

## Solutions to Conditional Probability Practice problems

(1)  $P(\text{w from A} \mid 2 \text{ w selected})$

$$= \frac{P(\text{w from A}, 2 \text{ w selected})}{P(2 \text{ w selected})} = \frac{\frac{2}{6} \cdot \frac{8}{12} \cdot \frac{3}{4} + \frac{2}{6} \cdot \frac{4}{12} \cdot \frac{1}{4}}{\frac{2}{6} \cdot \frac{8}{12} \cdot \frac{3}{4} + \frac{2}{6} \cdot \frac{4}{12} \cdot \frac{1}{4} + \frac{4}{6} \cdot \frac{8}{12} \cdot \frac{1}{4}}$$

(2)  $B_1 = \{\text{transferred ball white}\}$   
 $B_2 = \{\text{transferred ball red}\}$

$A = \{\text{final ball white}\}$

(a)  $P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) = \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{9}$

(b)  $\frac{\frac{2}{3} \cdot \frac{1}{3}}{\frac{4}{9}} = \frac{1}{2}$

(3) (a)  $0.8^3$  (b)  $3 \cdot 0.8^2 \cdot 0.2$

(c)  $B_1 = \{\text{sick}\}$ ,  $B_2 = \{\text{healthy}\}$ ,  $A = \{\text{pos. test}\}$

$$P(B_1|A) = \frac{P(B_1) \cdot P(A|B_1)}{P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2)} = \frac{0.05 \cdot 0.9}{0.05 \cdot 0.9 + 0.95 \cdot 0.2}$$

(d)  $B_1$  and  $B_2$  same as above,  $A = \{\text{pos. test twice}\}$

$$P(B_1|A) = \frac{0.05 \cdot (0.9)^2}{0.05 \cdot (0.9)^2 + 0.95 \cdot (0.2)^2}$$

(4)  $B_1 = \{\text{CA group chosen}\}$ ,  $B_2 = \{\text{NV gr. chosen}\}$ ,  $B_3 = \{\text{OR group chosen}\}$   
 $A = \{\text{all women}\}$

(a)  $P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$   
 $= \frac{1}{3} \cdot \frac{\binom{5}{3}}{\binom{10}{3}} + \frac{1}{3} \cdot \frac{\binom{7}{3}}{\binom{10}{3}} + \frac{1}{3} \cdot \frac{\binom{4}{3}}{\binom{10}{3}}$

(b)  $P(B_1|A) = \frac{\binom{5}{3}}{\binom{5}{3} + \binom{7}{3} + \binom{4}{3}} = \frac{10}{49}$

(5) (a)  $\frac{5}{8} \cdot \frac{6}{9} \cdot \frac{3}{10} \cdot \frac{4}{11}$

(b)  $B_1 = \{\text{1st white}\}$ ,  $B_2 = \{\text{1st black}\}$ ,  $A = \{\text{2nd white}\}$

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) = \frac{4}{9} \cdot \frac{3}{8} + \frac{3}{9} \cdot \frac{5}{8} = \frac{3}{8}$$

(c)  $4 \cdot 3 / (4 \cdot 3 + 3 \cdot 5) = 4/9$