Math 16A, Winter 2016.
Mar. 2, 2016.

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________________

ID#: ______________________

Instructions: Each of the first four problems is worth 15 points, while problems 5 and 6 are each worth 20 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. The proctor has been directed not to answer any interpretation questions.

Make sure that you have a total of 8 pages (including this one) with 6 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
5	
6	
TOTAL	

1.

Compute the derivatives of the following two functions. *Do not simplify!*

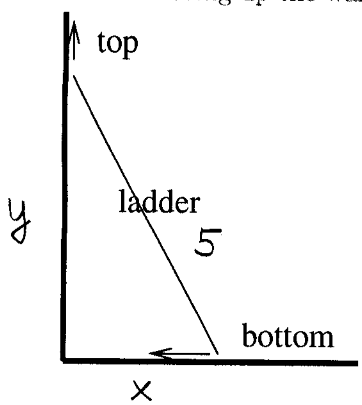
(a) $y = \dfrac{\sqrt{3x+4}}{x-1}$

$$y' = \frac{\frac{1}{2}(3x+4)^{-1/2} \cdot 3(x-1) - \sqrt{3x+4}}{(x-1)^2}$$

(b) $y = (x + \sin x)^{10}$

$$y' = 10(x + \sin x)^9 (1 + \cos x)$$

2. A 5-foot ladder is leaning against the wall. At one instance, the bottom of the ladder is 3 feet from the wall and is pushed towards the wall at the rate of 2 feet per second. At what rate is the top of the ladder moving up the wall at that instance?



$$x^2 + y^2 = 25$$

$$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = 0$$

Plug in $x = 3$, $\frac{dx}{dt} = -2$,

$y = \sqrt{25 - 3^2} = 4$, to

get

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dy}{dt} = -\frac{3}{4}(-2) = \frac{3}{2} \text{ (ft/sec)}$$

3. Find the equation of the tangent line to the curve $y^3 + 4\sqrt{3+y} = x^2$ at the point $(3, 1)$. You may leave the equation of the line in the point-slope form.

$$3y^2 y' + 4 \cdot \tfrac{1}{2}(3+y)^{-1/2} \cdot y' = 2x$$

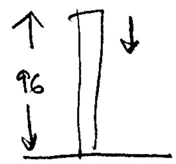
Plug in $x = 3, y = 1$:

$$3y' + 4 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}\, y' = 6$$

$$4y' = 6, \quad y' = \tfrac{3}{2} \leftarrow \text{slope}$$

<u>Line</u>: $y - 1 = \tfrac{3}{2}(x-3)$

4. You are standing on top of a 96 ft tall tower. You throw a rock straight *down* with velocity 16 ft/sec. How fast is the rock traveling in the moment when it hits the ground? Assume the acceleration of the rock is constantly -32 ft/sec^2. (Note that $96 = 6 \cdot 16$.)



$$h = -16t^2 - 16t + 96$$

$$\frac{dh}{dt} = -32t - 16$$

$h = 0$ when $-16t^2 - 16t + 96 = 0$

$$-16\,(t^2 + t - 6) = 0$$

$$-16\,(t-2)(t+3) = 0$$

$$t = 2 \text{ (sec.)}$$

At $t = 2$:

$$\frac{dh}{dt} = -64 - 16 = \underline{\underline{-80}} \text{ (ft/sec)}$$

5. *In all parts of this problem,* $f(x) = \frac{4x}{x^2+1}$.

(a) Determine the domain of the function $y = f(x)$.

All x.

(b) Determine the intervals on which $y = f(x)$ is increasing and the intervals on which it is decreasing. List all local extrema.

$$f'(x) = 4\,\frac{1(x^2+1) - x\cdot 2x}{(x^2+1)^2}$$

$$= 4\,\frac{1-x^2}{(x^2+1)^2} = 4\,\frac{(1-x)(1+x)}{(x^2+1)^2}$$

c. nos. : $x = -1, 1$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
sign of f'	$-$	$+$	$-$
f	$\searrow$	$\nearrow$	$\searrow$

local min at $(-1,-2)$

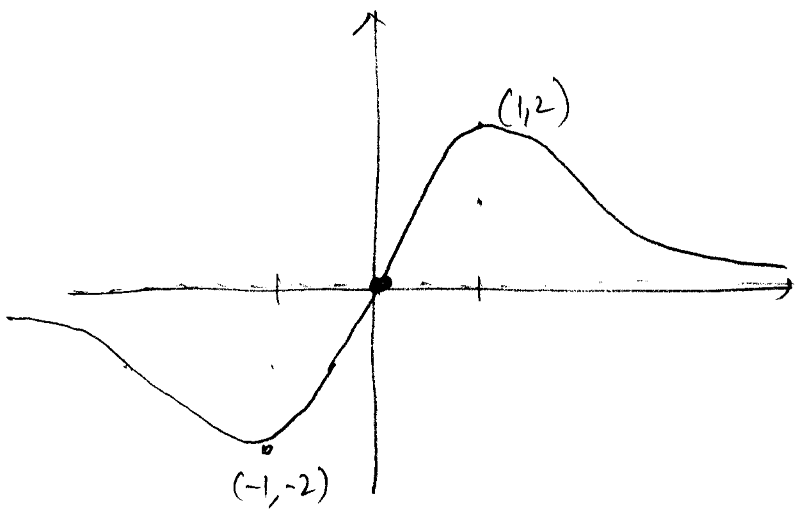
local max at $(1,2)$

(c) Determine the horizontal asymptote of this function.

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{4x}{x^2} = \underline{\underline{0}}$$

$y=0$ h.a.

(d) (Still $f(x) = \frac{4x}{x^2+1}$.) Sketch the graph of $y = f(x)$.



(e) Determine the range of this function.

$[-2, 2]$

(f) Determine the (global) maximum and minumum of this function on the interval $[0, 17]$.

max : $f(1) = 2$

min : $f(0) = 0$

6. A street vendor is about to start selling sandwiches. He can sell 300 sandwiches per day at the price of \$8 (per sandwich) and 200 sandwiches per day at the price of \$10. The only fixed daily cost for running the sandwich stand is the city fee of \$500. Each sandwich costs \$4 to make.

(a) Determine the demand function, assuming that it is linear. That is, express the selling price p in terms of the number x of sandwiches sold. Identify the proper interval for x.

x	p
300	8
200	10

$$p - 10 = \frac{10-8}{200-300}(x-200) = -\frac{1}{50}(x-200)$$

$$= -\frac{1}{50}x + 4$$

$$\underline{\underline{p = -\frac{1}{50}x + 14}} \qquad \underline{\underline{0 \le x \le 700}}$$

$(p=0$ when $x = 50 \cdot 14 = 700)$

(b) Express the vendor's profit P as a function of x.

$$R = xp = -\frac{1}{50}x^2 + 14x$$

$$C = 4x + 500$$

$$P = R - C = -\frac{1}{50}x^2 + 10x - 500$$

(c) Compute the marginal profit and determine intervals on which P increases and intervals on which it decreases.

$$\frac{dP}{dx} = -\frac{1}{25}x + 10 = 0 \quad \text{when} \quad x = 250$$

	$(0, 250)$	$(250, 700)$
$\frac{dP}{dx}$	$+$	$-$
P	↗	↘

(d) To maximize the profit, what is the number of sandwiches the vendor should make and at what price should they be sold?

number : $\underline{\underline{x = 250}}$ (sandwiches)

price : $p = -\frac{1}{50}250 + 14 = -5 + 14 = \underline{\underline{9}}$ (\$)