27. (a) \$2661.667 million   (b) \$15,970.002 million

29. $2\sqrt{x}\ln x - 4\sqrt{x} + C$ **31.** $xe^x - 2e^x + C$

33. $x^2e^{2x} - xe^{2x} + \frac{1}{2}e^{2x} + C$ **35.** \$45,317.31

37. \$432,979.25

39. (a) \$4423.98, \$3934.69, \$3517.56   (b) \$997,629.35

41. \$45,118.84   **43.** $\frac{1}{5}\ln\left|\frac{x}{x+5}\right| + C$

45. $\ln|x-5| + 3\ln|x+2| + C$

47. $x - \frac{25}{8}\ln|x+5| + \frac{9}{8}\ln|x-3| + C$

49. (a) $y = \dfrac{10,000}{1 + 7e^{-0.106873t}}$

(b)

Time, t	0	3	6	12	24
Sales, y	1250	1645	2134	3400	6500

(c) $t \approx 28$ weeks

51. $\sqrt{x^2+25} - 5\ln\left|\dfrac{5+\sqrt{x^2+25}}{x}\right| + C$

53. $\frac{1}{4}\ln\left|\dfrac{x-2}{x+2}\right| + C$ **55.** $\dfrac{8}{3}$

57. $2\sqrt{1+x} + \ln\left|\dfrac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right| + C$

59. $(x-5)^3e^{x-5} - 3(x-5)^2e^{x-5} + 6(x-6)e^{x-5} + C$

61. $\frac{1}{10}\ln\left|\dfrac{x-3}{x+7}\right| + C$

63. $\frac{1}{2}\left[(x-5)\sqrt{(x-5)^2 - 25}\right.$
    $\left. - 25\ln|(x-5) + \sqrt{(x-5)^2 - 25}|\right] + C$

65. 0.705   **67.** 0.741   **69.** 0.376   **71.** 0.289

73. 9.0997   **75.** 0.017   **77.** 1   **79.** Diverges

81. 2   **83.** 2   **85.** (a) \$494,525.28   (b) \$833,333.33

**87.** (a) 0.431   (b) 0.108   (c) 0.013

SAMPLE POST-GRAD EXAM QUESTIONS
(page 454)

1. a   **2.** d   **3.** a   **4.** b   **5.** c

6. d   **7.** b   **8.** a   **9.** b

CHAPTER 7

SECTION 7.1   *(page 462)*

Prerequisite Review

1. $2\sqrt{5}$ **2.** 5   **3.** 8   **4.** 8   **5.** (4, 7)

6. (1, 0)   **7.** (0, 3)   **8.** (−1, 1)

9. $(x-2)^2 + (y-3)^2 = 4$

10. $(x-1)^2 + (y-4)^2 = 25$

1.
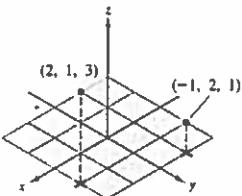
3.
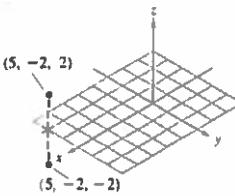
5. $3\sqrt{2}$ **7.** $\sqrt{206}$ **9.** (2, −5, 3)

11. $\left(\frac{1}{2}, \frac{1}{2}, -1\right)$ **13.** (6, −3, 5)   **15.** (1, 2, 1)

17. $3, 3\sqrt{5}, 6$; right triangle

19. $2, 2\sqrt{5}, 2\sqrt{2}$; neither right nor isosceles

21. $x^2 + (y-2)^2 + (z-2)^2 = 4$

23. $\left(x - \frac{3}{2}\right)^2 + (y-2)^2 + (z-1)^2 = \frac{21}{4}$

25. $(x-1)^2 + (y-1)^2 + (z-5)^2 = 9$

27. $(x-1)^2 + (y-3)^2 + z^2 = 10$

29. $(x+2)^2 + (y-1)^2 + (z-1)^2 = 1$

31. Center: $\left(\frac{5}{2}, 0, 0\right)$ **33.** Center: $(1, -3, -4)$
Radius: $\frac{5}{2}$ Radius: 5

**35.** Center: (1, 3, 2)
Radius: $\dfrac{5\sqrt{2}}{2}$

37.
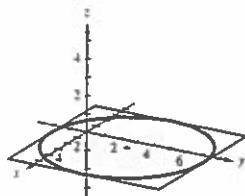
**39.**
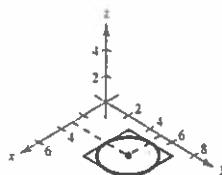
41.
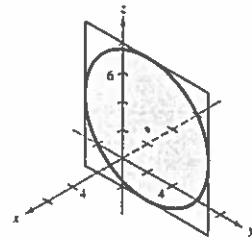
43. (a)
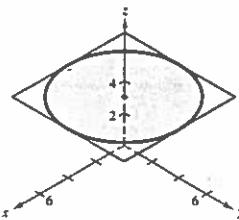
(b)
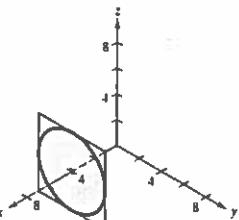
45. (a)
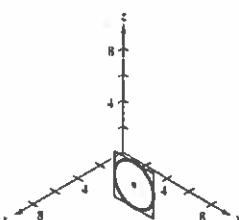
(b)
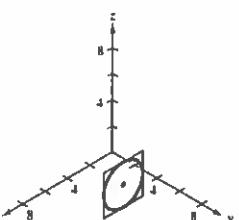
47. $(3, 3, 3)$

SECTION 7.2   (page 472)

Prerequisite Review

1. $(4, 0), (0, 3)$ **2.** $\left(-\frac{4}{3}, 0\right), (0, -8)$

3. $(1, 0), (0, -2)$ **4.** $(-5, 0), (0, -5)$

5. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 + 1 = 0$

6. $(x - 4)^2 + (y + 2)^2 - (z + 3)^2 = 0$

7. $(x + 1)^2 + (y - 1)^2 - z = 0$

8. $(x - 3)^2 + (y + 5)^2 + (z + 13)^2 = 1$

**9.** $x^2 - y^2 + z^2 = \frac{1}{4}$ **10.** $x^2 - y^2 + z^2 = 4$

1.

3.

5.

7.

9.

11.

13. Perpendicular   **15.** Parallel   **17.** Parallel

19. Neither parallel nor perpendicular   **21.** Perpendicular

23. $\dfrac{6\sqrt{14}}{7}$ **25.** $\dfrac{8\sqrt{14}}{7}$ **27.** $\dfrac{13\sqrt{29}}{29}$ **29.** $\dfrac{28\sqrt{29}}{29}$

31. c   **32.** e   **33.** f   **34.** g

35. d   **36.** b   **37.** a   **38.** h

39. Trace in xy-plane $(z = 0)$: $y = x^2$ (parabola)

Trace in plane $y = 1$: $x^2 - z^2 = 1$ (hyperbola)

Trace in yz-plane $(x = 0)$: $y = -z^2$ (parabola)

41. Trace in xy-plane $(z = 0)$: $\dfrac{x^2}{4} + y^2 = 1$ (ellipse)

Trace in xz-plane $(y = 0)$: $\dfrac{x^2}{4} + z^2 = 1$ (ellipse)

Trace in yz-plane $(x = 0)$: $y^2 + z^2 = 1$ (circle)

43. Ellipsoid   **45.** Hyperboloid of one sheet

47. Elliptic paraboloid   **49.** Hyperbolic paraboloid

51. Hyperboloid of two sheets   **53.** Elliptic cone

55. Hyperbolic paraboloid

57.
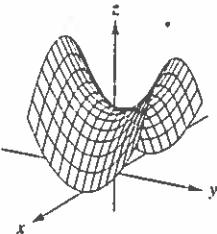
59.
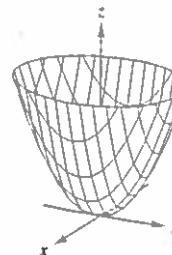
61. $\dfrac{x^2}{3963^2} + \dfrac{y^2}{3963^2} + \dfrac{z^2}{3950^2} = 1$

SECTIO

Prere

1. 11

4. 4

6. $(-c$

7. $[5,$

9. 55.0

1. (a) $\frac{3}{2}$

3. (a) 5

(d) $5e^y$

5. (a) $\frac{2}{3}$

9. (a) \$20

13. (a) x^2

15. Domain

$x^2 + y$

Range:

17. Domain

Range:

19. All poi

21. All poi

23. All poi

25. All poi

27. The hal

29. b   3

33. The lev

parallel

SECTION 7.3   (page 480)

Prerequisite Review

1. 11   2. -16 3. 7

4. 4   5. $(-\infty, \infty)$

6. $(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$

7. $[5, \infty)$ 8. $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

9. 55.0104   10. 6.9165

---

1. (a) $\frac{3}{2}$ (b) $-\frac{1}{4}$ (c) 6 (d) $\frac{5}{y}$ (e) $\frac{x}{2}$ (f) $\frac{5}{t}$

3. (a) 5   (b) $3e^2$ (c) $2e^{-1}$

   (d) $5e^y$ (e) xe^2 (f) te^t

5. (a) $\frac{2}{3}$ (b) 0   7. (a) 90π (b) 50π

9. (a) \$20,655   (b) \$1,397,673   11. (a) 0   (b) 6

13. (a) $x^2 + 2x\,\Delta x + (\Delta x)^2 - 2y$ (b) $-2, \Delta y \neq 0$

15. Domain: all points (x, y) inside and on the circle
    $x^2 + y^2 = 16$
   Range: $[0, 4]$

17. Domain: all points (x, y) such that $y \neq 0$
    Range: $(0, \infty)$

19. All points inside and on the ellipse $9x^2 + y^2 = 9$

21. All points (x, y) such that $y \neq 0$

23. All points (x, y) such that $x \neq 0$ nor $y \neq 0$

25. All points (x, y) such that $y \geq 0$

27. The half-plane below the line $y = -x + 4$

29. b   30. d   31. a   32. c

33. The level curves are   35. The level curves are circles.
    parallel lines.

---
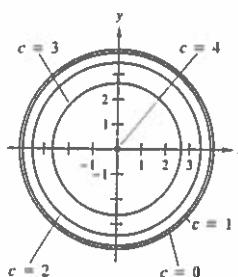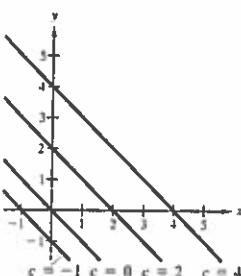
37. The level curves are hyperbolas.
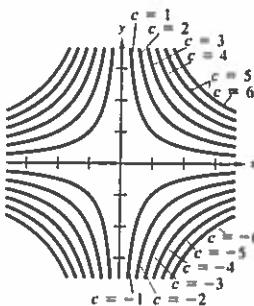
39. The level curves are circles.
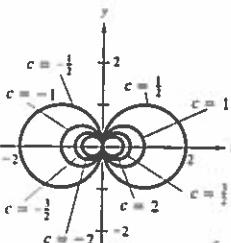
41. 135,540 units   43. \$21,960

45. (a) \$13,250.00   (b) \$15,925.00

47.

R \ I	0	0.03	0.05
0	\$2593.74	\$1929.99	\$1592.33
0.28	\$2004.23	\$1491.34	\$1230.42
0.35	\$1877.14	\$1396.77	\$1152.40

49. (a) The different colors represent various amplitudes.

    (b) No, the level curves are uneven and sporadically spaced.

SECTION 7.4   (page 491)

Prerequisite Review

1. $\dfrac{x}{\sqrt{x^2 + 3}}$ 2. $-6x(3 - x^2)^2$ 3. $e^{2t+1}(2t + 1)$

4. $\dfrac{e^{2x}(2 - 3e^{2x})}{\sqrt{1 - e^{2x}}}$ 5. $-\dfrac{2}{3 - 2x}$ 6. $\dfrac{3(t^2 - 2)}{2t(t^2 - 6)}$

7. $-\dfrac{10x}{(4x - 1)^3}$ 8. $-\dfrac{(x + 2)^2(x^2 + 8x + 27)}{(x^2 - 9)^3}$

9. $f'(2) = 8$ 10. $g'(2) = \frac{7}{2}$

---

1. $f_x(x, y) = 2$ 3. $f_x(x, y) = \dfrac{5}{2\sqrt{x}}$

   $f_y(x, y) = -3$ $f_y(x, y) = -12y$

5. $f_x(x, y) = \dfrac{1}{y}$

$f_y(x, y) = -\dfrac{x}{y^2}$

7. $f_x(x, y) = \dfrac{x}{\sqrt{x^2 + y^2}}$

$f_y(x, y) = \dfrac{y}{\sqrt{x^2 + y^2}}$

9. $\dfrac{\partial z}{\partial x} = 2xe^{2y}$

$\dfrac{\partial z}{\partial y} = 2x^2 e^{2y}$

11. $h_x(x, y) = -2xe^{-(x^2+y^2)}$

$h_y(x, y) = -2ye^{-(x^2+y^2)}$

13. $\dfrac{\partial z}{\partial x} = \dfrac{3y - x}{x^2 - y^2}$

$\dfrac{\partial z}{\partial y} = \dfrac{y - 3x}{x^2 - y^2}$

15. $f_x(x, y) = 3xye^{x-y}(2 + x)$

17. $g_x(x, y) = 3y^2 e^{y-x}(1 - x)$ **19.** 9

21. $f_x(x, y) = 6x + y, 13$; $f_y(x, y) = x - 2y, 0$

23. $f_x(x, y) = 3ye^{3xy}, 12$; $f_y(x, y) = 3xe^{3xy}, 0$

25. $f_x(x, y) = -\dfrac{y^2}{(x - y)^2}, -\dfrac{1}{4}$

$f_y(x, y) = \dfrac{x^2}{(x - y)^2}, \dfrac{1}{4}$

27. $f_x(x, y) = \dfrac{2x}{x^2 + y^2}, 2$

$f_y(x, y) = \dfrac{2y}{x^2 + y^2}, 0$

29. $w_x = 6xy - 5yz$

$w_y = 3x^2 - 5xz + 10z^2$

$w_z = -5xy + 20yz$

31. $w_x = \dfrac{y(y + z)}{(x + y + z)^2}$

$w_y = \dfrac{x(x + z)}{(x + y + z)^2}$

$w_z = -\dfrac{xy}{(x + y + z)^2}$

33. $w_x = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}, \dfrac{2}{3}$

$w_y = \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}, -\dfrac{1}{3}$

$w_z = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}, \dfrac{2}{3}$

35. $w_x = \dfrac{x}{x^2 + y^2 + z^2}, \dfrac{3}{25}$

$w_y = \dfrac{y}{x^2 + y^2 + z^2}, 0$

$w_z = \dfrac{z}{x^2 + y^2 + z^2}, \dfrac{4}{25}$

37. $w_x = 2z^2 + 3yz, 2$

$w_y = 3xz - 12yz, 30$

$w_z = 4xz + 3xy - 6y^2, -1$

39. $(-6, 4)$ **41.** $(1, 1)$

43. (a) 2   (b) -3 **45.** (a) 6   (b) -18

47. (a) $-\frac{3}{4}$ (b) 0   **49.** (a) -2 (b) -2

51. $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = -2$ **53.** $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = ye^{2xy}$

55. $\dfrac{\partial^2 z}{\partial x^2} = 6x$

$\dfrac{\partial^2 z}{\partial y^2} = -8$

$\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = 0$

57. $\dfrac{\partial^2 z}{\partial x^2} = 24x$

$\dfrac{\partial^2 z}{\partial y^2} = 6x - 24y$

$\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = 6y$

59. $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{2y^2}{(x - y)^3}$

$\dfrac{\partial^2 z}{\partial y^2} = \dfrac{2x^2}{(x - y)^3}$

$\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y}$

$= -\dfrac{2xy}{(x - y)^3}$

61. $\dfrac{\partial^2 z}{\partial x^2} = 0$

$\dfrac{\partial^2 z}{\partial y^2} = 2xe^{-y^2}(2y^2 - 1)$

$\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = -2ye^{-y^2}$

63. $f_{xx}(x, y) = 12x^2 - 6y^2, 12$

$f_{xy}(x, y) = -12xy, 0$

$f_{yy}(x, y) = -6x^2 + 2, -4$

$f_{yx}(x, y) = -12xy, 0$

65. $f_{xx}(x, y) = -\dfrac{1}{(x - y)^2}, -1$

$f_{xy}(x, y) = \dfrac{1}{(x - y)^2}, 1$

$f_{yy}(x, y) = -\dfrac{1}{(x - y)^2}, -1$

$f_{yx}(x, y) = \dfrac{1}{(x - y)^2}, 1$

67. At $(120, 160)$, $\dfrac{\partial C}{\partial x} \approx 154.77$

At $(120, 160)$, $\dfrac{\partial C}{\partial y} \approx 193.33$

69. (a) $f_x(x, y) = 60\left(\dfrac{y}{x}\right)^{0.4}$, $f_x(1000, 500) \approx 45.47$

(b) $f_y(x, y) = 40\left(\dfrac{x}{y}\right)^{0.6}$, $f_x(1000, 500) \approx 60.63$

71. (a) Complementary   (b) Substitute

(c) Complementary

73. An increase in either price will cause a decrease in the number of applicants.

75. (a) At $t = 90°$ and $h = 0.80$, $\dfrac{\partial A}{\partial t} = 1.845$.

At $t = 90°$ and $h = 0.80$, $\dfrac{\partial A}{\partial h} = 29.3$.

(b) The humidity has a greater effect since the coefficient of h is greater.

77. Answers will vary.

SECTI

Pre

1.

5.

8.

9.

10.

11.

12.

13.

14.

SECTION 7.5   (page 501)

Prerequisite Review

1. $(3, 2)$ **2.** $(11, 6)$ **3.** $(1, 4)$ **4.** $(4, 4)$

5. $(5, 2)$ **6.** $(3, -2)$ **7.** $(0, 0), (-1, 0)$

8. $(-2, 0), (2, -2)$

9. $\dfrac{\partial z}{\partial x} = 12x^2$ $\dfrac{\partial^2 z}{\partial y^2} = -6$

$\dfrac{\partial z}{\partial y} = -6y$ $\dfrac{\partial^2 z}{\partial x \partial y} = 0$

$\dfrac{\partial^2 z}{\partial x^2} = 24x$ $\dfrac{\partial^2 z}{\partial y \partial x} = 0$

10. $\dfrac{\partial z}{\partial x} = 10x^4$ $\dfrac{\partial^2 z}{\partial y^2} = -6y$

$\dfrac{\partial z}{\partial y} = -3y^2$ $\dfrac{\partial^2 z}{\partial x \partial y} = 0$

$\dfrac{\partial^2 z}{\partial x^2} = 40x^3$ $\dfrac{\partial^2 z}{\partial y \partial x} = 0$

11. $\dfrac{\partial z}{\partial x} = 4x^3 - \dfrac{\sqrt{xy}}{2x}$ $\dfrac{\partial^2 z}{\partial y^2} = \dfrac{\sqrt{xy}}{4y^2}$

$\dfrac{\partial z}{\partial y} = -\dfrac{\sqrt{xy}}{2y} + 2$ $\dfrac{\partial^2 z}{\partial x \partial y} = -\dfrac{\sqrt{xy}}{4xy}$

$\dfrac{\partial^2 z}{\partial x^2} = 12x^2 + \dfrac{\sqrt{xy}}{4x^2}$ $\dfrac{\partial^2 z}{\partial y \partial x} = -\dfrac{\sqrt{xy}}{4xy}$

12. $\dfrac{\partial z}{\partial x} = 4x - 3y$ $\dfrac{\partial^2 z}{\partial y^2} = 2$

$\dfrac{\partial z}{\partial y} = 2y - 3x$ $\dfrac{\partial^2 z}{\partial x \partial y} = -3$

$\dfrac{\partial^2 z}{\partial x^2} = 4$ $\dfrac{\partial^2 z}{\partial y \partial x} = -3$

13. $\dfrac{\partial z}{\partial x} = y^3 e^{xy^2}$ $\dfrac{\partial^2 z}{\partial y^2} = 4x^2 y^3 e^{xy^2} + 6xy e^{xy^2}$

$\dfrac{\partial z}{\partial y} = 2xy e^{xy^2} + e^{xy^2}$ $\dfrac{\partial^2 z}{\partial x \partial y} = 2xy^4 e^{xy^2} + 3y^2 e^{xy^2}$

$\dfrac{\partial^2 z}{\partial x^2} = y^5 e^{xy^2}$ $\dfrac{\partial^2 z}{\partial y \partial x} = 2xy^4 e^{xy^2} + 3y^2 e^{xy^2}$

14. $\dfrac{\partial z}{\partial x} = e^{xy}(xy + 1)$ $\dfrac{\partial^2 z}{\partial y^2} = x^3 e^{xy}$

$\dfrac{\partial z}{\partial y} = x^2 e^{xy}$ $\dfrac{\partial^2 z}{\partial x \partial y} = xe^{xy}(xy + 2)$

$\dfrac{\partial^2 z}{\partial x^2} = ye^{xy}(xy + 2)$ $\dfrac{\partial^2 z}{\partial y \partial x} = xe^{xy}(xy + 2)$

1. Critical point: $(-2, -4)$

No relative extrema

$(-2, -4, 1)$ is a saddle point.

3. Critical point: $(0, 0)$

Relative minimum: $(0, 0, 1)$

5. Relative minimum: $(1, 3, 0)$

7. Relative minimum: $(-1, 1, -4)$

9. Relative maximum: $(8, 16, 74)$

11. Relative minimum: $(2, 1, -7)$

13. Saddle point: $(-2, -2, -8)$

15. Saddle point: $(0, 0, 0)$

17. Relative maxima: $(0, \pm 1, 4)$

Relative minimum: $(0, 0, 0)$

Saddle points: $(\pm 1, 0, 1)$

19. Saddle point: $(0, 0, 1)$

21. Insufficient information

23. $f(x_0, y_0)$ is a saddle point.

25. Relative minima: $(a, 0, 0), (0, b, 0)$

Second-Partials Test fails at $(a, 0)$ and $(0, b)$.

27. Saddle point: $(0, 0, 0)$

Second-Partials Test fails at $(0, 0)$.

29. Relative minimum: $(0, 0, 0)$

Second-Partials Test fails at $(0, 0)$.

31. Relative minimum: $(1, -3, 0)$

33. $10, 10, 10$ **35.** $10, 10, 10$

37. $x_1 = 3, x_2 = 6$ **39.** $p_1 = 2500, p_2 = 3000$

41. $x_1 \approx 94, x_2 \approx 157$

43. 48 inches $\times$ 24 inches $\times$ 24 inches

45. Proof

47. $D_x(x, y) = 2x - 18 + 2y$

$D_y(x, y) = 4y - 24 + 2x$

To minimize the duration of the infection, 600 mg of the first drug and 300 mg of the second drug are necessary.

49. True

51. False. The origin is a minimum.

SECTION 7.6   (page 511)

Prerequisite Review

1. $\left(\frac{7}{8}, \frac{1}{12}\right)$ **2.** $\left(-\frac{1}{24}, -\frac{7}{8}\right)$ **3.** $\left(\frac{55}{12}, -\frac{25}{12}\right)$

4. $\left(\frac{22}{23}, -\frac{3}{23}\right)$ **5.** $\left(\frac{5}{3}, \frac{1}{3}, 0\right)$ **6.** $\left(\frac{14}{19}, -\frac{10}{19}, -\frac{32}{57}\right)$

7. $f_x = 2xy + y^2$ **8.** $f_x = 50y^2(x + y)$
$\quad f_y = x^2 + 2xy \qquad f_y = 50y(x + y)(x + 2y)$

9. $f_x = 3x^2 - 4xy + yz$ **10.** $f_x = yz + z^2$
$\quad f_y = -2x^2 + xz \qquad\quad f_y = xz + z^2$
$\quad f_z = xy \qquad\qquad\qquad f_z = xy + 2xz + 2yz$

1. $f(5, 5) = 25$ **3.** $f(2, 2) = 8$

5. $\bar{f}\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right) = \frac{1}{4}$ **7.** $f\left(\frac{25}{2}, \frac{25}{2}\right) = 231.25$

9. $f(1, 1) = 2$ **11.** $f(2, 2) = e^4$ **13.** $f(9, 6, 9) = 432$₁

15. $f\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3}$ **17.** $f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \sqrt{3}$

19. $f\left(\frac{4}{15}, \frac{8}{15}, \frac{4}{15}, \frac{2}{15}\right) = \frac{8}{15}$ **21.** $f(9, 6, 9) = 486$

23. $f\left(\sqrt{\frac{10}{3}}, \frac{1}{2}\sqrt{\frac{10}{3}}, \sqrt{\frac{5}{3}}\right) = \frac{5\sqrt{15}}{9}$

25. $x = 4, y = \frac{2}{3}, z = 2$ **27.** $40, 40, 40$ **29.** $\frac{S}{3}, \frac{S}{3}, \frac{S}{3}$

31. $\sqrt{5}$ **33.** $\sqrt{3}$

35. 36 inches $\times$ 18 inches $\times$ 18 inches

37. Length = width = $\sqrt[3]{360} \approx 7.1$ feet

Height $= \dfrac{480}{360^{2/3}} \approx 9.5$ feet

39. $x_1 = 752.5, x_2 = 1247.5$

To minimize cost, let $x_1 = 753$ units and $x_2 = 1247$ units.

41. (a) $x = 50\sqrt{2} \approx 71$ (b) Answers will vary.
$\quad y = 200\sqrt{2} \approx 283$

43. (a) $f\left(\frac{3125}{6}, \frac{6250}{3}\right) \approx 147{,}314$ (b) 1.473 (c) $184{,}142$

45. $x = \sqrt[3]{0.065} \approx 0.402$ liter
$\quad y = \frac{1}{2}\sqrt[3]{0.065} \approx 0.201$ liter
$\quad z = \frac{1}{3}\sqrt[3]{0.065} \approx 0.134$ liter

47. (a) 50 feet $\times$ 120 feet   (b) $\$2400$

49. Stock G: $\$138{,}333.33$
Stock P: $\$7000.00$
Stock S: $\$154{,}666.67$

51. Answers will vary.

SECTION 7.7   (page 521)

Prerequisite Review

1. 5.0225 **2.** 0.0189

3. $S_a = 2a - 4 - 4b$ **4.** $S_a = 8a - 6 - 2b$
$\quad S_b = 12b - 8 - 4a \qquad S_b = 18b - 4 - 2a$

5. 15 **6.** 42 **7.** $\frac{25}{12}$

8. 14 **9.** 31 **10.** 95

1. (a) $y = \frac{3}{4}x + \frac{4}{3}$ (b) $\frac{1}{6}$

3. (a) $y = -2x + 4$ (b) 2

5. $y = x + \frac{2}{3}$ **7.** $y = -2.3x - 0.9$ **9.** $y = \frac{7}{10}x + \frac{7}{5}$

11. $y = x + 4$ **13.** $y = -\frac{13}{20}x + \frac{7}{4}$

15. $y = \frac{37}{43}x + \frac{7}{43}$ **17.** $y = -\frac{175}{148}x + \frac{945}{148}$

19. $y = \frac{3}{7}x^2 + \frac{6}{5}x + \frac{26}{35}$ **21.** $y = 1.25x^2 - 1.75x + 0.25$
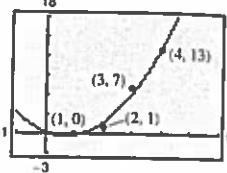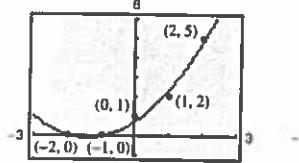
23. Linear: $y = 1.4x + 6$
Quadratic: $y = 0.12x^2 + 1.7x + 6$
The quadratic model is a better fit.

25. Linear: $y = -68.9x + 754$
Quadratic: $y = 2.82x^2 - 83.0x + 763$
The quadratic model is a better fit.

27. (a) $y = -240x + 685$ (b) 349 (c) $\$0.77$

29. (a) $y = 13.8x + 22.1$ (b) 44.18 bushels/acre

31. (a) $y = -0.48t + 19.74$; In 2010, $y \approx 0.54$ deaths
(b) $y = 0.0027t^2 - 0.51t + 19.03$
In 2010, $y \approx 2.95$ deaths

33. (a) $y = -\frac{25}{112}x^2 + \frac{541}{56}x - \frac{25}{14}$ (b) 40.9 miles per hour

35. Linear: $y = 3.757x + 9.03$
Quadratic: $y = 0.006x^2 + 3.625x + 9.43$
Either model is a good fit for the data.

37. Quadratic: $y = -0.087x^2 + 2.82x + 0.4$

39.

Po
r

43. y
se

45. Tr

SECT

P

1

6

11

13

15

1. $\frac{3x}{2}$

7. $\frac{1}{2}y$

13. 36

39.
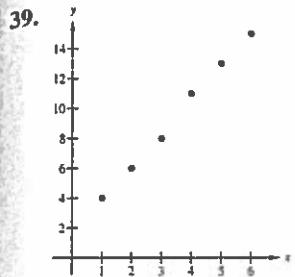
Positive correlation,
$r = 0.9981$

41.
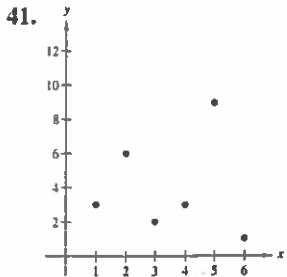

No correlation, $r = 0$

43. $y = -49.95t^2 + 4442.6t - 41,941$, where $t = 20$ represents 20-year-olds; $\approx \$43,291$

**45.** True   **47.** True

SECTION 7.8   (page 530)

Prerequisite Review

1. 1   **2.** 6   **3.** 42   **4.** $\frac{1}{2}$ **5.** $\frac{19}{4}$

6. $\frac{16}{3}$ **7.** $\frac{1}{7}$ **8.** 4   **9.** $\ln 5$ **10.** $\ln|e - 1|$

11. $\frac{e}{2}(e^4 - 1) \approx 72.8474$ **12.** $\frac{1}{2}\left(1 - \frac{1}{e^2}\right)$

**13.**

14.

15.

16.
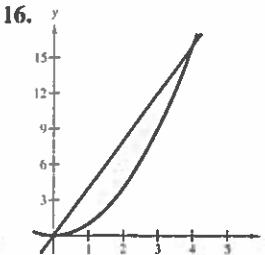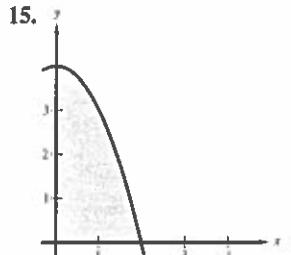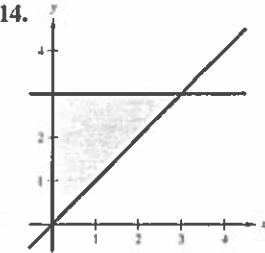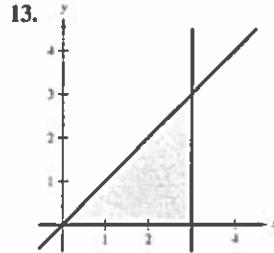
1. $\dfrac{3x^2}{2}$ **3.** $y \ln|2y|$ **5.** $\dfrac{x^2}{2}(9 - x^2)$

7. $\frac{1}{2}y[(\ln y)^2 - y^2]$ **9.** $x^2(1 - e^{-x^2} - x^2 e^{-x^2})$ **11.** 1

13. 36   **15.** $\frac{2}{3}$ **17.** $\frac{20}{3}$ **19.** 5   **21.** $\frac{16}{3}$ **23.** 4

25.
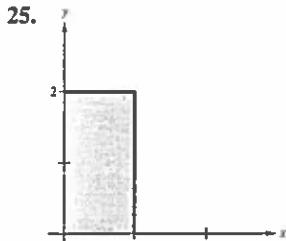
$$\int_0^1 \int_0^2 dy\, dx = \int_0^2 \int_0^1 dx\, dy = 2$$

27.
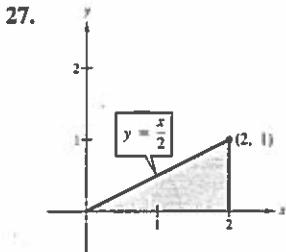
$$\int_0^1 \int_{2y}^2 dx\, dy = \int_0^2 \int_0^{x/2} dy\, dx = 1$$

29.
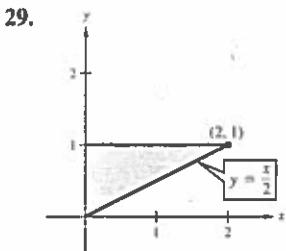
$$\int_0^2 \int_{x/2}^1 dy\, dx = \int_0^1 \int_0^{2y} dx\, dy = 1$$

31.
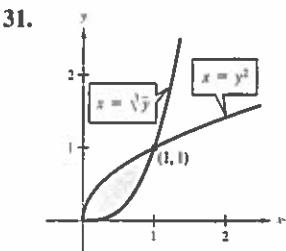
$$\int_0^1 \int_{y^2}^{\sqrt[3]{y}} dx\, dy = \int_0^1 \int_{x^3}^{\sqrt{x}} dy\, dx = \frac{5}{12}$$

33. $\frac{1}{2}(e^9 - 1) \approx 4051.042$ **35.** 24   **37.** $\frac{16}{3}$

39. $\frac{8}{3}$ **41.** $\frac{500}{3}$ **43.** $\frac{45}{14}$ **45.** 2   **47.** 0.6588

49. 8.1747   **51.** 0.4521   **53.** 1.1190

55. False, because $dA = dy\, dx = dx\, dy$, it doesn't matter in what order the integration is performed.

SECTION 7.9   *(page 538)*

Prerequisite Review

1.
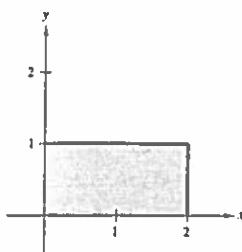
2.
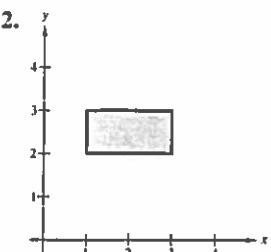
3.
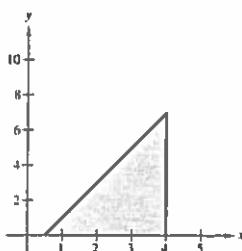
4.
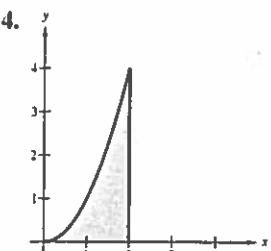
5. 1   **6.** 6   **7.** $\frac{1}{3}$ **8.** $\frac{40}{3}$

9. $\frac{28}{3}$ **10.** $\frac{7}{6}$

1.

10

3.

$\frac{1}{54}$

**5.**

$\frac{1}{3}$

7.

πa^2
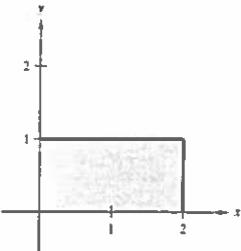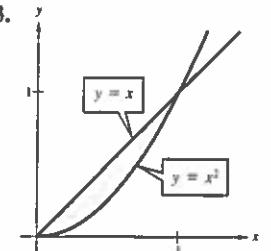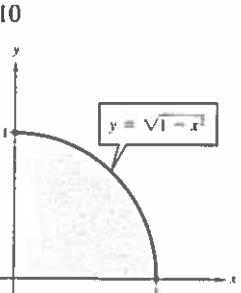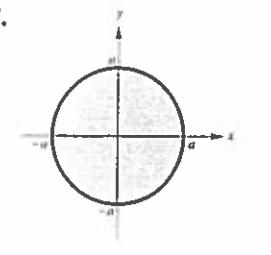
9. $\int_0^3 \int_0^5 xy\, dy\, dx = \int_0^5 \int_0^3 xy\, dx\, dy = \dfrac{225}{4}$

11. $\int_0^2 \int_x^{2x} \dfrac{y}{x^2+y^2}\, dy\, dx = \int_0^2 \int_{y/2}^y \dfrac{y}{x^2+y^2}\, dx\, dy$

$\qquad + \int_2^4 \int_{y/2}^2 \dfrac{y}{x^2+y^2}\, dx\, dy = \ln\dfrac{5}{2}$

13. $\int_0^{1/2} \int_0^{2x} e^{-x^2}\, dy\, dx = 0.2212$ **15.** 4

17. 22.5   **19.** 12   **21.** $\frac{3}{8}$ **23.** $\frac{40}{3}$

25. $\frac{1}{3}$ **27.** 4   **29.** $\frac{32}{3}$ **31.** 10,000

33. 2   **35.** $\frac{8}{3}$ **37.** \$75,125

REVIEW EXERCISES FOR CHAPTER 7
(page 544)

1.   (2, −1, 4)   **3.** $\sqrt{110}$ **5.** (−1, 4, 6)

(−1, 3, −3)
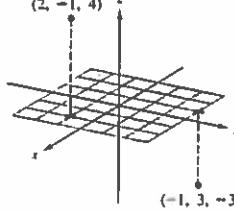
7. $x^2 + (y-1)^2 + z^2 = 25$

9. $(x-4)^2 + (y-6)^2 + (z-1)^2 = 6$

11. Center: $(-2, 1, 4)$; radius: 4

13.

15.

17.

19. Sphere   **21.** Ellipsoid   **23.** Elliptic Paraboloid

25. Top half of a circular cone

27. (a) 18   (b) 0   (c) −245   (d) −32

29. The domain is the set of all points inside or on the circle $x^2 + y^2 = 1$, and the range is [0, 1].

31. The

33. Th

35. (

37.

43.

47.

51.

53.

55

5

31. The level curves are lines of slope $-\frac{2}{5}$.
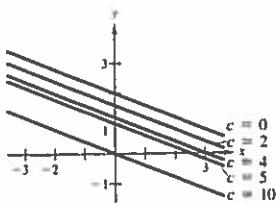
33. The level curves are hyperbolas.
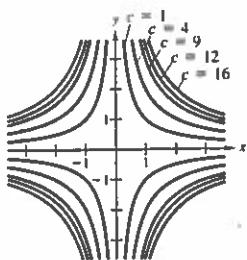
35. (a) As the color darkens from light green to dark green, the average yearly precipitation increases.

(b) The small eastern portion containing Davenport

(c) The northwestern portion containing Sioux City

37. Southwest   **39.** \$2.50   **41.** $f_x = 2xy + 3y + 2$

$f_y = x^2 + 3x - 5$

43. $z_x = 12x\sqrt{y} + \frac{3}{2}\sqrt{\frac{y}{x}} - 7y$ **45.** $f_x = \dfrac{2}{2x + 3y}$

$z_y = \frac{3x^2}{\sqrt{y}} + \frac{3}{2}\sqrt{\frac{x}{y}} - 7x$ $f_y = \dfrac{3}{2x + 3y}$

47. $f_x = 2xe^y - y^2e^x$ **49.** $w_x = yz^2$

$f_y = x^2e^y - 2ye^x$ $w_y = xz^2$

$w_z = 2xyz$

51. (a) $z_x = 3$ (b) $z_y = -4$

53. (a) $z_x = -2x$ (b) $z_y = -2y$

At $(1, 2, 3)$, $z_x = -2$.   At $(1, 2, 3)$, $z_y = -4$.

55. $f_{xx} = 6x$

$f_{yy} = -8x + 6y$

$f_{xy} = f_{yx} = -8y$

57. $f_{xx} = \dfrac{y^2 - 64}{(64 - x^2 - y^2)^{3/2}}$

$f_{yy} = \dfrac{x^2 - 64}{(64 - x^2 - y^2)^{3/2}}$

$f_{xy} = f_{yx} = \dfrac{-xy}{(64 - x^2 - y^2)^{3/2}}$

59. $C_x(250, 175) \approx 99.70$

$C_y(250, 175) \approx 140.01$

61. (a) $A_w = 43.095w^{-0.575}h^{0.725}$

$A_h = 73.515w^{0.425}h^{-0.275}$

(b) ≈ 47.35;

The surface area of an average human body increases approximately 47.35 square centimeters per pound for a human who weighs 180 pounds and is 70 inches tall.

63. Relative minimum: $(x, -x, 0)$

65. Saddle point: $\left(\frac{3}{2}, -\frac{3}{2}, \frac{27}{2}\right)$

67. Relative minimum: $\left(\frac{1}{6}, \frac{1}{12}, -\frac{1}{432}\right)$

Saddle point: $(0, 0, 0)$

69. Relative minimum: $(1, 1, -2)$

Relative maximum: $(-1, -1, 6)$

Saddle points: $(1, -1, 2), (-1, 1, 2)$

71. (a) $R = -x_1^2 - \frac{1}{2}x_2^2 + 100x_1 + 200x_2$

(b) $x_1 = 50, x_2 = 200$ (c) \$22,500.00

73. At $\left(\frac{4}{3}, \frac{1}{3}\right)$, the relative maximum is $\frac{16}{27}$.

At $(0, 1)$, the relative minimum is 0.

75. At $\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$, the relative maximum is $\frac{32}{27}$.

77. At $\left(\frac{4}{3}, \frac{10}{3}, \frac{14}{3}\right)$, the relative minimum is $34\frac{2}{3}$.

79. $x = 2\sqrt{2}, y = 2\sqrt{2}, z = \sqrt{2}$

81. $f(49.4, 253) \approx 13,202$

83. (a) $y = \frac{60}{59}x - \frac{15}{59}$ (b) 2.746

85. (a) $y = 14x + 19$ (b) 21.8 bushels/acre

87. $y = 1.71x^2 - 2.57x + 5.56$
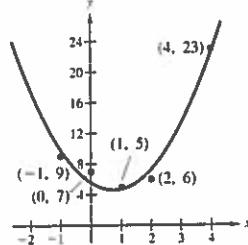
89. $\frac{29}{6}$ **91.** $\frac{7}{4}$

93. $\displaystyle\int_{-2}^{2}\int_{5}^{9-x^2} dy\, dx = \int_{5}^{9}\int_{-\sqrt{9-y}}^{\sqrt{9-y}} dx\, dy = \frac{32}{3}$

95. $\displaystyle\int_{-3}^{6}\int_{1/3(x+3)}^{\sqrt{x+3}} dy\, dx = \int_{0}^{3}\int_{3y-3}^{y^2-3} dx\, dy = \frac{9}{2}$

97. $\frac{4096}{9}$ **99.** 0.0833 mile

SAMPLE POST-GRAD EXAM QUESTIONS
(page 548)

1. b   2. b   3. c   4. d

5. a   6. c   7. b

3. If the essays are numbered 1, 2, 3, and 4,

$S = \{123, 124, 134, 234\}$.

5. $S = \{0, 1, 2, 3\}$

7.

x	0	1	2	3
$n(x)$	1	3	3	1

9.
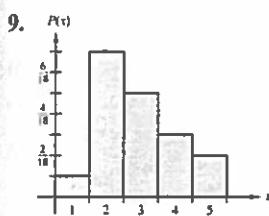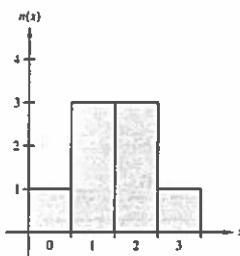
(a) $\frac{5}{6}$ (b) $\frac{5}{9}$

11. (a) $\frac{5}{36}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $\frac{1}{36}$

13. 19.5   15. (a) 20.5   (b) $15,375

17. $V(x) = 218{,}243.7500$ 19. $V(x) \approx 1.1611$

$\sigma \approx 467.1657$ $\sigma \approx 1.0775$

21.
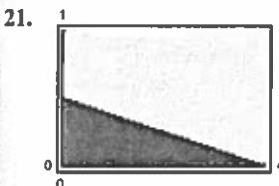
$$\int_0^4 \frac{1}{8}(4-x)\,dx = \left[\frac{1}{2}x - \frac{x^2}{16}\right]_0^4 = (2-1) = 1$$

23.
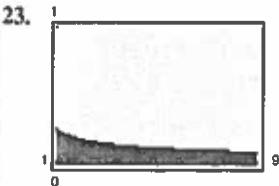
$$\int_1^9 \frac{1}{4\sqrt{x}}\,dx = \frac{1}{4}\left[2\sqrt{x}\right]_1^9 = 1$$

25. $\frac{9}{25}$ 27. $\frac{2}{3}$ 29. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

31. $\frac{1}{2}$ 33. 2.5   35. 6

37. Variance: $\frac{9}{20}$

Standard deviation: $\dfrac{3}{2\sqrt{5}}$

39. Variance: 4

Standard deviation: 2

41. $\frac{1}{2}$ 43. 2.7726   45. (a) 0.4866   (b) 0.2498

47. 0.00383   49. 0.3829

51.
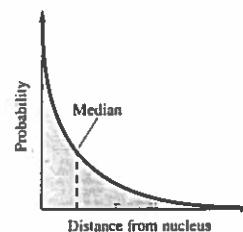
SAMPLE POST-GRAD EXAM QUESTIONS
(page 650)

1. c   2. b   3. c   4. e   5. a   6. c

CHAPTER 10
SECTION 10.1   (page 658)

Prerequisite Review

1. 0   2. 0   3. 2   4. ∞ 5. 0   6. 0

7. $\dfrac{n-2}{n}$ 8. $\dfrac{n-3}{n-4}$ 9. $\dfrac{3n^2+1}{n^3}$

10. $\dfrac{2n+1}{(n-1)(n+2)}$

---

1. 2, 4, 8, 16, 32   3. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ 5. 3, $\frac{9}{2}, \frac{27}{6}, \frac{81}{24}, \frac{243}{120}$

7. $-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}$ 9. Converges to 0

11. Converges to 1   13. Converges to $\frac{1}{2}$ 15. Diverges

17. Converges to 0   19. Diverges   21. Converges to 3

23. Converges to 0   25. Diverges   27. Diverges

29. Diverges   31. $3n - 2$ 33. $5n - 6$

35. $\dfrac{n+1}{n+2}$ 37. $\dfrac{(-1)^{n-1}}{2^{n-2}}$ 39. $\dfrac{n+1}{n}$ 41. $2(-1)^n$

43. $\dfrac{(-1)^n x^n}{n}$ 45. 2, 5, 8, 11, 14, 17, . . .

47. 1, $\frac{5}{3}, \frac{7}{3}$, 3, $\frac{11}{3}, \frac{13}{3}$, . . .   49. 3, $-\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, -\frac{3}{32}$, . . .

51. 2, 6, 18, 54, 162, 486, . . .   53. Geometric, $20\left(\frac{1}{2}\right)^{n-1}$

55. Arithmetic, $\frac{2}{3}n + 2$ **57.** $\frac{3n+1}{4n}$

59. $9045.00, $9090.23, $9135.68, $9181.35, $9227.26, $9273.40, $9319.76, $9366.36, $9413.20, $9460.26

61. (a)

| Year	1	2	3
Balance	$2200	$4620	$7282

Year	4	5	6
Balance	$10,210.20	$13,431.22	$16,974.34

    (b) $126,005.00   (c) $973,703.62

63. $S_6 = 240, S_7 = 440, S_8 = 810, S_9 = 1490, S_{10} = 2740$

65. (a) 2.40, 2.50, 2.77, 3.16, 3.58, 4.00, 4.38, 4.71, 4.98, 5.22, 5.41, 5.57, 5.71, 5.82, 5.92, 6.00

    (b)
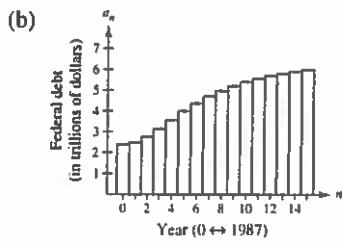
67. (a) $S_1 = 1$ (b) $S_{20} = 2870$
       $S_2 = 5$
       $S_3 = 14$
       $S_4 = 30$
 $S_5 = 55$

69. (a) $1.3(0.85)^n$ billion dollars

    (b)

Year	1	2
Budget amount	$1.105 billion	$0.939 billion

Year	3	4
Budget amount	$0.798 billion	$0.679 billion

    (c) Converges to 0

71. $2095

73. (a) $a_n = -0.265625n^3 + 5.32271n^2 - 9.7470n + 90.192$
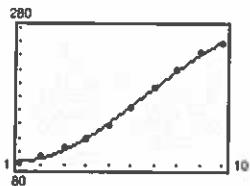
    (b) $157.7 billion

75.
$$a_1 = 2$$
$$a_{10} = 2.5937$$
$$a_{100} = 2.7048$$
$$a_{1000} = 2.7169$$
$$a_{10,000} = 2.7181$$

SECTION 10.2   (page 670)

Prerequisite Review

1. $\frac{77}{60}$ **2.** $\frac{73}{24}$ **3.** $\frac{31}{16}$ **4.** $\frac{40}{9}$ **5.** $\frac{21}{8}$
6. $\frac{31}{32}$ **7.** $\frac{3}{4}$ **8.** 0   **9.** 1   **10.** $\frac{1}{2}$

1. $S_1 = 1$
    $S_2 = \frac{5}{4} = 1.25$
    $S_3 = \frac{49}{36} \approx 1.361$
    $S_4 = \frac{205}{144} \approx 1.424$
    $S_5 = \frac{5269}{3600} \approx 1.464$

3. $S_1 = 3$
    $S_2 = \frac{9}{2} = 4.5$
    $S_3 = \frac{21}{4} = 5.25$
    $S_4 = \frac{45}{8} = 5.625$
    $S_5 = \frac{93}{16} = 5.8125$

5. nth-Term Test: $\lim\limits_{n\to\infty} \dfrac{n}{n+1} = 1 \neq 0$

7. nth-Term Test: $\lim\limits_{n\to\infty} \dfrac{n^2}{n^2+1} = 1 \neq 0$

9. Geometric series: $r = \frac{3}{2} > 1$

11. Geometric series: $r = 1.055 > 1$

13. $r = \frac{3}{4} < 1$ **15.** $r = 0.9 < 1$ **17.** 2   **19.** $\frac{2}{3}$

21. $4 + 2\sqrt{2} \approx 6.828$ **23.** $\frac{10}{9}$ **25.** $\frac{3}{2}$ **27.** $\frac{1}{2}$

29. $\dfrac{17}{6}$ **31.** $\lim\limits_{n\to\infty} \dfrac{n+10}{10n+1} = \dfrac{1}{10} \neq 0$; diverges

33. $\lim\limits_{n\to\infty} \dfrac{n!+1}{n!} = 1 \neq 0$; diverges

35. $\lim\limits_{n\to\infty} \dfrac{3n-1}{2n+1} = \dfrac{3}{2} \neq 0$; diverges

37. Geometric series: $r = 1.075 > 1$; diverges

39. Geometric series: $r = \frac{1}{4} < 1$; converges   **41.** $\frac{2}{3}$

43. $\frac{9}{11}$ **45.** (a) $80,000(1 - 0.9^n)$ (b) 80,000

47. ≈ 72.89 feet   **49.** $7808.24

51. $\sum\limits_{n=0}^{\infty} 100(0.75)^n = 400 million   **53.** $10,485.75

55. 2   **57.** ≈ 71.12 ppm

59. (a) $\sum\limits_{i=1}^{10} 880i = 8800

    (b) $\sum\limits_{i=1}^{168} 880i - 100,000 = 147,840 - 100,000$
$$= $47,840 \text{ more}$$

61. 6   **63.** ≈ 0.5431 **65.** $\dfrac{e^2}{e-1} \approx 4.3003$

67. False. $\lim\limits_{n\to 0}\dfrac{1}{n} = 0$, but $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ diverges.

SECTION 10.3   (page 679)

Prerequisite Review

1. $\dfrac{1}{n+1}$ **2.** $n+1$ **3.** $\dfrac{3n}{n+1}$ **4.** $\dfrac{n+1}{n^2}$

5. 1   **6.** 5   **7.** 1   **8.** $\frac{1}{3}$

9. Geometric series   **10.** Not a geometric series

---

1. p-series   **3.** Not a p-series   **5.** Not a p-series

7. Converges   **9.** Diverges   **11.** Converges

13. Diverges   **15.** Converges   **17.** Converges

19. Diverges   **21.** Converges   **23.** Diverges

25. Converges   **27.** Diverges   **29.** Converges

31. ≈ 1.1777; maximum error $\le \frac{1}{32}$.

33. ≈ 1.9953; maximum error $\le \dfrac{2}{\sqrt{10}} \approx 0.6325$.

35. $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\dfrac{1/[(n+1)^{3/2}]}{1/(n^{3/2})}$

$= \lim\limits_{n\to\infty}\left(\dfrac{n}{n+1}\right)^{3/2} = 1$

37. a; diverges: $p = \frac{3}{4} < 1$

38. d; diverges: $p = 1$, harmonic series

39. b; converges: $p = \frac{3}{2} > 1$

40. c; converges: $p = 2 > 1$

41. Diverges; nth-Term Test

43. Converges; p-Series Test; ≈ 3.6009

45. Converges; Geometric Series Test; $\frac{1}{5}$

47. Converges; p-Series Test; ≈ 0.4429

49. Diverges; Geometric Series Test

51. Diverges; Ratio Test

53. Diverges; Ratio Test   **55.** Converges; Ratio Test; $\frac{10}{3}$

57. $\sum\limits_{n=1}^{100}\dfrac{1}{n^2} \approx 1.635$, $\dfrac{\pi^2}{6} \approx 1.644934$

59. (a) $\sum\limits_{i=1}^{\infty}(0.1396n^2 + 0.309n + 12.32)$

(b) No, the Ratio Test yields a limit equal to 1.

SECTION 10.4   (page 689)

Prerequisite Review

1. $f(g(x)) = (x-1)^2$ **2.** $f(g(x)) = 6x+3$
$g(f(x)) = x^2 - 1$ $g(f(x)) = 6x + 1$

3. $f(g(x)) = \sqrt{x^2 + 4}$
$g(f(x)) = x + 4,\ x \ge -4$

4. $f(g(x)) = e^{x^2}$ **5.** $f'(x) = 5e^x$
$g(f(x)) = e^{2x}$ $f''(x) = 5e^x$
$f'''(x) = 5e^x$
$f^{(4)}(x) = 5e^x$

6. $f'(x) = \dfrac{1}{x}$ **7.** $f'(x) = 6e^{2x}$
$f''(x) = 12e^{2x}$
$f''(x) = -\dfrac{1}{x^2}$ $f'''(x) = 24e^{2x}$
$f'''(x) = \dfrac{2}{x^3}$ $f^{(4)}(x) = 48e^{2x}$
$f^{(4)}(x) = -\dfrac{6}{x^4}$

8. $f'(x) = \dfrac{1}{x}$ **9.** $\dfrac{n+1}{3}$ **10.** $\dfrac{n+3}{n+1}$
$f''(x) = -\dfrac{1}{x^2}$
$f'''(x) = \dfrac{2}{x^3}$
$f^{(4)}(x) = -\dfrac{6}{x^4}$

---

1. $1, \dfrac{x}{4}, \left(\dfrac{x}{4}\right)^2, \left(\dfrac{x}{4}\right)^3, \left(\dfrac{x}{4}\right)^4$

3. $-1, (x+1), -\dfrac{(x+1)^2}{2}, \dfrac{(x+1)^3}{6}, -\dfrac{(x+1)^4}{24}$

5. 2   **7.** 1   **9.** ∞ **11.** 0   **13.** 4

15. 5   **17.** 1   **19.** c **21.** ∞ **23.** ∞

25. $\sum\limits_{n=0}^{\infty}\dfrac{x^n}{n!},\ R = \infty$ **27.** $\sum\limits_{n=0}^{\infty}\dfrac{(2x)^n}{n!},\ R = \infty$

29. $\sum\limits_{n=0}^{\infty}(-1)^n x^n,\ R = 1$

31. $1 + \dfrac{1}{2}(x-1)\sum\limits_{n=2}^{\infty}\dfrac{(-1)^{n+1}1\cdot 3\cdot 5\cdots(2n-3)(x-1)^n}{2^n \cdot n!}$,
$R = 1$

33. $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{(n+2)(n+1)}{2}x^n,\ R = 1$

35. $1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^n$, $R = 1$

37. $R = 2$ (all parts)   **39.** $R = 1$ (all parts)

41. $\sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$ **43.** $3\sum_{n=0}^{\infty} \frac{x^{3n+2}}{n!}$ **45.** $\sum_{n=0}^{\infty} (-1)^n x^{4n}$

47. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n+1}$ **49.** $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}$

51. $\sum_{n=1}^{\infty} (-1)^{n+1} n(x-1)^{n-1}$

53. 1.6487   **55.** -0.6931 **57.** -2.3018

SECTION 10.5   (page 698)

Prerequisite Review

1. $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$ **2.** $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$

3. $4\sum_{n=0}^{\infty} (-1)^n (x-1)^n$

4. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$

5. $1 + \frac{x}{4} - \frac{3x^2}{4^2 2!} + \frac{3 \cdot 7 x^3}{4^3 3!} - \frac{3 \cdot 7 \cdot 11 x^4}{4^4 4!} + \cdots$

6. $1 + \frac{x}{2} - \frac{x^2}{2^2 2!} + \frac{1 \cdot 3 x^3}{2^3 3!} - \frac{1 \cdot 3 \cdot 5 x^4}{2^4 4!} + \cdots$

7. $\frac{47}{60}$ **8.** $\frac{311}{576}$ **9.** $\frac{5}{12}$ **10.** $\frac{77}{192}$

---

1. (a) $S_1(x) = 1 + x$ (b) $S_2(x) = 1 + x + \frac{x^2}{2}$

(c) $S_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

(d) $S_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

3. (a) $S_1(x) = 1 + \frac{x}{2}$ (b) $S_2(x) = 1 + \frac{x}{2} - \frac{x^2}{8}$

(c) $S_3(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(d) $S_4(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128}$

5. (a) $S_1(x) = x$ (b) $S_2(x) = x - x^2$

(c) $S_3(x) = x - x^2 + x^3$

(d) $S_4(x) = x - x^2 + x^3 - x^4$

7.

x	0	0.25	0.50	0.75	1.00
$f(x)$	1.0000	1.1331	1.2840	1.4550	1.6487
$S_1(x)$	1.0000	1.1250	1.2500	1.3750	1.5000
$S_2(x)$	1.0000	1.1328	1.2813	1.4453	1.6250
$S_3(x)$	1.0000	1.1331	1.2839	1.4541	1.6458
$S_4(x)$	1.0000	1.1331	1.2840	1.4549	1.6484

9. (a) $S_2(x) = 1 - x^2$ (b) $S_4(x) = 1 - x^2 + x^4$

(c) $S_6(x) = 1 - x^2 + x^4 - x^6$

(d) $S_8(x) = 1 - x^2 + x^4 - x^6 + x^8$

11. $S_4(x) = 1 - x^2 + x^4$ **13.** d   **14.** c   **15.** a

16. b   **17.** 0.607   **19.** 0.4055   **21.** 0.74286

23. 0.481   **25.** 7   **27.** $\frac{1}{6!} \approx 0.00139$

29. (a) Answers will vary.   (b) 1   (c) \$10

SECTION 10.6   (page 705)

Prerequisite Review

1. $f(2.4) = -0.04$ **2.** $f(-0.6) = 0.064$
    $f'(2.4) = 2.8$ $f'(-0.6) = 3.48$

3. $f(0.35) = 0.01$ **4.** $f(1.4) = 0.30$
    $f'(0.35) = 4.03$ $f'(1.4) = 12.88$

5. $4.9 \le x \le 5.1$ **6.** $0.798 \le x \le 0.802$

7. $5.97 \le x \le 6.03$ **8.** $-3.505 \le x \le -3.495$

**9.** $\left(\frac{\sqrt{13}+3}{2}, \sqrt{13}+2\right), \left(\frac{3-\sqrt{13}}{2}, 2-\sqrt{13}\right)$

10. $\left(\frac{1-\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right), \left(\frac{1+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$

**1.** 2.2364   **3.** 0.682   **5.** 1.25   **7.** 0.567   **9.** ± 0.753

11. $-4.596, -1.042, 5.638$ **13.** 2.926   **15.** 2.893

**17.** 11.8033   **19.** $\pm 1.9021, \pm 1.1756$

21. 0.9, 1.1, 1.9
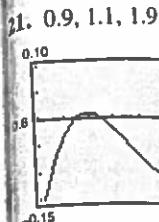
25. 0.2359, 1.33
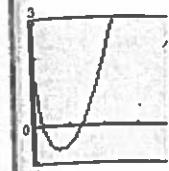
29. Newton's

31. Newton's
    $0 = x_2 = .$

33. $x_{n+1} = \frac{x_n^2}{}$

39. $f(x) = \frac{1}{x}$

    $f'(x) = -$

    Newton's

41. $x \approx 1.56$

45. 15.9 yea

47. False. L
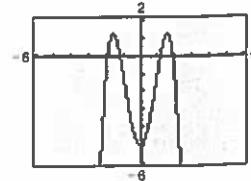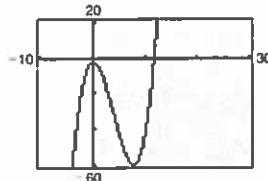
CHAPTER

1. $-\frac{1}{3}, \frac{1}{9},$

5. Conver

11. Conver

15. $(-1)^n$

17. (a) 15

21. 0.9, 1.1, 1.9
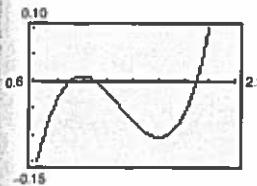
25. 0.2359, 1.3385

**23.** 1.1459, 7.8541
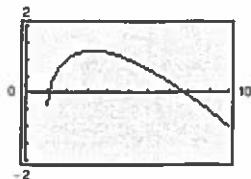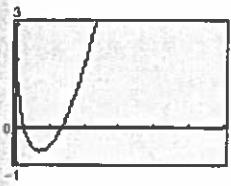
27. 0.8655
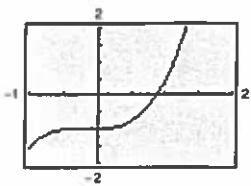
29. Newton's Method fails because $f'(x_1) = 0$.

31. Newton's Method fails because $1 = x_1 = x_3 = \ldots$; $0 = x_2 = x_4 = \ldots$. Therefore, the limit does not exist.

33. $x_{n+1} = \dfrac{x_n^2 + a}{2x_n}$ **35.** 2.646   **37.** 1.565

39. $f(x) = \dfrac{1}{x} - a$

$f'(x) = -\dfrac{1}{x^2}$

Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \dfrac{\dfrac{1}{x_n} - a}{-\dfrac{1}{x_n^2}}$

$= x_n(2 - ax_n)$

41. $x \approx 1.563$ miles down the coast   **43.** $t \approx 4.486$ hours

45. 15.9 years ≈ 2016

47. False. Let $f(x) = \dfrac{x^2 - 1}{x - 1}$.   **49.** True

CHAPTER 10 REVIEW EXERCISES   (page 712)

1. $-\dfrac{1}{3}, \dfrac{1}{9}, -\dfrac{1}{27}, \dfrac{1}{81}, -\dfrac{1}{243}$ **3.** $4, 8, 10\frac{2}{3}, 10\frac{2}{3}, \dfrac{128}{15}$

5. Converges to 0   **7.** Diverges   **9.** Converges to 5

11. Converges to 0   **13.** $\dfrac{n}{3n}$ or $\dfrac{1}{3}$, $n \neq 0$

15. $(-1)^n \dfrac{2^n}{3^{n+1}}$, $n = 0, 1, 2, \ldots$

17. (a) $15,000 + 10,000(n - 1)$ (b) $175,000

19. $1.07, $1.14, $1.23, $1.31, $1.40, $1.50, $1.61, $1.72, $1.84, $1.97

21. $S_0 = 1$
$S_1 = \dfrac{5}{2} = 2.5$
$S_2 = \dfrac{19}{4} = 4.75$
$S_3 = \dfrac{65}{8} = 8.125$
$S_4 = \dfrac{211}{16} = 13.1875$

23. $S_1 = \dfrac{1}{2} = 0.5$
$S_2 = \dfrac{11}{24} \approx 0.4583$
$S_3 = \dfrac{331}{720} \approx 0.4597$
$S_4 = \dfrac{18,535}{40,320} \approx 0.4597$
$S_5 = \dfrac{1,668,151}{3,628,800} \approx 0.4597$

25. Diverges   **27.** Converges

29. $\lim\limits_{n \to \infty} \dfrac{2n}{n + 5} = 2 \neq 0$ **31.** $\lim\limits_{n \to \infty} \left(\dfrac{5}{4}\right)^n = \infty \neq 0$

33. $\dfrac{5}{4}\left[1 - \left(\dfrac{1}{5}\right)^{N+1}\right]$ **35.** $2\left[1 - \left(\dfrac{1}{2}\right)^{N+1}\right] + \dfrac{4}{3}\left[1 - \left(\dfrac{1}{4}\right)^{N+1}\right]$

37. Diverges   **39.** Converges to $\dfrac{13}{4}$

41. (a) $D = -8 + 16 + 16(0.7) + 16(0.7)^2 + \cdots$
(b) $\dfrac{136}{3}$ feet

43. $\approx 2.782\%$ **45.** Converges   **47.** Converges

49. a   **50.** c   **51.** d   **52.** b

53. 1.0172; error $\leq \dfrac{1}{(5)4^5} \approx 1.9531 \times 10^{-4}$

55. 2.09074; error $\leq \dfrac{1}{(1/4)(6)^{1/4}} < 2.5558$

57. Converges   **59.** Diverges   **61.** Converges

63. $R = 1$ **65.** $R = 0$ **67.** $\sum\limits_{n=0}^{\infty} \left(-\dfrac{1}{2}\right)^n \dfrac{x^n}{n!}$

69. $-\sum\limits_{n=0}^{\infty} (x + 1)^n$ **71.** $\ln 2 + \sum\limits_{n=1}^{\infty} (-1)^{n+1} \dfrac{(x/2)^n}{n}$

73. $1 + 2x^2 + x^4 + \cdots$ **75.** $x^2 \sum\limits_{n=0}^{\infty} \dfrac{x^n}{n!} = \sum\limits_{n=0}^{\infty} \dfrac{x^{n+2}}{n!}$

77. $x^2 \sum\limits_{n=0}^{\infty} (-1)^n x^n = \sum\limits_{n=0}^{\infty} (-1)^n x^{n+2}$

79. $\dfrac{1}{9} - \dfrac{2}{27}x + \dfrac{1}{27}x^2 - \dfrac{4}{243}x^3 + \dfrac{5}{729}x^4 - \dfrac{2}{729}x^5 + \dfrac{7}{6561}x^6$

81. $\ln 3 + \dfrac{1}{3}(x - 1) - \dfrac{1}{18}(x - 1)^2 + \dfrac{1}{81}(x - 1)^3$
$- \dfrac{1}{324}(x - 1)^4 + \dfrac{1}{1215}(x - 1)^5 - \dfrac{1}{4374}(x - 1)^6$

83. 4.7705   **85.** 0.9163   **87.** $\dfrac{1}{32}$ **89.** 0.301

91. 0.1233   **93.** 0.5, $11.50   **95.** 0.313

97. 0.258   **99.** 1.341   **101.** 0.773

SAMPLE POST-GRAD EXAM QUESTIONS
(page 715)

1. b   **2.** b   **3.** b   **4.** b   **5.** c   **6.** d

APPENDIX A   (page A10)

1. Left Riemann sum: 0.518
   Right Riemann sum: 0.768
3. Left Riemann sum: 0.746
   Right Riemann sum: 0.646
5. Left Riemann sum: 0.859
   Right Riemann sum: 0.659
7. Midpoint Rule: 0.673
9. (a)
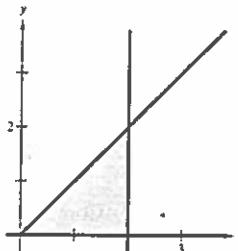
   (b) Answers will vary.
   (c) Answers will vary.
   (d) Answers will vary.

(e)

| n	5	10	50	100
Left sum, S_L	1.6	1.8	1.96	1.98
Right sum, S_R	2.4	2.2	2.04	2.02

(f) Answers will vary.

11. $\int_0^5 3\,dx$

13. $\int_{-4}^{4} (4 - |x|)\,dx = \int_{-4}^{0} (4 + x)\,dx + \int_{0}^{4} (4 - x)\,dx$

15. $\int_{-2}^{2} (4 - x^2)\,dx$ 17. $\int_{0}^{2} \sqrt{x + 1}\,dx$

19.

Rectangle

$A = 12$

21.

Triangle

$A = 8$

23.

Trapezoid

$A = 14$

25.

Triangle

$A = 1$

27.

Semicircle
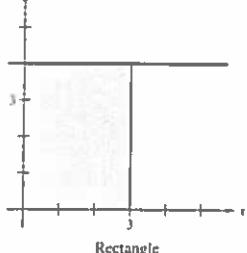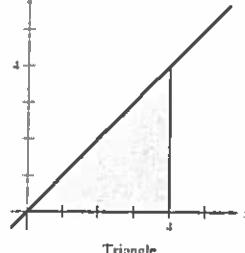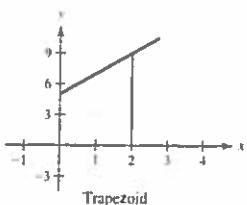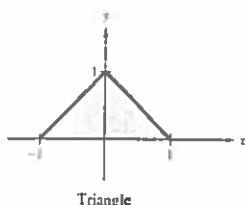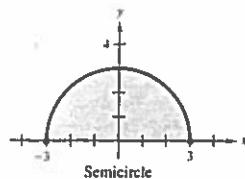
$A = \dfrac{9\pi}{2}$

29. Answers will vary.   31. >

APPENDIX C

Section C.1   (page A24)

Prerequisite Review

1. $y' = 6x + 2$ 2. $y' = -6x^2 - 8$
   $y'' = 6$ $y'' = -12x$

3. $y' = -6e^{2x}$ 4. $y' = -6xe^{x^2}$
   $y'' = -12e^{2x}$ $y'' = -6e^{x^2}(2x^2 + 1)$

5. $\dfrac{1 - x}{y}$ 6. $\dfrac{2}{3y^2 + 4}$ 7. $-\dfrac{y}{2x}$

8. $-\dfrac{y}{x}$ 9. $k = 2 \ln 3 - \ln \dfrac{17}{2} \approx 0.0572$

10. $k = \ln 10 - \dfrac{\ln 41}{2} \approx 0.4458$

---

1. $y = 3x^2$

3. $y' = -2e^{-2x}$ and $y' + 2y = -2e^{-2x} + 2(e^{-2x}) = 0$

5. $y' = 6x^2$ and $y' - \dfrac{3}{x}y = 6x^2 - \dfrac{3}{x}(2x^3) = 0$

7. $y'' = 2$ and $x^2y'' - 2y = x^2(2) - 2(x^2) = 0$

9. $y' = 4e^{2x}$, $y'' = 8e^{2x}$, and
   $y'' - y' - 2y = 8e^{2x} - 4e^{2x} - 2(2e^{2x}) = 0$

11. $\dfrac{dy}{dx} = -\dfrac{1}{x^2}$ 13. $\dfrac{dy}{dx} = 4Ce^{4x} = 4y$

15. $\dfrac{dy}{dt} = -\dfrac{1}{3}Ce^{-t/3}$ and

   $3\dfrac{dy}{dt} + y - 7 = 3\left(-\dfrac{1}{3}Ce^{-t/3}\right) + (Ce^{-t/3} + 7) - 7 = 0$

17. $xy' - 3x - 2y = x(2Cx - 3) - 3x - 2(Cx^2 - 3x) = 0$

19. $xy' + y = x\left(2x + 2 - \dfrac{C}{x^2}\right) + \left(x^2 + 2x + \dfrac{C}{x}\right)$
   $= x(3x + 4)$

21. $2y'' +$

23. $y' -$

25. $y' +$

27. $y' =$
   $x(y' -$

29. $2x +$
   $y' =$
   $=$

31. $x + y$
   $y'' =$
   $x^2y'' -$

33. Soluti

39. Soluti

45. $y = 3$

49.

53. $y = x^3$

57. $y = \dfrac{1}{2}$

59. $y = \dfrac{2}{5}($
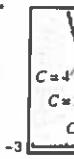
61. $y^2 = \dfrac{1}{4}$

65. (a) N
   (b)

   (c) N

21. $2y'' + 3y' - 2y = 2\left(\frac{1}{4}C_1e^{x/2} + 4C_2e^{-2x}\right)$
$$+ 3\left(\frac{1}{2}C_1e^{x/2} - 2C_2e^{-2x}\right)$$
$$- 2\left(C_1e^{x/2} + C_2e^{-2x}\right) = 0$$

23. $y' - \dfrac{ay}{x} = \left(\dfrac{4bx^3}{4-a} + aCx^{a-1}\right) - \dfrac{a}{x}\left(\dfrac{bx^4}{4-a} + Cx^a\right)$
$$= bx^3$$

25. $y' + 2xy = -\dfrac{4Cxe^{x^2}}{(1 - Ce^{x^2})^2} + 2x\left(\dfrac{2}{1 + Ce^{x^2}}\right) = xy^2$

27. $y' = \ln x + 1 + C$ and
$$x(y' - 1) - (y - 4) = x(\ln x + 1 + C - 1)$$
$$- (x \ln x + Cx + 4 - 4) = 0$$

29. $2x + 2yy' = Cy'$
$$y' = \frac{2x}{C - 2y} = \frac{2xy}{Cy - 2y^2}$$
$$= \frac{2xy}{(x^2 + y^2) - 2y^2} = \frac{2xy}{x^2 - y^2}$$

31. $x + y = \dfrac{C}{x}$
$$y'' = \frac{2C}{x^3}$$
$$x^2y'' - 2(x + y) = \frac{2C}{x} - \frac{2C}{x} = 0$$

33. Solution   **35.** Not a solution   **37.** Not a solution

39. Solution   **41.** $y = 3e^{-2x}$ **43.** $y = 5 + \ln\sqrt{|x|}$

45. $y = 3e^{4x} + 2e^{-3x}$ **47.** $y = \frac{4}{3}(3 - x)e^{2x/3}$

49.
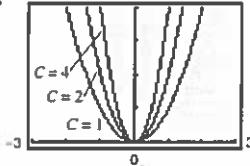
51.
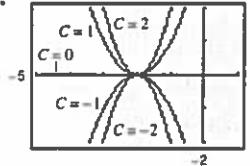
53. $y = x^3 + C$ **55.** $y = x + 3 \ln|x| + C$

57. $y = \dfrac{1}{2} \ln\left(\dfrac{x - 1}{x + 1}\right) + C$

59. $y = \frac{2}{5}(x - 3)^{3/2}(x + 2) + C$

61. $y^2 = \frac{1}{4}x^3$ **63.** $y = 3e^x$

65. (a) $N = 750 - 650e^{-0.0484t}$

(b)
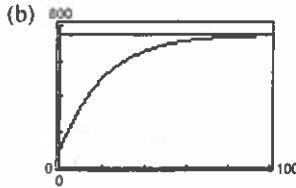
(c) $N \approx 214$

67.

Year, t	2	4	6	8	10
Units, x	3867	7235	10,169	12,725	14,951

69. Because
$$\frac{ds}{dh} = -\frac{13}{\ln 3}\left(\frac{1/2}{h/2}\right) = -\frac{13}{\ln 3}\frac{1}{h}, \text{ and } -\frac{13}{\ln 3}$$
is a constant, we can conclude that the equation is a solution of $ds/dh = k/h$ where $k = -13/(\ln 3)$.

71. $k = 0.07$

73. False. From Example 1, $y = e^x$ is a solution of $y'' - y = 0$, but $y = e^x + 1$ is not.

Section C.2   *(page A32)*

Prerequisite Review

1. $\frac{2}{5}x^{5/2} + C$ **2.** $\frac{1}{4}t^4 - \frac{3}{4}t^{4/3} + C$

3. $2 \ln|x - 5| + C$ **4.** $\frac{1}{4}\ln|2y^2 + 1| + C$

5. $\frac{1}{2}e^{2y} + C$ **6.** $-\frac{1}{2}e^{1-x^2} + C$ **7.** $C = -10$

8. $C = 5$ **9.** $k = \dfrac{\ln 5}{2} \approx 0.8047$

10. $k = -2 \ln 3 - \ln 2 \approx -2.8904$

1. Yes   **3.** Yes
$$(y + 3)\, dy = x\, dx \qquad dy = \left(\frac{1}{x} + 1\right) dx$$

5. No. The variables cannot be separated.

7. $y = x^2 + C$ **9.** $y = \sqrt[3]{x + C}$

11. $C = 2x^2 - (y + 1)^2$ **13.** $y = Ce^{x^2/2}$

15. $y^2 = \frac{1}{2}e^t + C$

17. $y = 1 - \left(C - \dfrac{x}{2}\right)^2$

19. $y = C(2 + x)^2$ **21.** $y = Cx$

23. $3y^2 + 2y^3 = 3x^2 + C$ **25.** $y = -e^{-x} - x + C$

27. $y^2 = 2e^x + 14$ **29.** $y = -4 - e^{-x^2/2}$

31. $P = 5e^{6t}$

33. $5y^2 = 6x^2 - 1$ or $6x^2 - 5y^2 = 1$
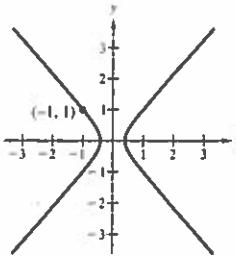
35. $v = 34.56(1 - e^{-0.1t})$ **37.** $T \approx 383.298°F$

39. (a) $T \approx 7.277°F$ (b) $t \approx 5.158$ hours

41. $N = 30 + Ce^{-kt}$ **43.** $y = Cx^{-k}$

Section C.3   (page A37)

Prerequisite Review

1. $e^x + 1$ **2.** $e^{3x} + 1$ **3.** $\dfrac{1}{x^3}$ **4.** $x^2 e^x$

5. $2e^x - e^{-x} + C$ **6.** $e^{3x}\left(\dfrac{x}{3} - \dfrac{1}{9}\right) + \dfrac{1}{2}e^{2x} + C$

7. $\dfrac{1}{2}\ln|2x + 5| + C$ **8.** $\dfrac{1}{2}\ln|x^2 + 2x + 3| + C$

9. $\dfrac{1}{12}(4x - 3)^3 + C$ **10.** $\dfrac{1}{6}(x^2 - 1)^3 + C$

---

1. $y' + \dfrac{-3}{2x^2}y = \dfrac{x}{2}$ **3.** $y' + \dfrac{1}{x}y = e^x$

5. $y' + \dfrac{1}{1 - x}y = \dfrac{1}{x - 1}$ **7.** $y = 2 + Ce^{-3x}$

9. $y = e^{-x}(x + C)$ **11.** $y = x^2 + 2x + \dfrac{C}{x}$

13. $y = \dfrac{1}{5} + Ce^{-(5/2)x^2}$ **15.** $y = \dfrac{x^3 - 3x + C}{3(x - 1)}$

17. $y = e^{1/x^2}\left(-\dfrac{1}{2x^2} + C\right)$ **19.** $y = Ce^{-x} + 4$

21. $y = Ce^{x^2} - 1$

23. c   **24.** d   **25.** a   **26.** b

27. $y = 3e^x$ **29.** $xy = 4$

31. $y = 1 + 5e^{-x^3}$ **33.** $y = x^2(5 - \ln|x|)$

35. $S = t + 95(1 - e^{-t/5})$

t	0	1	2	3	4	5
S	0	18.22	33.32	45.86	56.31	65.05

| t	6	7	8	9	10
S	72.39	78.57	83.82	88.30	92.14

37. $p = 400 - 3x$ **39.** $p = 15(4 + e^{-t})$

41. (a) $A = \dfrac{P}{r^2}(rt - 1 + e^{-rt})$ (b) $A \approx \$18,924,053.07$

43. $v = -\dfrac{gm}{k} + Ce^{-kt/m}$ **45.** Answers will vary.

Section C.4   (page A44)

Prerequisite Review

1. $y = \dfrac{3}{2}x^2 + C$ **2.** $y^2 = 3x + C$

3. $y = Ce^{x^2}$ **4.** $y^4 = \dfrac{1}{2}(x - 4)^2 + C$

5. $y = 2 + Ce^{-2x}$ **6.** $y = xe^{-2x} + Ce^{-2x}$

7. $y = 1 + Ce^{-x^2/2}$ **8.** $y = \dfrac{1}{4}x^2 + Cx^{-2}$

9. $\dfrac{dy}{dx} = Cx^2$ **10.** $\dfrac{dx}{dt} = C(x - t)$

---

1. $y = e^{(x \ln 2)/3} \approx e^{0.2310x}$ **3.** $y = 4e^{-(x \ln 4)/4}$
$\approx 4e^{-0.3466x}$

5. $y = \dfrac{1}{2}e^{(\ln 2)x} \approx \dfrac{1}{2}e^{0.6931x}$ **7.** $\$4451.08$

9. $S = L(1 - e^{-kt})$ **11.** $y = \dfrac{20}{1 + 19e^{-0.5889x}}$

13. $y = \dfrac{5000}{1 + 19e^{-0.10156x}}$ **15.** $N = \dfrac{500}{1 + 4e^{-0.2452t}}$

17. $\dfrac{dP}{dn} = kP(L - P)$, $P = \dfrac{CL}{e^{-Lkn} + C}$

19. $y = \dfrac{360}{8 + 41t}$ **21.** $y = 500e^{-1.6094e^{-0.1451t}}$
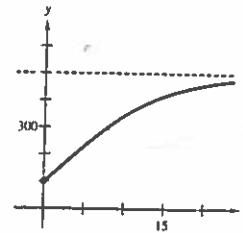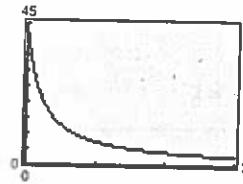
23. 34 beavers   **25.** 92%

27. (a) $y = Ce^{kt}$ (b) ≈ 6.2 hours

29. 38.843 pounds per gallon   **31.** ≈ 3.15 hours

33. $P = Ce^{kt} - \dfrac{N}{k}$ **35.** $A = \dfrac{P}{r}(e^{rt} - 1)$

37. $\$7,305,295.15$ **39.** (a) $C = C_0 e^{-Rt/V}$ (b) 0

41. (a) $C(t) = \dfrac{Q}{R}(1 - e^{-Rt/V})$ (b) $\dfrac{Q}{R}$