Math 17C, Spring 2011.
May 25, 2011.

MIDTERM EXAM 3

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ____________________

ID#: ____________________

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. Unless directed to do so, do *not* evaluate complicated expressions to give the result as a decimal number.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

1. A group of 40 people consists of 18 Californians, 10 Nevadans and 12 Oregonians. Select a committee of 8 of them at random.

(a) Compute the probability that the committee consists entirely of Californians.

$$\frac{\binom{18}{8}}{\binom{40}{8}}$$

(b) Compute the probability that the committee consists of 3 Californians, 3 Nevadans and 2 Oregonians.

$$\frac{\binom{18}{3}\binom{10}{3}\binom{12}{2}}{\binom{40}{8}}$$

(c) Compute the probability that the committee consists of representatives of a single state (i.e., consists entirely of Californians, or entirely of Nevadans, or entirely of Oregonians).

$$\frac{\binom{18}{8}+\binom{10}{8}+\binom{12}{8}}{\binom{40}{8}}$$

(d) Compute the conditional probability that there are no Californians of the committee, given that there are exactly 5 Oregonians on the committee.

$B = \{5 \text{ Oreg.}\} = \{5 \text{ Oreg.}, 3 \text{ others}\}$

$A = \{\text{no Calif}\}$

$A \cap B = \{5 \text{ Oreg.}, 3 \text{ Nev}\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\cancel{\binom{12}{5}}\binom{10}{3}\Big/\cancel{\binom{40}{8}}}{\cancel{\binom{12}{5}}\binom{28}{3}\Big/\cancel{\binom{40}{8}}} = \frac{\binom{10}{3}}{\binom{28}{3}}$$

2.

(a) Roll a fair die twice. Compute the probability that the two numbers you roll are the same.

$$\frac{6}{36} = \frac{1}{6}$$

(b) Roll a fair die 10 times. Compute the probability that you roll exactly five 1's and exactly five 2's.

$$\frac{\binom{10}{5}}{6^{10}}$$ ← no. of possible positions of 5 1's

(c) Roll a fair die 10 times. Compute the probability that you roll exactly three 1's and exactly five 2's.

position of 1's   position of 2's

$$\frac{\binom{10}{3}\binom{7}{5}4^2}{6^{10}}$$ ← remaining 2 numbers are 1,…,4

(d) Roll a fair die 6 times. Compute the probability that the numbers you roll are all different.

$$\frac{6!}{6^6}$$

3. A bag contains 60 golf balls: 15 yellow, 25 red, and 20 green golf balls.
(a) Select four balls from the bag one by one *without* replacement. Compute the probability that first ball selected is yellow, the second is green, the third is red and the fourth is again yellow.

$$\frac{15}{60}, \frac{20}{59}, \frac{25}{58}, \frac{14}{57}$$

(b) Select four balls from the bag one by one *without* replacement. Compute the probability that all four are green.

$$\frac{20}{60} \cdot \frac{19}{59} \cdot \frac{18}{58} \cdot \frac{17}{57} \quad , \text{ or } \quad \frac{\binom{20}{4}}{\binom{60}{4}}$$

(c) Select four balls from the bag one by one *with* replacement. Compute the probability in (a), i.e., that first ball selected is yellow, the second is green, the third is red and the fourth is again yellow.

$$\frac{15}{60} \cdot \frac{20}{60} \cdot \frac{25}{60} \cdot \frac{15}{60} = \left(\frac{1}{4}\right)^2 \cdot \frac{1}{3} \cdot \frac{5}{12}$$

4. You have a deck of 13 cards labeled with values 1, 2, ..., 13. Shuffle this deck.
(a) Compute the probability that the cards 1 and 2 are together, i.e., next to each other in any order, in the deck. Give the result as a simple fraction.

$$\frac{2!\ 12!}{13!} = \frac{2}{13}$$

(b) Compute the probability that the cards 1 and 2 are *not* together in the deck.

$$1 - \frac{2}{13} = \frac{11}{13}$$

(c) Compute the probability that the four cards 1, 2, 3, and 4 are together in the deck.

$$\frac{4!\ 10!}{13!}$$

(d) Compute the conditional probability that that the four cards 1, 2, 3, and 4 are together in the deck, given that 1 and 2 are together in the deck. Give the result as a simple fraction.

order 1, 2, 3, 4 so that 1 and 2 are together

$$\frac{\dfrac{\cancel{2!}\cdot 3!\ 10!}{\cancel{13!}}}{\dfrac{\cancel{2!}\ 12!}{\cancel{13!}}} = \frac{3!\ 10!}{12!} = \frac{6}{11\cdot 12} = \underline{\underline{\frac{1}{22}}}$$