$

$2 \le t \le 13.$

Graph the model and the data in the same viewing window.

(d) Find the years when the revenue per share was increasing and decreasing.

(e) Find the years when the rate of change of the revenue per share was increasing and decreasing.

(f) Briefly explain your results for parts (d) and (e).

96. ***Medicine*** The effectiveness E of a pain-killing drug t hours after entering the bloodstream is modeled by

$$E = 22.5t + 7.5t^2 - 2.5t^3, \quad 0 \le t \le 4.5.$$

(a) Use a graphing utility to graph the equation. Choose an appropriate window.

(b) Find the maximum effectiveness the pain-killing drug attains over the interval $[0, 4.5]$.

97. ***Surface Area and Volume*** The diameter of a sphere is measured to be 18 inches with a possible error of 0.05 inch. Use differentials to approximate the possible error in the surface area and the volume of the sphere.

98. ***Demand*** A company finds that the demand for its product is modeled by

$$p = 85 - 0.125x.$$

If x changes from 7 to 8, what is the corresponding change in p? Compare the values of Δp and dp.

99. ***Economics: Revenue*** Consider the following cost and demand information for a monopoly (in dollars). Complete the table, and then use the information to answer the questions. *(Source: Adapted from Taylor,* Economics, *Fourth Edition)*

Quantity of output	Price	Total revenue	Marginal revenue
1	14.00		
2	12.00		
3	10.00		
4	8.50		
5	7.00		
6	5.50		

(a) Use the *regression* feature of a graphing utility to find a quadratic model for the total revenue data.

(b) From the total revenue model you found in part (a), use derivatives to find an equation for the marginal revenue. Now use the values for output in the table and compare the results with the values in the marginal revenue column of the table. How close was your model?

(c) What quantity maximizes total revenue for the monopoly?