Math 16A, Winter 2016.
Feb. 3, 2016.

MIDTERM EXAM 1

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): _____

ID#: _____

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. The proctor has been directed not to answer any interpretation questions.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

1.

(a) A line has x–intercept $(4, 0)$ and slope $-1/2$. Find its y-intercept.

$y - 0 = -\frac{1}{2}(x-4)$

$y = -\frac{1}{2}x + 2$ y-int: $(0, 2)$

(b) A circle has center at $(0, 2)$ and goes through the point $(1, 4)$. Find the equation of this circle.

$r^2 = (1-0)^2 + (4-2)^2 = 5$

Circle: $(x-0)^2 + (y-2)^2 = 5$

$x^2 + (y-2)^2 = 5$

(c) Find all points of intersection (if there are any) between the line from (a) and the circle from (b).

$y = -\frac{1}{2}x + 2$ $\quad y - 2 = -\frac{1}{2}x$

$x^2 + (y-2)^2 = 5$

$x^2 + \frac{1}{4}x^2 = 5$

$\frac{5}{4}x^2 = 5$ $\quad x^2 = 4$, $x = \pm 2$

$x = -2$, $y = 3$; $\quad x = 2$, $y = 1$

Points: $(-2, 3)$, $(2, 1)$

2. Consider the function $f(x) = \dfrac{x^2 - 4x}{x^2 - 4}$. Determine the domain, intercepts, and vertical and horizontal asymptotes. (Include computation of limits at vertical asymptotes.) Determine also any points where the graph of $y = f(x)$ intersects its horizontal asymptote. Then sketch the graph of this function on which all obtained points and asymptotes are clearly marked.

Domain: $x \neq -2$, $x = 2$

Intercepts: $(0, 0)$, $(4, 0)$

$\lim_{x \to \pm\infty} \dfrac{x^2 - 4x}{x^2 - 4} = 1$ $\quad y = 1$ h.a.

$\lim_{x \to -2+} \dfrac{x(x-4)}{(x-2)(x+2)} = -\infty$ ($x \approx -2$, -4, $x - 2$ small > 0)

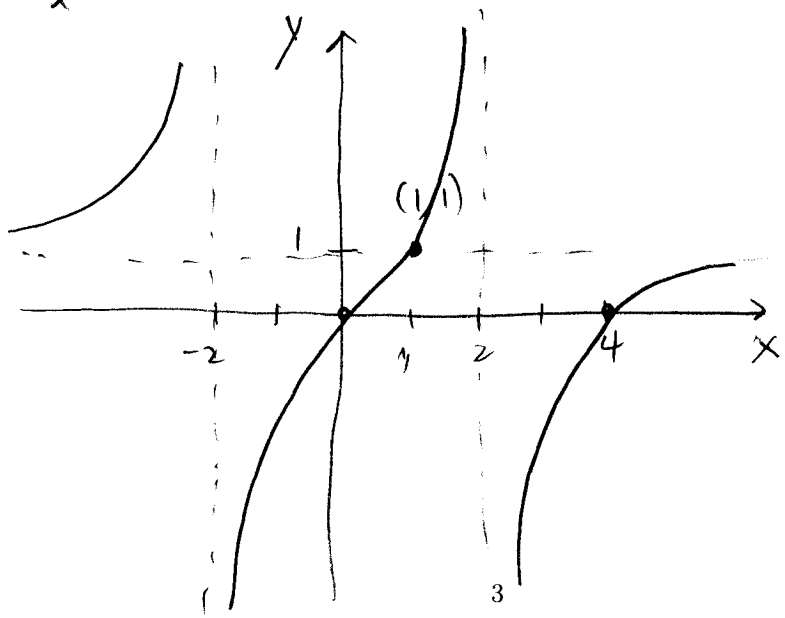
$\lim_{x \to -2-} f(x) = +\infty$

$\lim_{x \to 2+} \dfrac{x(x-4)}{(x-2)(x+2)} = -\infty$ ($x \approx 2$, $x - 2$ small > 0, ≈ 4)

$\lim_{x \to 2-} f(x) = +\infty$

Intersection with h.a.:

$\dfrac{x^2 - 4x}{x^2 - 4} = 1$; $x^2 - 4x = x^2 - 4$, $x = 1$: $(1, 1)$



3. Compute the following limits. Give each answer as a finite number, $+\infty$, or $-\infty$.

(a) $\lim_{x\to 4} \dfrac{\sqrt{x+5}}{\sqrt{x-3}} = \dfrac{3}{1} = 3$

(b) $\lim_{x\to 4} \dfrac{x-3-\sqrt{x-3}}{x-4} \cdot \dfrac{(x-3+\sqrt{x-3})}{(x-3+\sqrt{x-3})}$

$$= \lim_{x\to 4} \frac{(x-3)^2 - (x-3)}{(x-4)(x-3+\sqrt{x-3})}$$

$$= \lim_{x\to 4} \frac{(x-3)\cancel{(x-4)}}{\cancel{(x-4)}(x-3+\sqrt{x-3})} = \frac{1}{1+1} = \frac{1}{2}$$

(c) $\lim_{x\to\infty} \dfrac{x-4\sqrt{x}}{3x+\sqrt{x}+4} = \dfrac{1}{3}$

4. *In all parts of this problem,* $f(x) = \sqrt{x} + 2x - 4$ and $g(x) = 2|x-2| + x + 2$.

(a) Determine the domain of the function $y = f(x)$.

Domain: $x \geq 0$

(b) Compute $g(f(1))$. $f(1) = -1$, so $g(f(1)) = g(-1) = 2 \cdot 3 - 1 + 2 = 7$

(c) A line is tangent to the graph of $y = f(x)$ and perpendicular to the line $x + 3y + 7 = 0$. Determine the equation of this line (in the slope-intercept form).

$x + 3y + 7 = 0$, $y = -\frac{1}{3}x - \frac{7}{3}$. Slope of a perp. line: 3

Solve $f'(x) = 3$

$\frac{1}{2\sqrt{x}} + 2 = 3$ $\quad \frac{1}{2\sqrt{x}} = 1$, $\sqrt{x} = \frac{1}{2}$, $x = \frac{1}{4}$

$f(\frac{1}{4}) = \frac{1}{2} + \frac{1}{2} - 4 = -3$

Point: $(\frac{1}{4}, -3)$

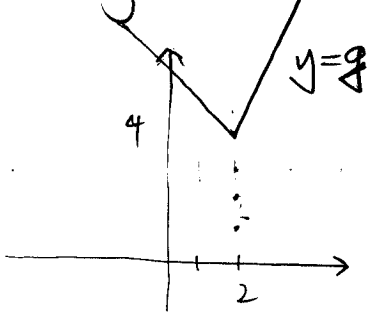
Line: $y + 3 = 3(x - \frac{1}{4})$, $y = 3x - \frac{3}{4} - 3 = 3x - \frac{15}{4}$

(d) Discuss continuity and differentiability of $y = g(x)$, and determine the range of this function.

$$g(x) = \begin{cases} 3x - 2 & x \geq 2 \\ -x + 6 & x < 2 \end{cases}$$

Cont. for all x.

Diff. at all x, except for $x = 2$, where right derivative is 3 and left derivative is -1.



Range: $[4, \infty)$

(i.e. all $y \geq 4$)