Math 17C, Spring 2011.
Apr. 20, 2011.

MIDTERM EXAM 1

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ____________________

ID#: ____________________

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed.

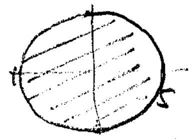
Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

1. Consider the function $f(x,y) = \sqrt{25 - x^2 - y^2}$.

(a) Determine and sketch the domain D of f.

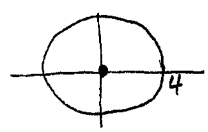
$D = \{(x,y) : x^2+y^2 \le 25\}$ = interior & boundary of the circle of radius 5 with center at $(0,0)$. 6

D: 

(b) Sketch the level curve $f(x,y) = 3$.

$\sqrt{25-x^2-y^2} = 3$, $25 - x^2 + y^2 = 9$, $x^2+y^2 = 16$

6



(c) Determine all critical points of f in the interior of D.

$$\frac{\partial f}{\partial x} = \frac{-2x}{2\sqrt{25-x^2-y^2}}$$

$x = y = 0$

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$$\frac{\partial f}{\partial y} = \frac{-2y}{2\sqrt{25-x^2-y^2}}$$

$(0,0)$ is the only c.p.

(d) Determine the range of f.

On the boundary of D, $x^2+y^2 = 25$ and f is 0.

At $(0,0)$, $f(0,0) = 5$. 6

Maximum of f is 5, minimum is 0,

so range is $[0, 5]$.

2. Consider the function $f(x,y) = x^3y^2 - 2x$.

(a) Determine the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(1,2,2)$.

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 2 \qquad \text{At } (x=1, y=2): \; = 10$$

5

$$\frac{\partial f}{\partial y} = 2x^3y \qquad = 4$$

2 9

$$z - 2 = 10(x-1) + 4(y-2); \quad \underline{z - 10x - 4y + 16 = 0}$$

(b) Determine the direction $\vec{u}$ in which the directional derivative $D_{\vec{u}}f(1,2)$ is the largest.

4

$$\nabla f(1,2) = \begin{bmatrix} 10 \\ 4 \end{bmatrix}, \text{ its length } \sqrt{10^2+4^2} = \sqrt{116}$$

4

$$\text{Answer: } \begin{bmatrix} 10/\sqrt{116} \\ 4/\sqrt{116} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5/\sqrt{29} \\ 2/\sqrt{29} \end{bmatrix}}}$$

8

(c) Determine a direction $\vec{u}$ in which the directional derivative $D_{\vec{u}}f(1,2) = 0$.

$$D_{\vec{u}} f = \nabla f \cdot \vec{u} = 0$$

If $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $5u_1 + 2u_2 = 0$;

8

take vector $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ 4 and normalize to

get $\underline{\begin{bmatrix} -2/\sqrt{29} \\ 5/\sqrt{29} \end{bmatrix}}$. 4

3. Consider $f(x, y) = x^2y + x^2 + 2y^2$.

(a) Find all critical points of f.

$$\frac{\partial f}{\partial x} = 2xy + 2x = 2x(y+1) = 0$$

$$\frac{\partial f}{\partial y} = x^2 + 4y = 0$$

6

$y = -\frac{1}{4}x^2$

Critical pts.:

13

if $x=0$, $y=0$

if $y+1=0$, $y=-1$, $x^2=4$, $x=\pm 2$.

$(0,0)$

$(2,-1)$

$(-2,-1)$

2

(b) For each critical point, determine whether it is a local maximum, a local minimum, or a saddle point.

$f_{xx} = 2y+2$

$f_{yy} = 4$

$f_{xy} = 2x$

6

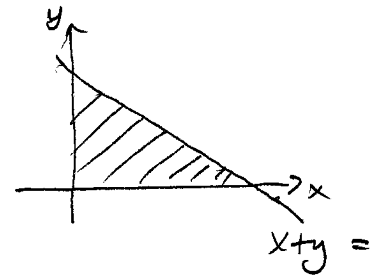
c.p.	D	f_{xx}	
$(0,0)$	8	2	local min. 2
$(2,-1)$	-16		saddle 2
$(-2,-1)$	-16		saddle 2

12

$D = f_{xx}f_{yy} - f_{xy}^2$

4. Determine the maximum of the expression x^2yz, where $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + y + z = 1$.

$$f(x,y) = x^2 y (1-x-y) \quad \text{on } D:$$ 5



f is 0 on the boundary of D; the maximum will be achieved at an interior critical pt. 5

$$\frac{\partial f}{\partial x} = y\,(2x(1-x-y) - x^2)$$
$$= xy\,(2 - 2x - 2y - x)$$
$$= xy\,(2 - 3x - 2y)$$ 5

$$\frac{\partial f}{\partial y} = x^2(1-x-y-y)$$
$$= x^2(1 - x - 2y)$$ 5

$$3x + 2y = 2$$
$$x + 2y = 1$$

$2x = 1$, $\underline{x = \frac{1}{2}}$ 5

$4y = 1$, $\underline{y = \frac{1}{4}}$

$\underline{z} = 1 - x - y = \underline{\frac{1}{4}}$

$$\underline{x^2 y z = \frac{1}{64}}$$