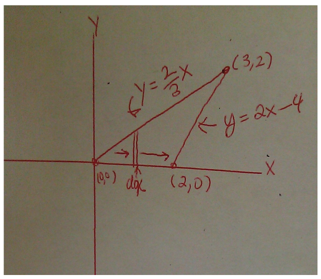
This will be the solutions to #1:

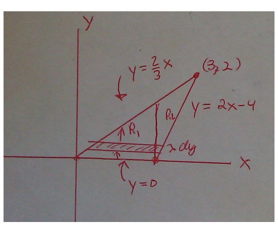
#1 Let $\mathbf{R}$ be the triangular region with vertices $(0,0),(2,0)$, and $(3,2)$. Compute the volume of the solid above $\mathbf{R}$ whose top is given by the surface $z = 2 + x + y + xy$.



$$\iint\limits_{\mathbf{R}} 2 + x + y + xy \mathrm{d}x\, \mathrm{d}y$$

Please refer to the picture above. We are moving the dx in the x direction, so the first curve we see is $x = \frac{3}{2}y$. This is the lower bound. The second curve we see is $x = \frac{1}{2}y + 2$, giving us the upper bound. Thus,

$$\iint\limits_{\mathbf{R}} 2 + x + y + xy\, \mathrm{d}x\, \mathrm{d}y = \int_0^2 \int_{\frac{3}{2}y}^{\frac{1}{2}y+2} 2 + x + y + xy\, \mathrm{d}x\, \mathrm{d}y$$



If we change the order of the integral we will need to divide the region $\mathbf{R}$ into two regions $\mathbf{R}_1$ and $\mathbf{R}_2$. So,

$$\iint\limits_{\mathbf{R}} 2+x+y+xy \mathrm{d}y\, \mathrm{d}x = \iint\limits_{\mathbf{R}_1} 2+x+y+xy \mathrm{d}y\, \mathrm{d}x + \iint\limits_{\mathbf{R}_2} 2+x+y+xy \mathrm{d}y\, \mathrm{d}x$$

Moving dy in the y direction for $\mathbf{R}_1$ we see that the lower bound equation should by $y = 0$ and the upper bound should be $y = \frac{2}{3}x$. For $\mathbf{R}_2$ the lower bound is $y = 2x - 4$ and the upper bound is $y = \frac{2}{3}x$. Thus,

$$\iint\limits_{\mathbf{R}_1} 2 + x + y + xy \mathrm{d}y \, \mathrm{d}x + \iint\limits_{\mathbf{R}_2} 2 + x + y + xy \mathrm{d}y \, \mathrm{d}x =$$

$$\int_0^2 \int_0^{\frac{2}{3}x} 2 + x + y + xy \, \mathrm{d}y \, \mathrm{d}x + \int_2^3 \int_{2x-4}^{\frac{2}{3}x} 2 + x + y + xy \, \mathrm{d}y \, \mathrm{d}x$$

Both integrals should be equal.