

(35) Objective:  $V = xyz$

Constraint:  $x + 2y + 2z = 108$

$$F = xyz - \lambda(x + 2y + 2z - 108)$$

$$F_x = yz - \lambda = 0$$

$$F_y = xz - 2\lambda = 0$$

$$F_z = xy - 2\lambda = 0$$

$$F_\lambda = -(x + 2y + 2z - 108) = 0$$

Multiply  $F_x$  by  $x$ ,  $F_y$  by  $y$  and  $F_z$  by  $z$  to conclude that

$$\frac{xyz}{x} = x, \quad \frac{xyz}{y} = 2y, \quad \frac{xyz}{z} = 2z$$

and so  $x = 2y = 2z$ . As  $x + 2y + 2z = 108$ ,

$$x = 2y = 2z = \frac{108}{3} = 36$$

It follows that  $x = 36, y = 18, z = 18$ ,

(36)  $F = xyz - \lambda(5xy + 2xz + 2yz - C)$

$$F_x = yz - \lambda(5y + 2z) = 0$$

$$F_y = xz - \lambda(5x + 2z) = 0$$

$$F_z = xy - \lambda(2x + 2y) = 0$$

$$F_\lambda = -(5xy + 2xz + 2yz - C) = 0$$

Multiply the 1st eq. by  $x$ , 2nd by  $y$  and subtract:  $\lambda(2yz - 2xz) = 0$ ,

$$2xz = 2yz, \quad \underline{x = y}$$

Multiply the 1st eq. by  $x$ , 3rd by  $z$  and subtract:

$$5xy = 2yz, \quad \underline{5x = 2z}$$

Multiply the 2nd eq. by  $y$ , 3rd by  $z$  and subtract:

$$5xy = 2xz, \quad \underline{5y = 2z}$$

We get  $x = y = \frac{2}{5}z$ . So:  $5 \cdot \left(\frac{2}{5}z\right)^2 + 2 \cdot \frac{2}{5}z \cdot z + 2 \cdot \frac{2}{5}z \cdot z = C$ ,

$$z^2 = \frac{5C}{12}, \quad \left| z = \sqrt{\frac{5C}{12}}, \quad x = \frac{2}{5} \sqrt{\frac{5C}{12}}, \quad y = \frac{2}{5} \sqrt{\frac{5C}{12}} \right|$$