

ANSWERS TO ODD-NUMBERED EXERCISES

Chapter 1

SECTION 1.1, pp. 11–13

1. $D: (-\infty, \infty)$, $R: [1, \infty)$ 3. $D: [-2, \infty)$, $R: [0, \infty)$

5. $D: (-\infty, 3) \cup (3, \infty)$, $R: (-\infty, 0) \cup (0, \infty)$

7. (a) Not a function of x because some values of x have two values of y

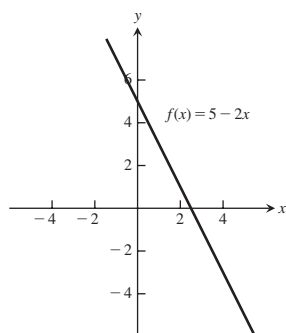
(b) A function of x because for every x there is only one possible y

9. $A = \frac{\sqrt{3}}{4}x^2$, $p = 3x$

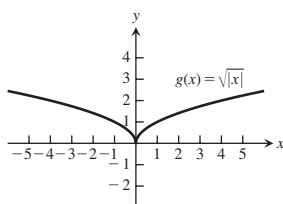
11. $x = \frac{d}{\sqrt{3}}$, $A = 2d^2$, $V = \frac{d^3}{3\sqrt{3}}$

13. $L = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

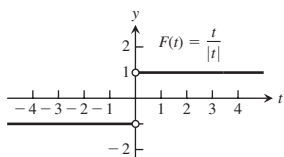
15. $(-\infty, \infty)$



17. $(-\infty, \infty)$

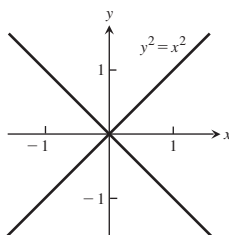
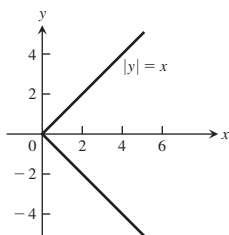


19. $(-\infty, 0) \cup (0, \infty)$

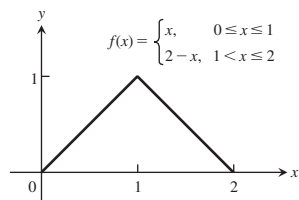


21. $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

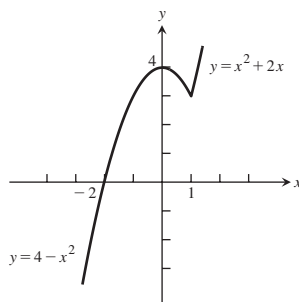
23. (a) For each positive value of x , there are two values of y . (b) For each value of $x \neq 0$, there are two values of y .



25.



27.



29. (a) $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x + 2, & 1 < x \leq 2 \end{cases}$

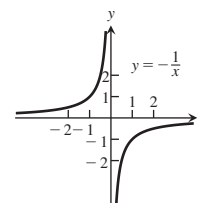
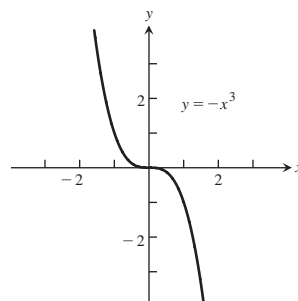
(b) $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

31. (a) $f(x) = \begin{cases} -x, & -1 \leq x < 0 \\ 1, & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$

(b) $f(x) = \begin{cases} \frac{1}{2}x, & -2 \leq x \leq 0 \\ -2x + 2, & 0 < x \leq 1 \\ -1, & 1 < x \leq 3 \end{cases}$

33. (a) $0 \leq x < 1$ (b) $-1 < x \leq 0$ 35. Yes

37. Symmetric about the origin 39. Symmetric about the origin

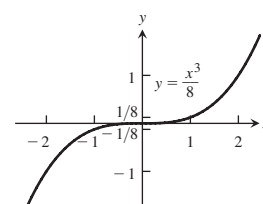
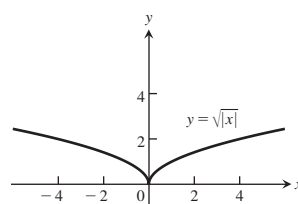


Inc. $-\infty < x < 0$ and $0 < x < \infty$

Dec. $-\infty < x < \infty$

41. Symmetric about the y-axis

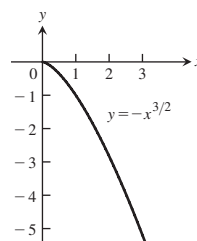
43. Symmetric about the origin



Dec. $-\infty < x \leq 0$;

Inc. $0 \leq x < \infty$

45. No symmetry



Dec. $0 \leq x < \infty$

47. Even 49. Even 51. Odd 53. Even

55. Neither 57. Neither 59. Odd 61. Even

63. $t = 180$ 65. $s = 2.4$ 67. $V = x(14 - 2x)(22 - 2x)$

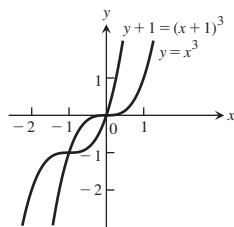
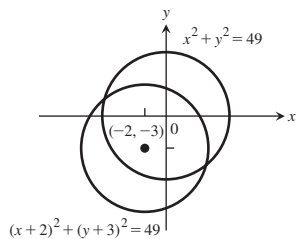
69. (a) h (b) f (c) g 71. (a) $(-2, 0) \cup (4, \infty)$

75. $C = 5(2 + \sqrt{2})h$

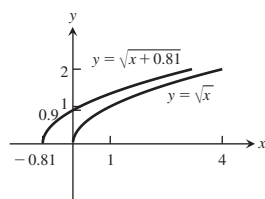
SECTION 1.2, pp. 18-21

1. $D_f: -\infty < x < \infty$, $D_g: x \geq 1$, $R_f: -\infty < y < \infty$,
 $R_g: y \geq 0$, $D_{f+g} = D_{fg} = D_g$, $R_{f+g}: y \geq 1$, $R_{fg}: y \geq 0$
3. $D_f: -\infty < x < \infty$, $D_g: -\infty < x < \infty$, $R_f: y = 2$, $R_g: y \geq 1$,
 $D_{f/g}: -\infty < x < \infty$, $R_{f/g}: 0 < y \leq 2$, $D_{g/f}: -\infty < x < \infty$,
 $R_{g/f}: y \geq 1/2$
5. (a) 2 (b) 22 (c) $x^2 + 2$ (d) $x^2 + 10x + 22$ (e) 5
 (f) -2 (g) $x + 10$ (h) $x^4 - 6x^2 + 6$
7. $13 - 3x$ 9. $\sqrt{\frac{5x+1}{4x+1}}$
11. (a) $f(g(x))$ (b) $j(g(x))$ (c) $g(g(x))$ (d) $j(j(x))$
 (e) $g(h(f(x)))$ (f) $h(j(f(x)))$
13.

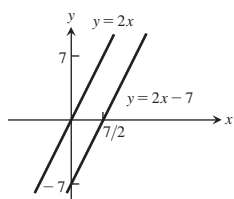
$g(x)$	$f(x)$	$(f \circ g)(x)$
(a) $x - 7$	$\sqrt{x}$	$\sqrt{x - 7}$
(b) $x + 2$	$\frac{3x}{x - 1}$	$\frac{3x + 6}{x - 1}$
(c) x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
(d) $\frac{x}{x - 1}$	$\frac{x}{x - 1}$	x
(e) $\frac{1}{x - 1}$	$1 + \frac{1}{x}$	x
(f) $\frac{1}{x}$	$\frac{1}{x}$	x
15. (a) 1 (b) 2 (c) -2 (d) 0 (e) -1 (f) 0
17. (a) $f(g(x)) = \sqrt{\frac{1}{x} + 1}$, $g(f(x)) = \frac{1}{\sqrt{x + 1}}$
 (b) $D_{f \circ g} = (-\infty, -1] \cup (0, \infty)$, $D_{g \circ f} = (-1, \infty)$
 (c) $R_{f \circ g} = [0, 1) \cup (1, \infty)$, $R_{g \circ f} = (0, \infty)$
19. $g(x) = \frac{2x}{x - 1}$ 21. $V(t) = 4t^2 - 8t + 6$
23. (a) $y = -(x + 7)^2$ (b) $y = -(x - 4)^2$
25. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3
27. $(x + 2)^2 + (y + 3)^2 = 49$ 29. $y + 1 = (x + 1)^3$



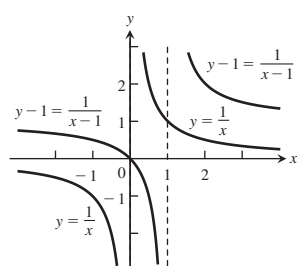
31. $y = \sqrt{x + 0.81}$



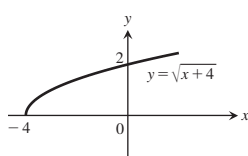
33. $y = 2x$



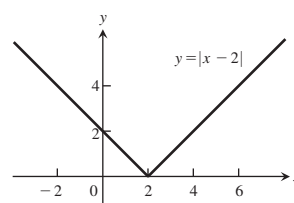
35. $y - 1 = \frac{1}{x - 1}$



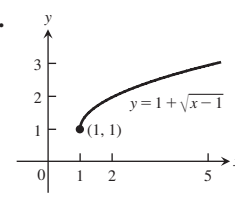
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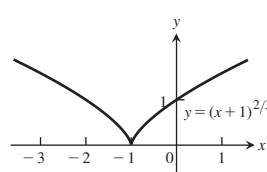
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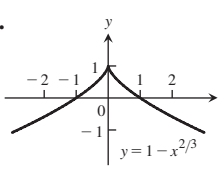
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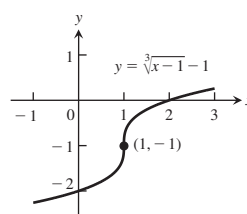
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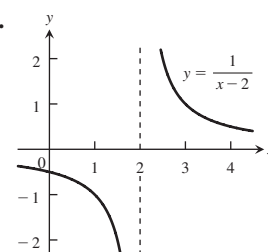
45.



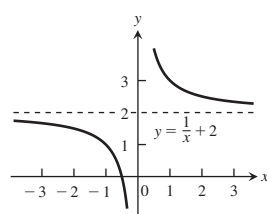
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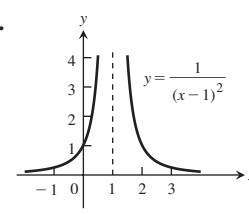
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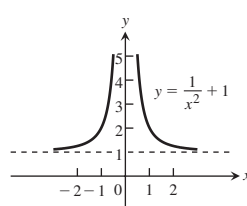
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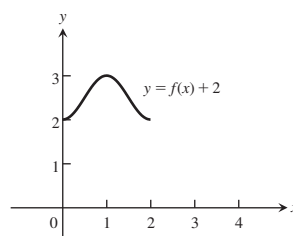
53.



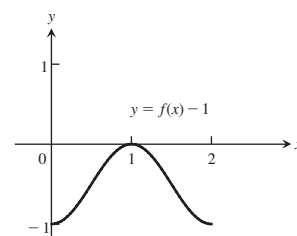
55.



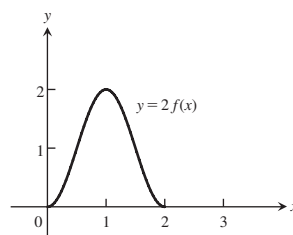
57. (a) $D: [0, 2]$, $R: [2, 3]$



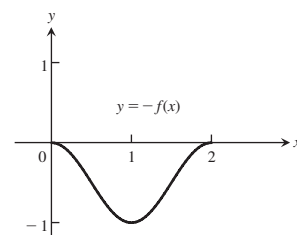
(b) $D: [0, 2]$, $R: [-1, 0]$



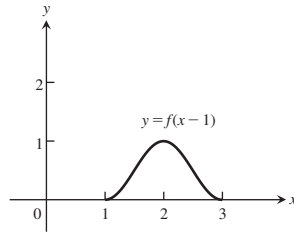
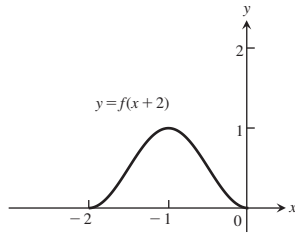
(c) $D: [0, 2]$, $R: [0, 2]$



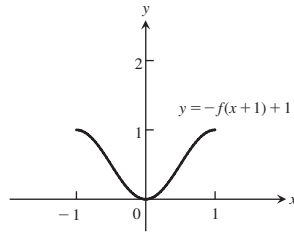
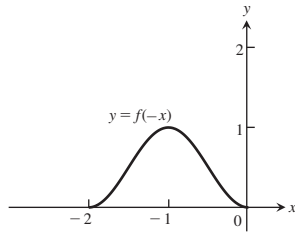
(d) $D: [0, 2]$, $R: [-1, 0]$



(e) $D: [-2, 0], R: [0, 1]$ (f) $D: [1, 3], R: [0, 1]$

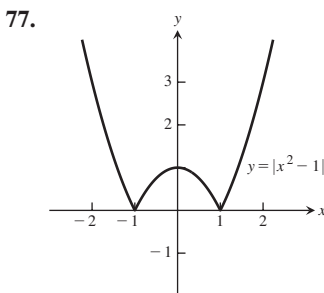
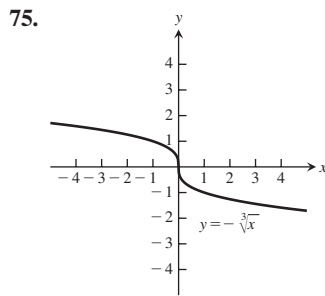
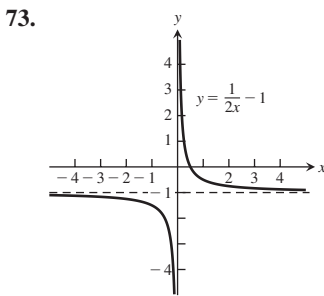
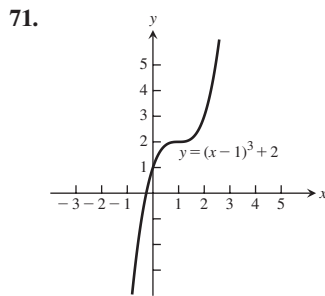
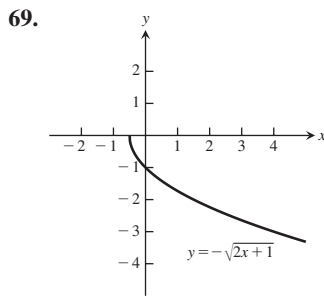


(g) $D: [-2, 0], R: [0, 1]$ (h) $D: [-1, 1], R: [0, 1]$



59. $y = 3x^2 - 3$ 61. $y = \frac{1}{2} + \frac{1}{2x^2}$ 63. $y = \sqrt{4x + 1}$

65. $y = \sqrt{4 - \frac{x^2}{4}}$ 67. $y = 1 - 27x^3$



79. (a) Odd (b) Odd (c) Odd (d) Even (e) Even
(f) Even (g) Even (h) Even (i) Odd

SECTION 1.3, pp. 27-29

1. (a) 8π m (b) $\frac{55\pi}{9}$ m 3. 8.4 in.

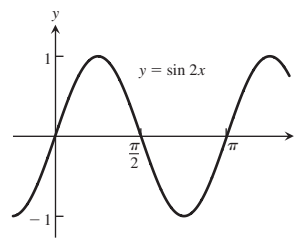
θ	$-\pi$	$-2\pi/3$	0	$\pi/2$	$3\pi/4$
$\sin \theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos \theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	0	$\sqrt{3}$	0	UND	-1
$\cot \theta$	UND	$\frac{1}{\sqrt{3}}$	UND	0	-1
$\sec \theta$	-1	-2	1	UND	$-\sqrt{2}$
$\csc \theta$	UND	$-\frac{2}{\sqrt{3}}$	UND	1	$\sqrt{2}$

7. $\cos x = -4/5, \tan x = -3/4$

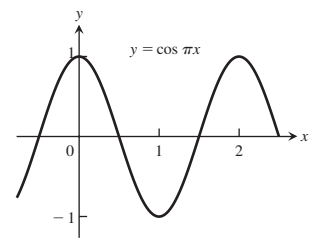
9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

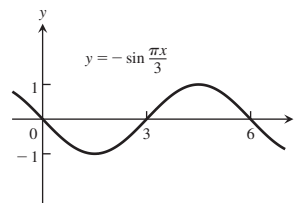
13. Period π



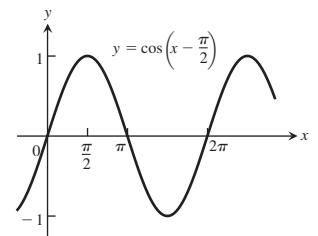
15. Period 2



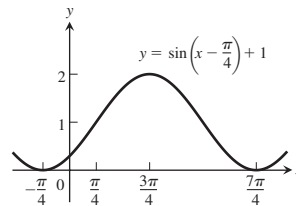
17. Period 6



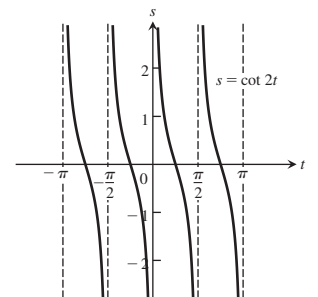
19. Period 2π



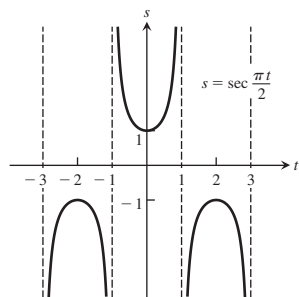
21. Period 2π



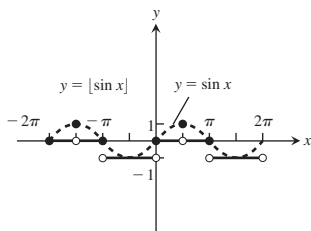
23. Period $\pi/2$, symmetric about the origin



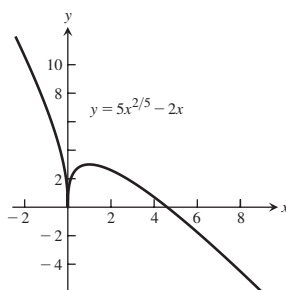
25. Period 4, symmetric about the y-axis



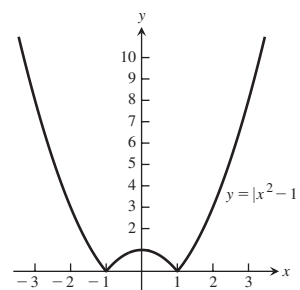
29. $D : (-\infty, \infty)$,
 $R : y = -1, 0, 1$



13. $[-2, 8]$ by $[-5, 10]$



15. $[-3, 3]$ by $[0, 10]$



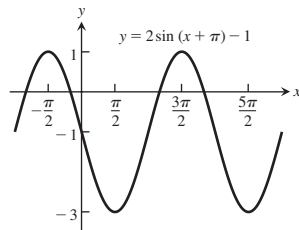
39. $-\cos x$ 41. $-\cos x$ 43. $\frac{\sqrt{6} + \sqrt{2}}{4}$ 45. $\frac{\sqrt{2} + \sqrt{6}}{4}$

47. $\frac{2 + \sqrt{2}}{4}$ 49. $\frac{2 - \sqrt{3}}{4}$ 51. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

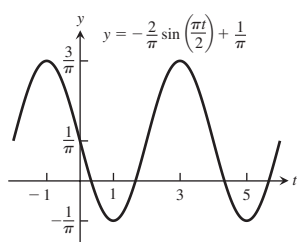
53. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ 59. $\sqrt{7} \approx 2.65$ 63. $a = 1.464$

65. $r = \frac{\alpha \sin \theta}{1 - \sin \theta}$

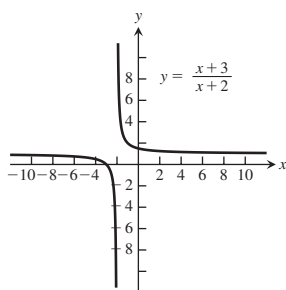
67. $A = 2, B = 2\pi$,
 $C = -\pi, D = -1$



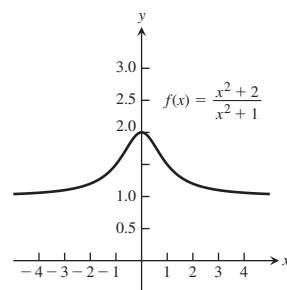
69. $A = -\frac{2}{\pi}, B = 4$,
 $C = 0, D = \frac{1}{\pi}$



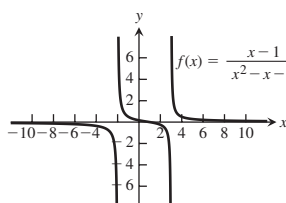
17. $[-10, 10]$ by $[-10, 10]$



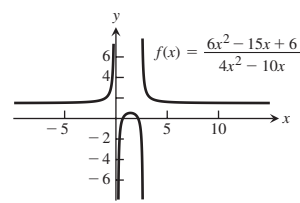
19. $[-4, 4]$ by $[0, 3]$



21. $[-10, 10]$ by $[-6, 6]$



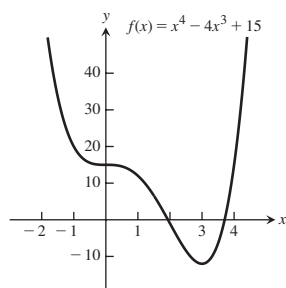
23. $[-6, 10]$ by $[-6, 6]$



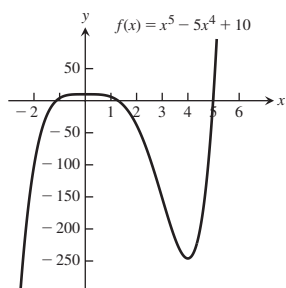
SECTION 1.4, p. 33

1. d 3. d

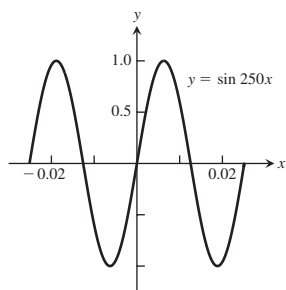
5. $[-3, 5]$ by $[-15, 40]$



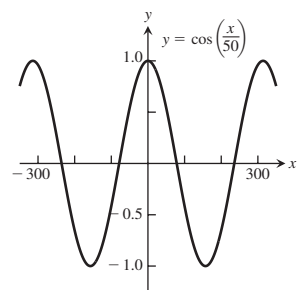
7. $[-3, 6]$ by $[-250, 50]$



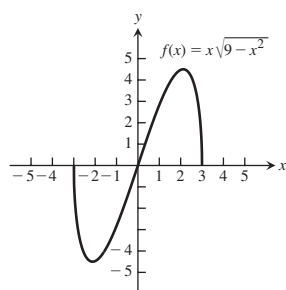
25. $\left[-\frac{\pi}{125}, \frac{\pi}{125}\right]$ by
 $[-1.25, 1.25]$



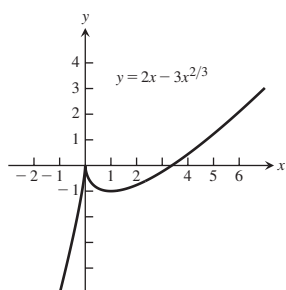
27. $[-100\pi, 100\pi]$ by
 $[-1.25, 1.25]$



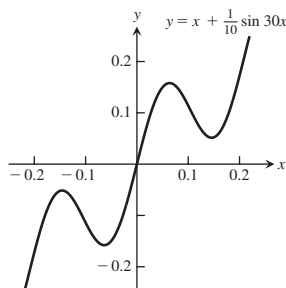
9. $[-5, 5]$ by $[-6, 6]$



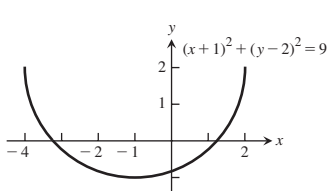
11. $[-2, 6]$ by $[-5, 4]$



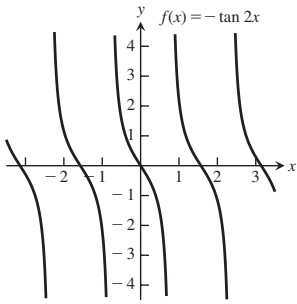
29. $\left[-\frac{\pi}{15}, \frac{\pi}{15}\right]$ by $[-0.25, 0.25]$



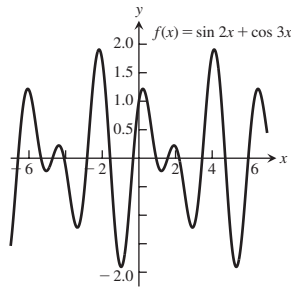
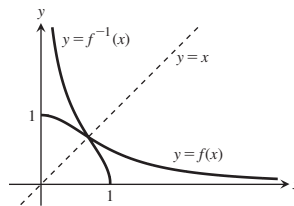
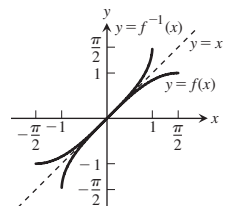
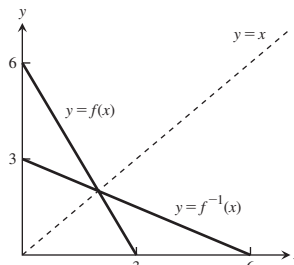
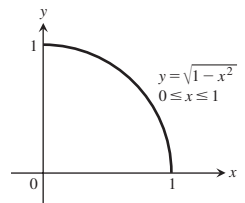
- 31.



33.

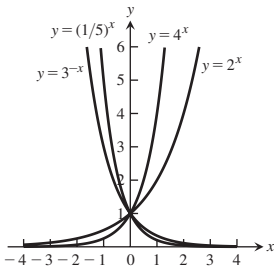


35.

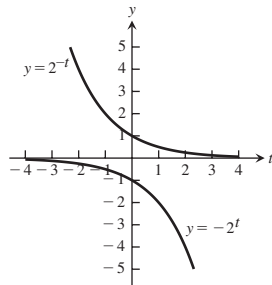
11. $D: (0, 1]$ $R: [0, \infty)$ 13. $D: [-1, 1]$ $R: [-\pi/2, \pi/2]$ 15. $D: [0, 6]$ $R: [0, 3]$ 17. Symmetric about the line $y = x$ 

SECTION 1.5, pp. 37-38

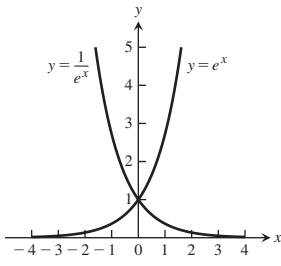
1.



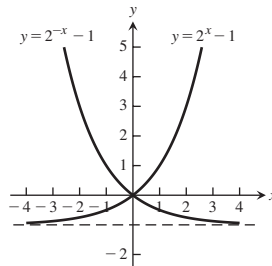
3.



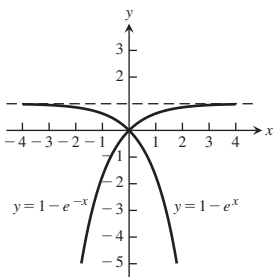
5.



7.



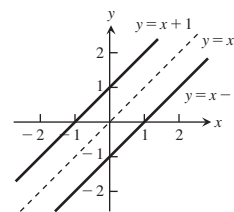
9.

11. $16^{1/4} = 2$ 13. $4^{1/2} = 2$ 15. 5 17. $14^{\sqrt{3}}$ 19. 421. $D: -\infty < x < \infty$; $R: 0 < y < 1/2$ 23. $D: -\infty < t < \infty$; $R: 1 < y < \infty$ 25. $x \approx 2.3219$ 27. $x \approx -0.6309$ 29. After 19 years31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$ (b) About 38 days later33. ≈ 11.433 years, or when interest is paid35. $2^{48} \approx 2.815 \times 10^{14}$

SECTION 1.6, pp. 48-50

1. One-to-one 3. Not one-to-one 5. One-to-one

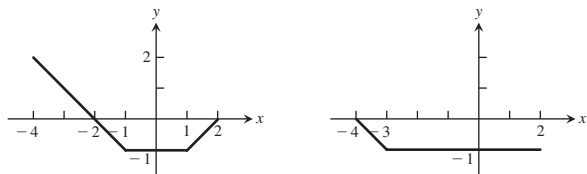
7. Not one-to-one 9. One-to-one

19. $f^{-1}(x) = \sqrt{x-1}$ 21. $f^{-1}(x) = \sqrt[3]{x+1}$ 23. $f^{-1}(x) = \sqrt{x-1}$ 25. $f^{-1}(x) = \sqrt[5]{x}$; $D: -\infty < x < \infty$; $R: -\infty < y < \infty$ 27. $f^{-1}(x) = \sqrt[3]{x-1}$; $D: -\infty < x < \infty$; $R: -\infty < y < \infty$ 29. $f^{-1}(x) = \frac{1}{\sqrt{x}}$; $D: x > 0$; $R: y > 0$ 31. $f^{-1}(x) = \frac{2x+3}{x-1}$; $D: -\infty < x < \infty, x \neq 1$;
 $R: -\infty < y < \infty, y \neq 2$ 33. $f^{-1}(x) = 1 - \sqrt{x+1}$; $D: -1 \leq x < \infty$;
 $R: -\infty < y \leq 1$ 35. $f^{-1}(x) = \frac{2x+b}{x-1}$;
 $D: -\infty < x < \infty, x \neq 1$, $R: -\infty < y < \infty, y \neq 2$ 37. (a) $f^{-1}(x) = \frac{1}{m}x$ (b) The graph of f^{-1} is the line through the origin with slope $1/m$.39. (a) $f^{-1}(x) = x - 1$ (b) $f^{-1}(x) = x - b$. The graph of f^{-1} is a line parallel to the graph of f . The graphs of f and f^{-1} lie on opposite sides of the line $y = x$ and are equidistant from that line.(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.41. (a) $\ln 3 - 2\ln 2$ (b) $2(\ln 2 - \ln 3)$ (c) $-\ln 2$ (d) $\frac{2}{3}\ln 3$ (e) $\ln 3 + \frac{1}{2}\ln 2$ (f) $\frac{1}{2}(3\ln 3 - \ln 2)$ 43. (a) $\ln 5$ (b) $\ln(x-3)$ (c) $\ln\left(\frac{2t^2}{b}\right)$

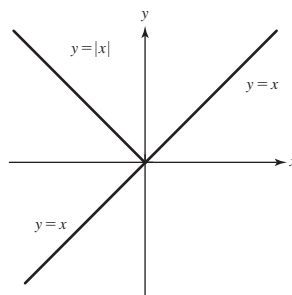
45. (a) 7.2 (b) $\frac{1}{x^2}$ (c) $\frac{x}{y}$ 47. (a) 1 (b) 1 (c) $-x^2 - y^2$
 49. e^{2t+4} 51. $e^{5t} + b$ 53. $y = 2xe^x + 1$
 55. (a) $k = \ln 2$ (b) $k = (1/10) \ln 2$ (c) $k = 1000 \ln a$
 57. (a) $t = -10 \ln 3$ (b) $t = -\frac{\ln 2}{k}$ (c) $t = \frac{\ln 4}{\ln 2}$
 59. $4(\ln x)^2$ 61. $t = \ln 3$ 63. $t = e^2/(e^2 - 1)$
 65. (a) 7 (b) $\sqrt{2}$ (c) 75 (d) 2 (e) 0.5 (f) -1
 67. (a) $\sqrt{x}$ (b) x^2 (c) $\sin x$ 69. (a) $\frac{\ln 3}{\ln 2}$ (b) 3 (c) 2
 71. (a) $-\pi/6$ (b) $\pi/4$ (c) $-\pi/3$ 73. (a) π (b) $\pi/2$
 75. Yes, $g(x)$ is also one-to-one.
 77. Yes, $f \circ g$ is also one-to-one.
 79. (a) $f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$ (b) $f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$
 (c) $f^{-1}(x) = \ln\left(\frac{x+1}{1-x}\right)$ (d) $f^{-1}(x) = e^{\frac{2x}{x+1}}$
 81. (a) $y = \ln x - 3$ (b) $y = \ln(x - 1)$
 (c) $y = 3 + \ln(x + 1)$ (d) $y = \ln(x - 2) - 4$
 (e) $y = \ln(-x)$ (f) $y = e^x$
 83. ≈ -0.7667
 85. (a) Amount = $8\left(\frac{1}{2}\right)^{t/12}$ (b) 36 hours
 87. ≈ 44.081 years

PRACTICE EXERCISES, pp. 51–53

1. $A = \pi r^2$, $C = 2\pi r$, $A = \frac{C^2}{4\pi}$ 3. $x = \tan \theta$, $y = \tan^2 \theta$
 5. Origin 7. Neither 9. Even 11. Even
 13. Odd 15. Neither
 17. (a) Even (b) Odd (c) Odd (d) Even (e) Even
 19. (a) Domain: all reals (b) Range: $[-2, \infty)$
 21. (a) Domain: $[-4, 4]$ (b) Range: $[0, 4]$
 23. (a) Domain: all reals (b) Range: $(-3, \infty)$
 25. (a) Domain: all reals (b) Range: $[-3, 1]$
 27. (a) Domain: $(3, \infty)$ (b) Range: all reals
 29. (a) Domain: $(-\infty, -1]$ and $[3, \infty)$ (b) Range: $(-\infty, 5]$
 31. (a) Domain: $(-\infty, 0)$ and $(0, \infty)$ (b) Range: $[-4, 4]$
 33. (a) Increasing (b) Neither (c) Decreasing (d) Increasing
 35. (a) Domain: $[-4, 4]$ (b) Range: $[0, 2]$
 37. $f(x) = \begin{cases} 1-x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$
 39. (a) 1 (b) $\frac{1}{\sqrt{2.5}} = \sqrt{\frac{2}{5}}$ (c) $x, x \neq 0$
 (d) $\frac{1}{\sqrt{1/\sqrt{x+2}+2}}$
 41. (a) $(f \circ g)(x) = -x, x \geq -2$, $(g \circ f)(x) = \sqrt{4-x^2}$
 (b) Domain $(f \circ g)$: $[-2, \infty)$, domain $(g \circ f)$: $[-2, 2]$
 (c) Range $(f \circ g)$: $(-\infty, 2]$, range $(g \circ f)$: $[0, 2]$
 43.

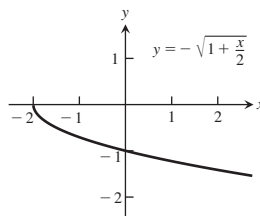


45. Replace the portion for $x < 0$ with the mirror image of the portion for $x > 0$ to make the new graph symmetric with respect to the y -axis.

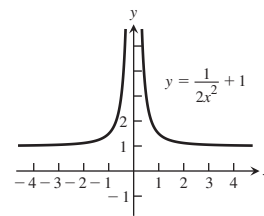


47. Reflects the portion for $y < 0$ across the x -axis
 49. Reflects the portion for $y < 0$ across the x -axis
 51. Adds the mirror image of the portion for $x > 0$ to make the new graph symmetric with respect to the y -axis
 53. (a) $y = g(x - 3) + \frac{1}{2}$ (b) $y = g\left(x + \frac{2}{3}\right) - 2$
 (c) $y = g(-x)$ (d) $y = -g(x)$ (e) $y = 5g(x)$
 (f) $y = g(5x)$

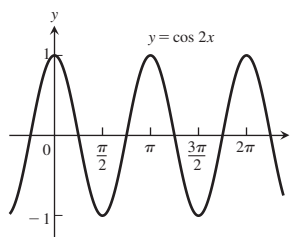
55.



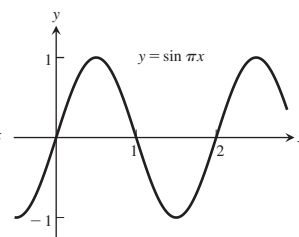
57.



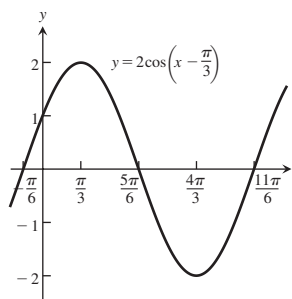
59. Period π



61. Period 2



63.



65. (a) $a = 1$ $b = \sqrt{3}$ (b) $a = 2\sqrt{3}/3$ $c = 4\sqrt{3}/3$
 67. (a) $a = \frac{b}{\tan B}$ (b) $c = \frac{a}{\sin A}$
 69. ≈ 16.98 m 71. (b) 4π
 73. (a) Domain: $-\infty < x < \infty$ (b) Domain: $x > 0$
 75. (a) Domain: $-3 \leq x \leq 3$ (b) Domain: $0 \leq x \leq 4$

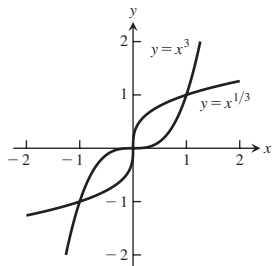
77. $(f \circ g)(x) = \ln(4 - x^2)$ and domain: $-2 < x < 2$;
 $(g \circ f)(x) = 4 - (\ln x)^2$ and domain: $x > 0$;
 $(f \circ f)(x) = \ln(\ln x)$ and domain: $x > 1$;
 $(g \circ g)(x) = -x^4 + 8x^2 - 12$ and domain: $-\infty < x < \infty$.

83. (a) $D: (-\infty, \infty)$ $R: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) $D: [-1, 1]$ $R: [-1, 1]$

85. (a) No (b) Yes

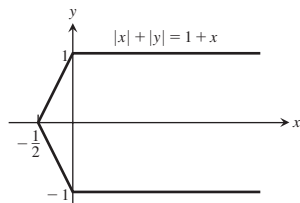
87. (a) $f(g(x)) = (\sqrt[3]{x})^3 = x$, $g(f(x)) = \sqrt[3]{x^3} = x$
 (b)



ADDITIONAL AND ADVANCED EXERCISES, pp. 53–55

1. Yes. For instance: $f(x) = 1/x$ and $g(x) = 1/x$, or $f(x) = 2x$ and $g(x) = x/2$, or $f(x) = e^x$ and $g(x) = \ln x$.
 3. If $f(x)$ is odd, then $g(x) = f(x) - 2$ is not odd. Nor is $g(x)$ even, unless $f(x) = 0$ for all x . If f is even, then $g(x) = f(x) - 2$ is also even.

5.



19. (a) Domain: all reals. Range: If $a > 0$, then (d, ∞) ; if $a < 0$, then $(-\infty, d)$.
 (b) Domain: (c, ∞) , range: all reals
 21. (a) $y = 100,000 - 10,000x$, $0 \leq x \leq 10$ (b) After 4.5 years
 23. After $\frac{\ln(10/3)}{\ln 1.08} \approx 15.6439$ years. (If the bank only pays interest at the end of the year, it will take 16 years.)
 25. $x = 2$, $x = 1$ 27. $1/2$

Chapter 2

SECTION 2.1, pp. 61–63

1. (a) 19 (b) 1
 3. (a) $-\frac{4}{\pi}$ (b) $-\frac{3\sqrt{3}}{\pi}$ 5. 1
 7. (a) 4 (b) $y = 4x - 9$
 9. (a) 2 (b) $y = 2x - 7$
 11. (a) 12 (b) $y = 12x - 16$
 13. (a) -9 (b) $y = -9x - 2$
 15. (a) $-1/4$ (b) $y = -x/4 - 1$
 17. (a) $1/4$ (b) $y = x/4 + 1$

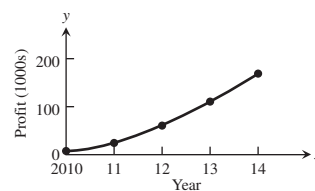
19. Your estimates may not completely agree with these.

(a)	PQ_1	PQ_2	PQ_3	PQ_4
	43	46	49	50

The appropriate units are m/sec.

- (b) ≈ 50 m/sec or 180 km/h

21. (a)



(b) $\approx \$56,000/\text{year}$

(c) $\approx \$42,000/\text{year}$

23. (a) $0.414213, 0.449489, (\sqrt{1+h}-1)/h$ (b) $g(x) = \sqrt{x}$

$1+h$	1.1	1.01	1.001	1.0001
$\sqrt{1+h}$	1.04880	1.004987	1.0004998	1.0000499
$(\sqrt{1+h}-1)/h$	0.4880	0.4987	0.4998	0.499

1.00001	1.000001
1.000005	1.0000005
0.5	0.5

(c) 0.5 (d) 0.5

25. (a) 15 mph, 3.3 mph, 10 mph (b) 10 mph, 0 mph, 4 mph
 (c) 20 mph when $t = 3.5$ h

SECTION 2.2, pp. 71–74

1. (a) Does not exist. As x approaches 1 from the right, $g(x)$ approaches 0. As x approaches 1 from the left, $g(x)$ approaches 1. There is no single number L that all the values $g(x)$ get arbitrarily close to as $x \rightarrow 1$.
 (b) 1 (c) 0 (d) $1/2$
 3. (a) True (b) True (c) False (d) False (e) False
 (f) True (g) True (h) False (i) True (j) True (k) False
 5. As x approaches 0 from the left, $x/|x|$ approaches -1 . As x approaches 0 from the right, $x/|x|$ approaches 1. There is no single number L that the function values all get arbitrarily close to as $x \rightarrow 0$.
 7. Nothing can be said. 9. No; no; no 11. -4 13. -8
 15. 3 17. $-25/2$ 19. 16 21. $3/2$ 23. $1/10$
 25. -7 27. $3/2$ 29. $-1/2$ 31. -1 33. $4/3$
 35. $1/6$ 37. 4 39. $1/2$ 41. $3/2$ 43. -1
 45. 1 47. $1/3$ 49. $\sqrt{4-\pi}$
 51. (a) Quotient Rule (b) Difference and Power Rules
 (c) Sum and Constant Multiple Rules
 53. (a) -10 (b) -20 (c) -1 (d) $5/7$
 55. (a) 4 (b) -21 (c) -12 (d) $-7/3$
 57. 2 59. 3 61. $1/(2\sqrt{7})$ 63. $\sqrt{5}$
 65. (a) The limit is 1.
 67. (a) $f(x) = (x^2 - 9)/(x + 3)$

x	-3.1	-3.01	-3.001	-3.0001	-3.00001	-3.000001
$f(x)$	-6.1	-6.01	-6.001	-6.0001	-6.00001	-6.000001

x	-2.9	-2.99	-2.999	-2.9999	-2.99999	-2.999999
$f(x)$	-5.9	-5.99	-5.999	-5.9999	-5.99999	-5.999999

(c) $\lim_{x \rightarrow -3} f(x) = -6$

69. (a) $G(x) = (x + 6)/(x^2 + 4x - 12)$

x	-5.9	-5.99	-5.999	-5.9999
$G(x)$	-.126582	-.1251564	-.1250156	-.1250015

-5.99999	-5.999999
-.1250001	-.1250000

x	-6.1	-6.01	-6.001	-6.0001
$G(x)$	-.123456	-.124843	-.124984	-.124998

-6.00001	-6.000001
-.124999	-.124999

(c) $\lim_{x \rightarrow -6} G(x) = -1/8 = -0.125$

71. (a) $f(x) = (x^2 - 1)/(|x| - 1)$

x	-1.1	-1.01	-1.001	-1.0001	-1.00001	-1.000001
$f(x)$	2.1	2.01	2.001	2.0001	2.00001	2.000001

x	-.9	-.99	-.999	-.9999	-.99999	-.999999
$f(x)$	1.9	1.99	1.999	1.9999	1.99999	1.999999

(c) $\lim_{x \rightarrow -1} f(x) = 2$

73. (a) $g(\theta) = (\sin \theta)/\theta$

θ	.1	.01	.001	.0001	.00001	.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

θ	-.1	-.01	-.001	-.0001	-.00001	-.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

$\lim_{\theta \rightarrow 0} g(\theta) = 1$

75. (a) $f(x) = x^{1/(1-x)}$

x	.9	.99	.999	.9999	.99999	.999999
$f(x)$	.348678	.366032	.367695	.367861	.367877	.367879

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$f(x)$	.385543	.369711	.368063	.367897	.367881	.367878

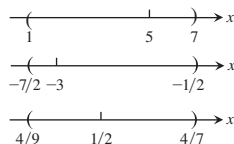
$\lim_{x \rightarrow 1} f(x) \approx 0.36788$

77. $c = 0, 1, -1$; the limit is 0 at $c = 0$, and 1 at $c = 1, -1$.

79. 7 81. (a) 5 (b) 5 83. (a) 0 (b) 0

SECTION 2.3, pp. 79–82

1. $\delta = 2$ 3. $\delta = 1/2$ 5. $\delta = 1/18$ 7. $\delta = 0.1$ 9. $\delta = 7/16$ 11. $\delta = \sqrt{5} - 2$
 13. $\delta = 0.36$ 15. $(3.99, 4.01)$, $\delta = 0.01$
 17. $(-0.19, 0.21)$, $\delta = 0.19$ 19. $(3, 15)$, $\delta = 5$
 21. $(10/3, 5)$, $\delta = 2/3$
 23. $(-\sqrt{4.5}, -\sqrt{3.5})$, $\delta = \sqrt{4.5} - 2 \approx 0.12$
 25. $(\sqrt{15}, \sqrt{17})$, $\delta = \sqrt{17} - 4 \approx 0.12$
 27. $(2 - \frac{0.03}{m}, 2 + \frac{0.03}{m})$, $\delta = \frac{0.03}{m}$
 29. $(\frac{1}{2} - \frac{c}{m}, \frac{1}{2} + \frac{c}{m})$, $\delta = \frac{c}{m}$



31. $L = -3$, $\delta = 0.01$ 33. $L = 4$, $\delta = 0.05$

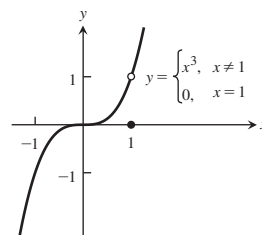
35. $L = 4$, $\delta = 0.75$

55. $[3.384, 3.387]$. To be safe, the left endpoint was rounded up and the right endpoint rounded down.

59. The limit does not exist as x approaches 3.

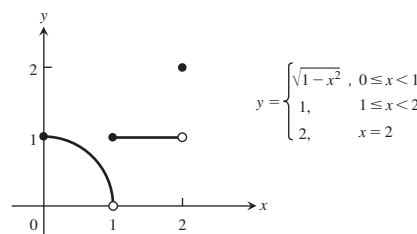
SECTION 2.4, pp. 88–90

1. (a) True (b) True (c) False (d) True
 (e) True (f) True (g) False (h) False
 (i) False (j) False (k) True (l) False
 3. (a) 2, 1 (b) No, $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$
 (c) 3, 3 (d) Yes, 3
 5. (a) No (b) Yes, 0 (c) No
 7. (a) (b) 1, 1 (c) Yes, 1



9. (a) $D: 0 \leq x \leq 2$, $R: 0 < y \leq 1$ and $y = 2$

(b) $[0, 1) \cup (1, 2]$ (c) $x = 2$ (d) $x = 0$



11. $\sqrt{3}$ 13. 1 15. $2/\sqrt{5}$ 17. (a) 1 (b) -1
 19. (a) 1 (b) -1 21. (a) 1 (b) $2/3$ 23. 1 25. $3/4$
 27. 2 29. $1/2$ 31. 2 33. 0 35. 1 37. $1/2$
 39. 0 41. $3/8$ 43. 3 45. 0
 51. $\delta = \epsilon^2$, $\lim_{x \rightarrow 5^+} \sqrt{x-5} = 0$
 55. (a) 400 (b) 399 (c) The limit does not exist.

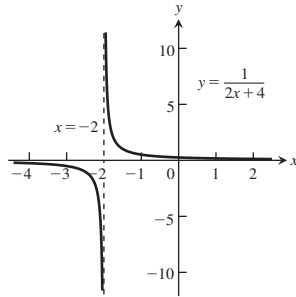
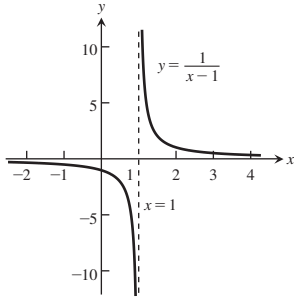
SECTION 2.5, pp. 100–102

1. No; not defined at $x = 2$
 3. Continuous 5. (a) Yes (b) Yes (c) Yes (d) Yes
 7. (a) No (b) No 9. 0
 11. 1, nonremovable; 0, removable 13. All x except $x = 2$
 15. All x except $x = 3$, $x = 1$ 17. All x
 19. All x except $x = 0$ 21. All x except $n\pi/2$, n any integer
 23. All x except $n\pi/2$, n an odd integer 25. All $x \geq -3/2$
 27. All x 29. All x 31. All x except $x = 1$
 33. 0; continuous at $x = \pi$ 35. 1; continuous at $y = 1$
 37. $\sqrt{2}/2$; continuous at $t = 0$ 39. 1; continuous at $x = 0$
 41. $g(3) = 6$ 43. $f(1) = 3/2$ 45. $a = 4/3$
 47. $a = -2, 3$ 49. $a = 5/2, b = -1/2$
 73. $x \approx 1.8794, -1.5321, -0.3473$ 75. $x \approx 1.7549$
 77. $x \approx 3.5156$ 79. $x \approx 0.7391$

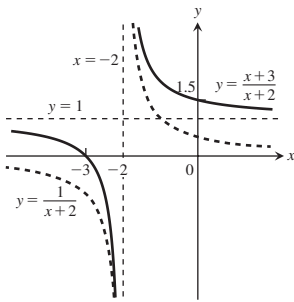
SECTION 2.6, pp. 112–115

1. (a) 0 (b) -2 (c) 2 (d) Does not exist (e) -1
 (f) ∞ (g) Does not exist (h) 1 (i) 0
 3. (a) -3 (b) -3 5. (a) $1/2$ (b) $1/2$ 7. (a) $-5/3$
 (b) $-5/3$ 9. 0 11. -1 13. (a) $2/5$ (b) $2/5$

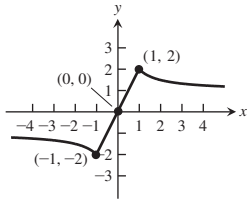
15. (a) 0 (b) 0 17. (a) 7 (b) 7 19. (a) 0 (b) 0
 21. (a) ∞ (b) ∞ 23. 2 25. ∞ 27. 0 29. 1
 31. ∞ 33. 1 35. $1/2$ 37. ∞ 39. $-\infty$
 41. $-\infty$ 43. ∞ 45. (a) ∞ (b) $-\infty$ 47. ∞
 49. ∞ 51. $-\infty$ 53. (a) ∞ (b) $-\infty$ (c) $-\infty$ (d) ∞
 55. (a) $-\infty$ (b) ∞ (c) 0 (d) $3/2$
 57. (a) $-\infty$ (b) $1/4$ (c) $1/4$ (d) $1/4$ (e) It will be $-\infty$.
 59. (a) $-\infty$ (b) ∞ 61. (a) ∞ (b) ∞ (c) ∞ (d) ∞
 63. 65.



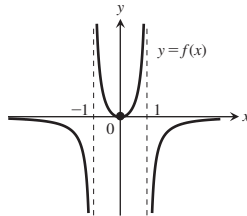
67.

69. Domain: $(-\infty, \infty)$, Range: $[4, 7)$ 71. Domain: $(-\infty, \infty)$, Range: $(-1, 4)$ 73. Domain: $(-\infty, 0)$ and $(0, \infty)$, Range: $(-\infty, -1)$ and $(1, \infty)$

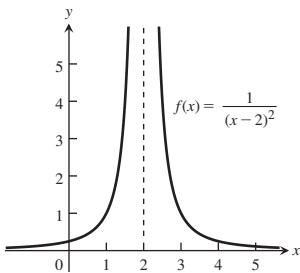
75. Here is one possibility.



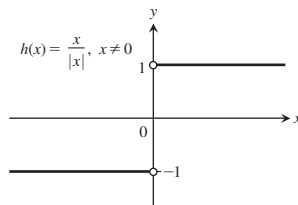
77. Here is one possibility.



79. Here is one possibility.



81. Here is one possibility.

85. At most one 87. 0 89. $-3/4$ 91. $5/2$ 99. (a) For every positive real number B there exists a corresponding number $\delta > 0$ such that for all x

$$c - \delta < x < c \Rightarrow f(x) > B.$$

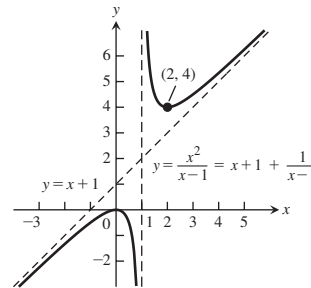
(b) For every negative real number $-B$ there exists a corresponding number $\delta > 0$ such that for all x

$$c < x < c + \delta \Rightarrow f(x) < -B.$$

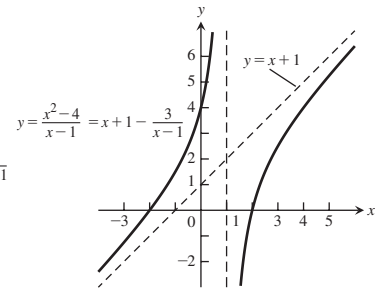
(c) For every negative real number $-B$ there exists a corresponding number $\delta > 0$ such that for all x

$$c - \delta < x < c \Rightarrow f(x) < -B.$$

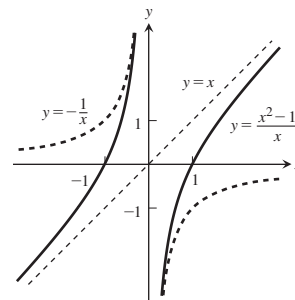
105.



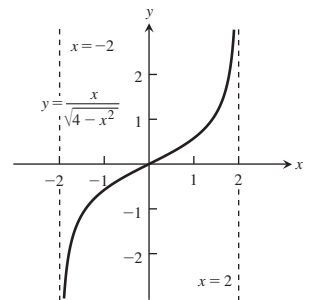
107.



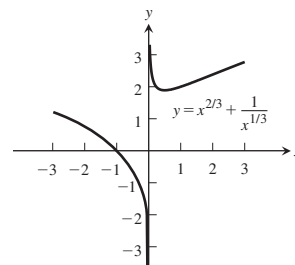
109.



111.

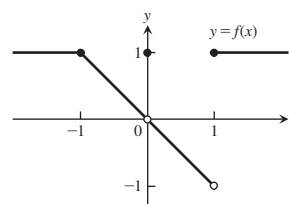


113.

115. At ∞ : ∞ , at $-\infty$: 0

PRACTICE EXERCISES, pp. 116-117

1. At $x = -1$: $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) = 1$, so $\lim_{x \rightarrow -1} f(x) = 1 = f(-1)$; continuous at $x = -1$
 At $x = 0$: $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 0$, so $\lim_{x \rightarrow 0} f(x) = 0$. However, $f(0) \neq 0$, so f is discontinuous at $x = 0$. The discontinuity can be removed by redefining $f(0)$ to be 0.
 At $x = 1$: $\lim_{x \rightarrow 1^-} f(x) = -1$ and $\lim_{x \rightarrow 1^+} f(x) = 1$, so $\lim_{x \rightarrow 1} f(x)$ does not exist. The function is discontinuous at $x = 1$, and the discontinuity is not removable.



3. (a) -21 (b) 49 (c) 0 (d) 1 (e) 1 (f) 7
 (g) -7 (h) $-\frac{1}{7}$ 5. 4
 7. (a) $(-\infty, +\infty)$ (b) $[0, \infty)$ (c) $(-\infty, 0)$ and $(0, \infty)$
 (d) $[0, \infty)$
 9. (a) Does not exist (b) 0 11. $\frac{1}{2}$ 13. $2x$ 15. $-\frac{1}{4}$
 17. $2/3$ 19. $2/\pi$ 21. 1 23. 4 25. $-\infty$
 27. 0 29. 2 31. 0
 35. No in both cases, because $\lim_{x \rightarrow 1} f(x)$ does not exist, and $\lim_{x \rightarrow -1} f(x)$ does not exist.
 37. Yes, f does have a continuous extension, to $a = 1$ with $f(1) = 4/3$.
 39. No 41. $2/5$ 43. 0 45. $-\infty$ 47. 0 49. 1
 51. 1 53. $-\pi/2$ 55. (a) $x = 3$ (b) $x = 1$ (c) $x = -4$
 57. Domain: $[-4, 2)$ and $(2, 4]$, Range: $(-\infty, \infty)$

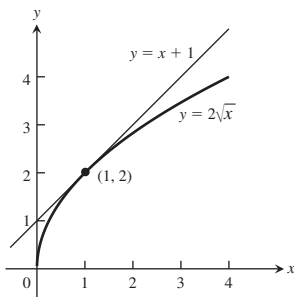
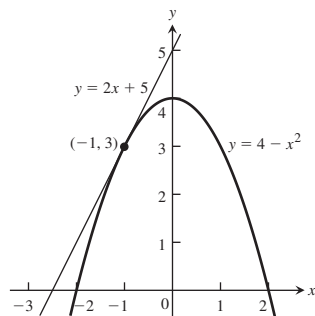
ADDITIONAL AND ADVANCED EXERCISES, pp. 118-120

3. 0; the left-hand limit was taken because the function is undefined for $v > c$.
 5. $65 < t < 75$; within 5°F 13. (a) B (b) A (c) A (d) A
 21. (a) $\lim_{a \rightarrow 0} r_+(a) = 0.5$, $\lim_{a \rightarrow -1^+} r_+(a) = 1$
 (b) $\lim_{a \rightarrow 0} r_-(a)$ does not exist, $\lim_{a \rightarrow -1^+} r_-(a) = 1$
 25. 0 27. 1 29. 4 31. $y = 2x$ 33. $y = x$, $y = -x$
 37. $-4/3$
 39. (a) Domain: $\{0, 1, 1/2, 1/3, 1/4, \dots\}$
 (b) The domain intersects any interval (a, b) containing 0.
 (c) 0
 41. (a) Domain: $(-\infty, -1/\pi] \cup [-1/(2\pi), -1/(3\pi)] \cup [-1/(4\pi), -1/(5\pi)] \cup \dots \cup [1/(5\pi), 1/(4\pi)] \cup [1/(3\pi), 1/(2\pi)] \cup [1/\pi, \infty)$
 (b) The domain intersects any interval (a, b) containing 0.
 (c) 0

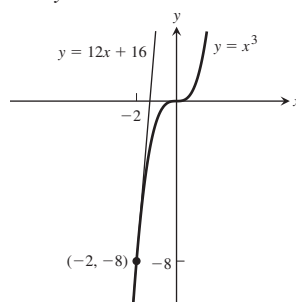
Chapter 3

SECTION 3.1, pp. 123-125

1. $P_1: m_1 = 1$, $P_2: m_2 = 5$ 3. $P_1: m_1 = 5/2$, $P_2: m_2 = -1/2$
 5. $y = 2x + 5$ 7. $y = x + 1$



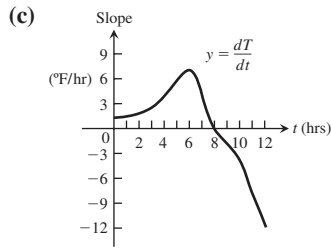
9. $y = 12x + 16$



11. $m = 4$, $y - 5 = 4(x - 2)$
 13. $m = -2$, $y - 3 = -2(x - 3)$
 15. $m = 12$, $y - 8 = 12(t - 2)$
 17. $m = \frac{1}{4}$, $y - 2 = \frac{1}{4}(x - 4)$
 19. $m = -1$ 21. $m = -1/4$
 23. (a) It is the rate of change of the number of cells when $t = 5$.
 The units are the number of cells per hour.
 (b) $P'(3)$ because the slope of the curve is greater there.
 (c) $51.72 \approx 52$ cells/h
 25. $(-2, -5)$ 27. $y = -(x + 1)$, $y = -(x - 3)$
 29. 19.6 m/sec 31. 6π 35. Yes 37. Yes
 39. (a) Nowhere 41. (a) At $x = 0$ 43. (a) Nowhere
 45. (a) At $x = 1$ 47. (a) At $x = 0$

SECTION 3.2, pp. 130-134

1. $-2x$, 6, 0, -2 3. $-\frac{2}{t^3}$, 2, $-\frac{1}{4}$, $-\frac{2}{3\sqrt{3}}$
 5. $\frac{3}{2\sqrt{3\theta}}$, $\frac{3}{2\sqrt{3}}$, $\frac{1}{2}$, $\frac{3}{2\sqrt{2}}$ 7. $6x^2$ 9. $\frac{1}{(2t + 1)^2}$
 11. $\frac{3}{2}q^{1/2}$ 13. $1 - \frac{9}{x^2}$, 0 15. $3t^2 - 2t$, 5
 17. $\frac{-4}{(x - 2)\sqrt{x - 2}}$, $y - 4 = -\frac{1}{2}(x - 6)$ 19. 6
 21. $1/8$ 23. $\frac{-1}{(x + 2)^2}$ 25. $\frac{-1}{(x - 1)^2}$ 27. (b) 29. (d)
 31. (a) $x = 0, 1, 4$
 (b)
 33.
 35. (a) i) 1.5°F/hr ii) 2.9°F/hr
 iii) 0°F/hr iv) -3.7°F/hr

(b) 7.3°F/hr at 12 P.M., -11°F/hr at 6 P.M.

37. Since $\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = 1$

while $\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = 0$,

$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$ does not exist and $f(x)$ is not differentiable at $x = 0$.

39. Since $\lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h} = 2$ while

$\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h} = \frac{1}{2}$, $f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

does not exist and $f(x)$ is not differentiable at $x = 1$.

41. Since $f(x)$ is not continuous at $x = 0$, $f(x)$ is not differentiable at $x = 0$.

43. Since $\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = 3$ while

$\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = 0$, f is not differentiable at $x = 0$.

45. (a) $-3 \leq x \leq 2$ (b) None (c) None

47. (a) $-3 \leq x < 0, 0 < x \leq 3$ (b) None (c) $x = 0$

49. (a) $-1 \leq x < 0, 0 < x \leq 2$ (b) $x = 0$ (c) None

SECTION 3.3, pp. 142-144

1. $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$

3. $\frac{ds}{dt} = 15t^2 - 15t^4$, $\frac{d^2s}{dt^2} = 30t - 60t^3$

5. $\frac{dy}{dx} = 4x^2 - 1 + 2e^x$, $\frac{d^2y}{dx^2} = 8x + 2e^x$

7. $\frac{dw}{dz} = -\frac{6}{z^3} + \frac{1}{z^2}$, $\frac{d^2w}{dz^2} = \frac{18}{z^4} - \frac{2}{z^3}$

9. $\frac{dy}{dx} = 12x - 10 + 10x^{-3}$, $\frac{d^2y}{dx^2} = 12 - 30x^{-4}$

11. $\frac{dr}{ds} = \frac{-2}{3s^3} + \frac{5}{2s^2}$, $\frac{d^2r}{ds^2} = \frac{2}{s^4} - \frac{5}{s^3}$

13. $y' = -5x^4 + 12x^2 - 2x - 3$

15. $y' = 3x^2 + 10x + 2 - \frac{1}{x^2}$ 17. $y' = \frac{-19}{(3x-2)^2}$

19. $g'(x) = \frac{x^2 + x + 4}{(x+0.5)^2}$ 21. $\frac{dv}{dt} = \frac{t^2 - 2t - 1}{(1+t^2)^2}$

23. $f'(s) = \frac{1}{\sqrt{s}(\sqrt{s}+1)^2}$ 25. $v' = -\frac{1}{x^2} + 2x^{-3/2}$

27. $y' = \frac{-4x^3 - 3x^2 + 1}{(x^2-1)^2(x^2+x+1)^2}$ 29. $y' = -2e^{-x} + 3e^{3x}$

31. $y' = 3x^2e^x + x^3e^x$ 33. $y' = \frac{9}{4}x^{5/4} - 2e^{-2x}$

35. $\frac{ds}{dt} = 3t^{1/2}$ 37. $y' = \frac{2}{7x^{5/7}} - ex^{e-1}$ 39. $\frac{dr}{ds} = \frac{se^s - e^s}{s^2}$

41. $y' = 2x^3 - 3x - 1$, $y'' = 6x^2 - 3$, $y''' = 12x$, $y^{(4)} = 12$, $y^{(n)} = 0$ for $n \geq 5$

43. $y' = 3x^2 + 8x + 1$, $y'' = 6x + 8$, $y''' = 6$, $y^{(n)} = 0$ for $n \geq 4$

45. $y' = 2x - 7x^{-2}$, $y'' = 2 + 14x^{-3}$

47. $\frac{dr}{d\theta} = 3\theta^{-4}$, $\frac{d^2r}{d\theta^2} = -12\theta^{-5}$ 49. $\frac{dw}{dz} = -z^{-2} - 1$, $\frac{d^2w}{dz^2} = 2z^{-3}$

51. $\frac{dw}{dz} = 6ze^{2z}(1+z)$, $\frac{d^2w}{dz^2} = 6e^{2z}(1+4z+2z^2)$

53. (a) 13 (b) -7 (c) $7/25$ (d) 20

55. (a) $y = -\frac{x}{8} + \frac{5}{4}$ (b) $m = -4$ at $(0, 1)$

(c) $y = 8x - 15$, $y = 8x + 17$

57. $y = 4x$, $y = 2$ 59. $a = 1$, $b = 1$, $c = 0$

61. $(2, 4)$ 63. $(0, 0)$, $(4, 2)$ 65. $y = -16x + 24$

67. (a) $y = 2x + 2$ (c) $(2, 6)$ 69. 50 71. $a = -3$

73. $P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

75. The Product Rule is then the Constant Multiple Rule, so the latter is a special case of the Product Rule.

77. (a) $\frac{d}{dx}(uvw) = uvw' + uv'w + u'vw$

(b) $\frac{d}{dx}(u_1u_2u_3u_4) = u_1u_2u_3u_4' + u_1u_2u_3'u_4 + u_1u_2'u_3u_4 + u_1'u_2u_3u_4$

(c) $\frac{d}{dx}(u_1 \cdots u_n) = u_1u_2 \cdots u_{n-1}u_n' + u_1u_2 \cdots u_{n-2}u_{n-1}'u_n + \cdots + u_1'u_2 \cdots u_n$

79. $\frac{dP}{dV} = -\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$

SECTION 3.4, pp. 150-154

1. (a) -2 m, -1 m/sec

(b) 3 m/sec, 1 m/sec; 2 m/sec², 2 m/sec²

(c) Changes direction at $t = 3/2$ sec

3. (a) -9 m, -3 m/sec

(b) 3 m/sec, 12 m/sec; 6 m/sec², -12 m/sec²

(c) No change in direction

5. (a) -20 m, -5 m/sec

(b) 45 m/sec, $(1/5)$ m/sec; 140 m/sec², $(4/25)$ m/sec²

(c) No change in direction

7. (a) $a(1) = -6$ m/sec², $a(3) = 6$ m/sec²

(b) $v(2) = 3$ m/sec (c) 6 m

9. Mars: ≈ 7.5 sec, Jupiter: ≈ 1.2 sec

11. $g_s = 0.75$ m/sec²

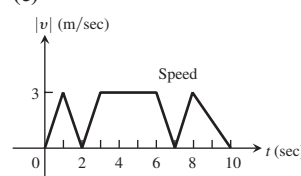
13. (a) $v = -32t$, $|v| = 32t$ ft/sec, $a = -32$ ft/sec²

(b) $t \approx 3.3$ sec

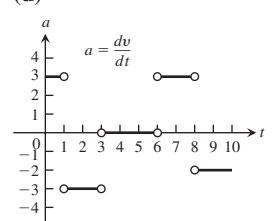
(c) $v \approx -107.0$ ft/sec

15. (a) $t = 2$, $t = 7$ (b) $3 \leq t \leq 6$

(c)



(d)



17. (a) 190 ft/sec (b) 2 sec (c) 8 sec, 0 ft/sec
 (d) 10.8 sec, 90 ft/sec (e) 2.8 sec
 (f) Greatest acceleration happens 2 sec after launch
 (g) Constant acceleration between 2 and 10.8 sec, -32 ft/sec^2

19. (a) $\frac{4}{7} \text{ sec}$, 280 cm/sec (b) 560 cm/sec, 980 cm/sec²
 (c) 29.75 flashes/sec

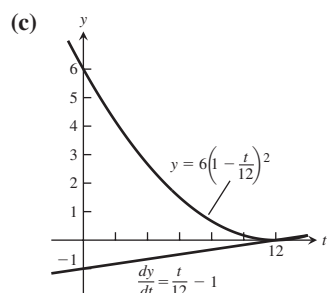
21. C = position, A = velocity, B = acceleration

23. (a) \$110/machine (b) \$80 (c) \$79.90

25. (a) $b'(0) = 10^4 \text{ bacteria/h}$ (b) $b'(5) = 0 \text{ bacteria/h}$
 (c) $b'(10) = -10^4 \text{ bacteria/h}$

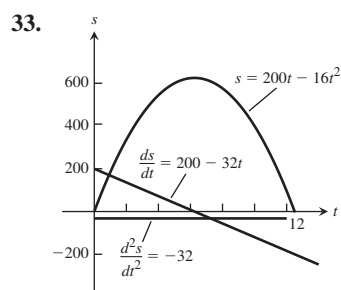
27. (a) $\frac{dy}{dt} = \frac{t}{12} - 1$

- (b) The largest value of $\frac{dy}{dt}$ is 0 m/h when $t = 12$ and the smallest value of $\frac{dy}{dt}$ is -1 m/h when $t = 0$.

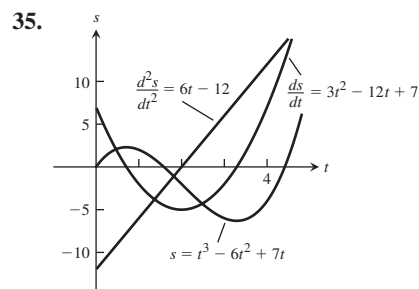


29. 4.88 ft, 8.66 ft, additional ft to stop car for 1 mph speed increase

31. $t = 25 \text{ sec}$, $D = \frac{6250}{9} \text{ m}$



- (a) $v = 0$ when $t = 6.25 \text{ sec}$
 (b) $v > 0$ when $0 \leq t < 6.25 \Rightarrow$ the object moves up; $v < 0$ when $6.25 < t \leq 12.5 \Rightarrow$ the object moves down.
 (c) The object changes direction at $t = 6.25 \text{ sec}$.
 (d) The object speeds up on $(6.25, 12.5]$ and slows down on $[0, 6.25)$.
 (e) The object is moving fastest at the endpoints $t = 0$ and $t = 12.5$ when it is traveling 200 ft/sec. It's moving slowest at $t = 6.25$ when the speed is 0.
 (f) When $t = 6.25$ the object is $s = 625 \text{ m}$ from the origin and farthest away.



- (a) $v = 0$ when $t = \frac{6 \pm \sqrt{15}}{3} \text{ sec}$

- (b) $v < 0$ when $\frac{6 - \sqrt{15}}{3} < t < \frac{6 + \sqrt{15}}{3} \Rightarrow$
 the object moves left; $v > 0$ when $0 \leq t < \frac{6 - \sqrt{15}}{3}$ or
 $\frac{6 + \sqrt{15}}{3} < t \leq 4 \Rightarrow$ the object moves right.

- (c) The object changes direction at $t = \frac{6 \pm \sqrt{15}}{3} \text{ sec}$.

- (d) The object speeds up on $\left(\frac{6 - \sqrt{15}}{3}, 2\right) \cup \left(\frac{6 + \sqrt{15}}{3}, 4\right]$
 and slows down on $\left[0, \frac{6 - \sqrt{15}}{3}\right) \cup \left(2, \frac{6 + \sqrt{15}}{3}\right)$.

- (e) The object is moving fastest at $t = 0$ and $t = 4$ when it is moving 7 units/sec and slowest at $t = \frac{6 \pm \sqrt{15}}{3} \text{ sec}$.

- (f) When $t = \frac{6 + \sqrt{15}}{3}$ the object is at position $s \approx -6.303$ units and farthest from the origin.

SECTION 3.5, pp. 158–160

1. $-10 - 3 \sin x$ 3. $2x \cos x - x^2 \sin x$

5. $-\csc x \cot x - \frac{2}{\sqrt{x}} - \frac{7}{e^x}$ 7. $\sin x \sec^2 x + \sin x$

9. $(e^{-x} \sec x)(1 - x + x \tan x)$ 11. $\frac{-\csc^2 x}{(1 + \cot x)^2}$

13. $4 \tan x \sec x - \csc^2 x$ 15. 0

17. $3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x$

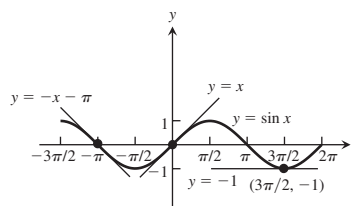
19. $\sec^2 t + e^{-t}$ 21. $\frac{-2 \csc t \cot t}{(1 - \csc t)^2}$ 23. $-\theta(\theta \cos \theta + 2 \sin \theta)$

25. $\sec \theta \csc \theta (\tan \theta - \cot \theta) = \sec^2 \theta - \csc^2 \theta$ 27. $\sec^2 q$

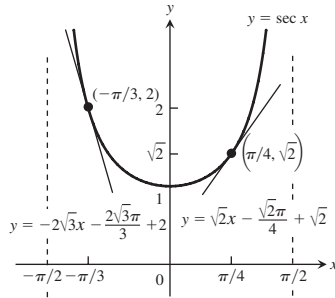
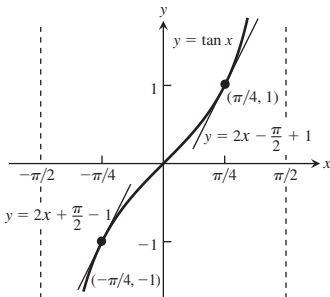
29. $\sec^2 q$ 31. $\frac{q^3 \cos q - q^2 \sin q - q \cos q - \sin q}{(q^2 - 1)^2}$

33. (a) $2 \csc^3 x - \csc x$ (b) $2 \sec^3 x - \sec x$

35.



37.

39. Yes, at $x = \pi$ 41. No 43. Yes, at $x = 0, \pi$, and 2π 45. $\left(-\frac{\pi}{4}, -1\right); \left(\frac{\pi}{4}, 1\right)$ 47. (a) $y = -x + \pi/2 + 2$ (b) $y = 4 - \sqrt{3}$ 49. 0 51. $\sqrt{3}/2$ 53. -1 55. 057. $-\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec², $\sqrt{2}$ m/sec³59. $c = 9$ 61. (a) $\sin x$ (b) $3 \cos x - \sin x$
(c) $73 \sin x + x \cos x$ 63. (a) i) 10 cm ii) 5 cm iii) $-5\sqrt{2} \approx -7.1$ cm
(b) i) 0 cm/sec ii) $-5\sqrt{3} \approx -8.7$ cm/sec
iii) $-5\sqrt{2} \approx -7.1$ cm/sec

SECTION 3.6, pp. 166–169

1. $12x^3$ 3. $3 \cos(3x + 1)$ 5. $\frac{\cos x}{2\sqrt{\sin x}}$ 7. $2\pi x \sec^2(\pi x^2)$ 9. With $u = (2x + 1)$, $y = u^5$: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 5u^4 \cdot 2 = 10(2x + 1)^4$ 11. With $u = (1 - (x/7))$, $y = u^{-7}$: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -7u^{-8} \cdot \left(-\frac{1}{7}\right) = \left(1 - \frac{x}{7}\right)^{-8}$ 13. With $u = ((x^2/8) + x - (1/x))$, $y = u^4$: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 4u^3 \cdot \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right) = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$ 15. With $u = \tan x$, $y = \sec u$: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\sec u \tan u)(\sec^2 x) = \sec(\tan x) \tan(\tan x) \sec^2 x$ 17. With $u = \tan x$, $y = u^3$: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 3u^2 \sec^2 x = 3 \tan^2 x (\sec^2 x)$ 19. $y = e^u$, $u = -5x$, $\frac{dy}{dx} = -5e^{-5x}$ 21. $y = e^u$, $u = 5 - 7x$, $\frac{dy}{dx} = -7e^{(5-7x)}$ 23. $-\frac{1}{2\sqrt{3-t}}$ 25. $\frac{4}{\pi}(\cos 3t - \sin 5t)$ 27. $\frac{\csc \theta}{\cot \theta + \csc \theta}$ 29. $2x \sin^4 x + 4x^2 \sin^3 x \cos x + \cos^2 x + 2x \cos^3 x \sin x$ 31. $(3x - 2)^5 - \frac{1}{x^3 \left(4 - \frac{1}{2x^2}\right)^2}$ 33. $\frac{(4x + 3)^3(4x + 7)}{(x + 1)^4}$ 35. $(1 - x)e^{-x} + 3x^2 e^{x^3}$ 37. $\left(\frac{5}{2}x^2 - 3x + 3\right)e^{5x/2}$ 39. $\sqrt{x} \sec^2(2\sqrt{x}) + \tan(2\sqrt{x})$ 41. $\frac{x \sec x \tan x + \sec x}{2\sqrt{7} + x \sec x}$ 43. $\frac{2 \sin \theta}{(1 + \cos \theta)^2}$ 45. $-2 \sin(\theta^2) \sin 2\theta + 2\theta \cos(2\theta) \cos(\theta^2)$ 47. $\left(\frac{t+2}{2(t+1)^{3/2}}\right) \cos\left(\frac{t}{\sqrt{t+1}}\right)$ 49. $2\theta e^{-\theta^2} \sin(e^{-\theta^2})$ 51. $2\pi \sin(\pi t - 2) \cos(\pi t - 2)$ 53. $\frac{8 \sin(2t)}{(1 + \cos 2t)^5}$ 55. $10t^{10} \tan^9 t \sec^2 t + 10t^9 \tan^{10} t$ 57. $\frac{dy}{dt} = -2\pi \sin(\pi t - 1) \cdot \cos(\pi t - 1) \cdot e^{\cos^2(\pi t - 1)}$ 59. $\frac{-3t^6(t^2 + 4)}{(t^3 - 4t)^4}$ 61. $-2 \cos(\cos(2t - 5))(\sin(2t - 5))$ 63. $\left(1 + \tan^4\left(\frac{t}{12}\right)\right)^2 \left(\tan^3\left(\frac{t}{12}\right) \sec^2\left(\frac{t}{12}\right)\right)$ 65. $-\frac{t \sin(t^2)}{\sqrt{1 + \cos(t^2)}}$ 67. $6 \tan(\sin^3 t) \sec^2(\sin^3 t) \sin^2 t \cos t$ 69. $3(2t^2 - 5)^3(18t^2 - 5)$ 71. $\frac{6}{x^3} \left(1 + \frac{1}{x}\right) \left(1 + \frac{2}{x}\right)$ 73. $2 \csc^2(3x - 1) \cot(3x - 1)$ 75. $16(2x + 1)^2(5x + 1)$ 77. $2(2x^2 + 1)e^{x^2}$ 79. $f'(x) = 0$ for $x = 1, 4$; $f''(x) = 0$ for $x = 2, 4$ 81. $5/2$ 83. $-\pi/4$ 85. 0 87. -589. (a) $2/3$ (b) $2\pi + 5$ (c) $15 - 8\pi$ (d) $37/6$ (e) -1
(f) $\sqrt{2}/24$ (g) $5/32$ (h) $-5/(3\sqrt{17})$ 91. 5 93. (a) 1 (b) 1 95. $y = 1 - 4x$ 97. (a) $y = \pi x + 2 - \pi$ (b) $\pi/2$

99. It multiplies the velocity, acceleration, and jerk by 2, 4, and 8, respectively.

101. $v(6) = \frac{2}{5}$ m/sec, $a(6) = -\frac{4}{125}$ m/sec²

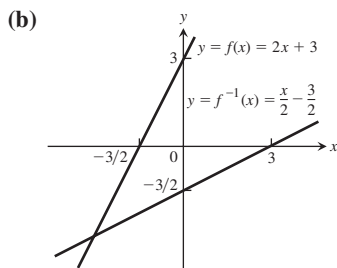
SECTION 3.7, pp. 172–174

1. $\frac{-2xy - y^2}{x^2 + 2xy}$ 3. $\frac{1 - 2y}{2x + 2y - 1}$ 5. $\frac{-2x^3 + 3x^2y - xy^2 + x}{x^2y - x^3 + y}$ 7. $\frac{1}{y(x + 1)^2}$ 9. $\cos y \cot y$ 11. $\frac{-\cos^2(xy) - y}{x}$ 13. $\frac{-y^2}{y \sin\left(\frac{1}{y}\right) - \cos\left(\frac{1}{y}\right) + xy}$

15. $\frac{2e^{2x} - \cos(x + 3y)}{3 \cos(x + 3y)}$ 17. $-\frac{\sqrt{r}}{\sqrt{\theta}}$ 19. $\frac{-r}{\theta}$
21. $y' = -\frac{x}{y}, y'' = \frac{-y^2 - x^2}{y^3}$
23. $\frac{dy}{dx} = \frac{xe^{x^2} + 1}{y}, \frac{d^2y}{dx^2} = \frac{(2x^2y^2 + y^2 - 2x)e^{x^2} - x^2e^{2x^2} - 1}{y^3}$
25. $y' = \frac{\sqrt{y}}{\sqrt{y} + 1}, y'' = \frac{1}{2(\sqrt{y} + 1)^3}$
27. $y' = \frac{3x^2}{1 - \cos y}, y'' = \frac{6x(1 - \cos y)^2 - 9x^4 \sin y}{(1 - \cos y)^3}$
29. -2 31. $(-2, 1): m = -1, (-2, -1): m = 1$
33. (a) $y = \frac{7}{4}x - \frac{1}{2}$ (b) $y = -\frac{4}{7}x + \frac{29}{7}$
35. (a) $y = 3x + 6$ (b) $y = -\frac{1}{3}x + \frac{8}{3}$
37. (a) $y = \frac{6}{7}x + \frac{6}{7}$ (b) $y = -\frac{7}{6}x - \frac{7}{6}$
39. (a) $y = -\frac{\pi}{2}x + \pi$ (b) $y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$
41. (a) $y = 2\pi x - 2\pi$ (b) $y = -\frac{x}{2\pi} + \frac{1}{2\pi}$
43. Points: $(-\sqrt{7}, 0)$ and $(\sqrt{7}, 0)$, Slope: -2
45. $m = -1$ at $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$, $m = \sqrt{3}$ at $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$
47. $(-3, 2): m = -\frac{27}{8}; (-3, -2): m = \frac{27}{8}; (3, 2): m = \frac{27}{8};$
 $(3, -2): m = -\frac{27}{8}$
49. $(3, -1)$
55. $\frac{dy}{dx} = -\frac{y^3 + 2xy}{x^2 + 3xy^2}, \frac{dx}{dy} = -\frac{x^2 + 3xy^2}{y^3 + 2xy}, \frac{dx}{dy} = \frac{1}{dy/dx}$

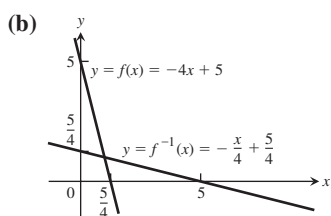
SECTION 3.8, pp. 183-184

1. (a) $f^{-1}(x) = \frac{x}{2} - \frac{3}{2}$



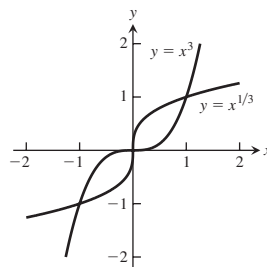
(c) $2, 1/2$

3. (a) $f^{-1}(x) = -\frac{x}{4} + \frac{5}{4}$



(c) $-4, -1/4$

5. (b)



(c) Slope of f at $(1, 1)$: 3; slope of g at $(1, 1)$: $1/3$; slope of f at $(-1, -1)$: 3; slope of g at $(-1, -1)$: $1/3$

(d) $y = 0$ is tangent to $y = x^3$ at $x = 0$; $x = 0$ is tangent to $y = \sqrt[3]{x}$ at $x = 0$.

7. $1/9$ 9. 3 11. $\frac{1}{x} + 1$ 13. $2/t$ 15. $-1/x$
17. $\frac{1}{\theta + 1} - e^\theta$ 19. $3/x$ 21. $2(\ln t) + (\ln t)^2$
23. $x^3 \ln x$ 25. $\frac{1 - \ln t}{t^2}$ 27. $\frac{1}{x(1 + \ln x)^2}$ 29. $\frac{1}{x \ln x}$
31. $2 \cos(\ln \theta)$ 33. $-\frac{3x + 2}{2x(x + 1)}$ 35. $\frac{2}{t(1 - \ln t)^2}$
37. $\frac{\tan(\ln \theta)}{\theta}$ 39. $\frac{10x}{x^2 + 1} + \frac{1}{2(1 - x)}$
41. $\left(\frac{1}{2}\right)\sqrt{x(x + 1)}\left(\frac{1}{x} + \frac{1}{x + 1}\right) = \frac{2x + 1}{2\sqrt{x(x + 1)}}$
43. $\left(\frac{1}{2}\right)\sqrt{\frac{t}{t + 1}}\left(\frac{1}{t} - \frac{1}{t + 1}\right) = \frac{1}{2\sqrt{t}(t + 1)^{3/2}}$
45. $\sqrt{\theta + 3}(\sin \theta)\left(\frac{1}{2(\theta + 3)} + \cot \theta\right)$
47. $t(t + 1)(t + 2)\left[\frac{1}{t} + \frac{1}{t + 1} + \frac{1}{t + 2}\right] = 3t^2 + 6t + 2$
49. $\frac{\theta + 5}{\theta \cos \theta}\left[\frac{1}{\theta + 5} - \frac{1}{\theta} + \tan \theta\right]$
51. $\frac{x\sqrt{x^2 + 1}}{(x + 1)^{2/3}}\left[\frac{1}{x} + \frac{x}{x^2 + 1} - \frac{2}{3(x + 1)}\right]$
53. $\frac{1}{3}\sqrt[3]{\frac{x(x - 2)}{x^2 + 1}}\left(\frac{1}{x} + \frac{1}{x - 2} - \frac{2x}{x^2 + 1}\right)$ 55. $-2 \tan \theta$
57. $\frac{1 - t}{t}$ 59. $1/(1 + e^\theta)$ 61. $e^{\cos t}(1 - t \sin t)$
63. $\frac{ye^y \cos x}{1 - ye^y \sin x}$ 65. $\frac{dy}{dx} = \frac{y^2 - xy \ln y}{x^2 - xy \ln x}$ 67. $2^x \ln x$
69. $\left(\frac{\ln 5}{2\sqrt{s}}\right)5^{\sqrt{s}}$ 71. $\pi x^{(\pi - 1)}$ 73. $\frac{1}{\theta \ln 2}$ 75. $\frac{3}{x \ln 4}$
77. $\frac{2(\ln r)}{r(\ln 2)(\ln 4)}$ 79. $\frac{-2}{(x + 1)(x - 1)}$
81. $\sin(\log_7 \theta) + \frac{1}{\ln 7} \cos(\log_7 \theta)$ 83. $\frac{1}{\ln 5}$
85. $\frac{1}{t}(\log_2 3)3^{\log_2 t}$ 87. $\frac{1}{t}$ 89. $(x + 1)^x\left(\frac{x}{x + 1} + \ln(x + 1)\right)$
91. $(\sqrt{t})^t\left(\frac{\ln t}{2} + \frac{1}{2}\right)$ 93. $(\sin x)^x(\ln \sin x + x \cot x)$
95. $(x^{\ln x})\left(\frac{\ln x^2}{x}\right)$ 97. $\frac{3y - xy \ln y}{x^2 - x}$ 99. $\frac{1 - xy \ln y}{x^2(1 + \ln y)}$

SECTION 3.9, pp. 189–191

1. (a) $\pi/4$ (b) $-\pi/3$ (c) $\pi/6$
 3. (a) $-\pi/6$ (b) $\pi/4$ (c) $-\pi/3$
 5. (a) $\pi/3$ (b) $3\pi/4$ (c) $\pi/6$
 7. (a) $3\pi/4$ (b) $\pi/6$ (c) $2\pi/3$
 9. $1/\sqrt{2}$ 11. $-1/\sqrt{3}$ 13. $\pi/2$ 15. $\pi/2$ 17. $\pi/2$
 19. 0 21. $\frac{-2x}{\sqrt{1-x^4}}$ 23. $\frac{\sqrt{2}}{\sqrt{1-2t^2}}$
 25. $\frac{1}{|2s+1|\sqrt{s^2+s}}$ 27. $\frac{-2x}{(x^2+1)\sqrt{x^4+2x^2}}$
 29. $\frac{-1}{\sqrt{1-t^2}}$ 31. $\frac{-1}{2\sqrt{t}(1+t)}$ 33. $\frac{1}{(\tan^{-1}x)(1+x^2)}$
 35. $\frac{-e^t}{|e^t|\sqrt{(e^t)^2-1}} = \frac{-1}{\sqrt{e^{2t}-1}}$ 37. $\frac{-2s^2}{\sqrt{1-s^2}}$ 39. 0
 41. $\sin^{-1}x$ 43. 0 45. $\frac{8\sqrt{2}}{4+3\pi}$
 51. (a) Defined; there is an angle whose tangent is 2.
 (b) Not defined; there is no angle whose cosine is 2.
 53. (a) Not defined; no angle has secant 0.
 (b) Not defined; no angle has sine $\sqrt{2}$.
 63. (a) Domain: all real numbers except those having the form $\frac{\pi}{2} + k\pi$ where k is an integer; range: $-\pi/2 < y < \pi/2$
 (b) Domain: $-\infty < x < \infty$; range: $-\infty < y < \infty$
 65. (a) Domain: $-\infty < x < \infty$; range: $0 \leq y \leq \pi$
 (b) Domain: $-1 \leq x \leq 1$; range: $-1 \leq y \leq 1$
 67. The graphs are identical.

SECTION 3.10, pp. 196–199

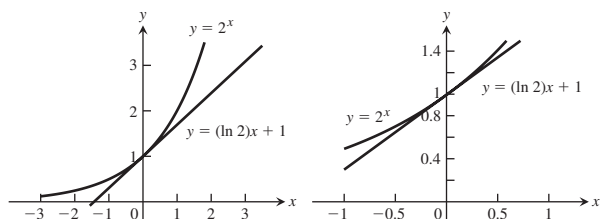
1. $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ 3. 10 5. -6 7. $-3/2$
 9. $31/13$ 11. (a) $-180 \text{ m}^2/\text{min}$ (b) $-135 \text{ m}^3/\text{min}$
 13. (a) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ (b) $\frac{dV}{dt} = 2\pi hr \frac{dr}{dt}$
 (c) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi hr \frac{dr}{dt}$
 15. (a) 1 volt/sec (b) $-\frac{1}{3}$ amp/sec
 (c) $\frac{dR}{dt} = \frac{1}{I} \left(\frac{dV}{dt} - \frac{V}{I} \frac{dI}{dt} \right)$
 (d) $3/2$ ohms/sec, R is increasing.
 17. (a) $\frac{ds}{dt} = \frac{x}{\sqrt{x^2+y^2}} \frac{dx}{dt}$
 (b) $\frac{ds}{dt} = \frac{x}{\sqrt{x^2+y^2}} \frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2}} \frac{dy}{dt}$
 (c) $\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$
 19. (a) $\frac{dA}{dt} = \frac{1}{2} ab \cos \theta \frac{d\theta}{dt}$
 (b) $\frac{dA}{dt} = \frac{1}{2} ab \cos \theta \frac{d\theta}{dt} + \frac{1}{2} b \sin \theta \frac{da}{dt}$
 (c) $\frac{dA}{dt} = \frac{1}{2} ab \cos \theta \frac{d\theta}{dt} + \frac{1}{2} b \sin \theta \frac{da}{dt} + \frac{1}{2} a \sin \theta \frac{db}{dt}$
 21. (a) $14 \text{ cm}^2/\text{sec}$, increasing (b) 0 cm/sec, constant
 (c) $-14/13 \text{ cm}^2/\text{sec}$, decreasing
 23. (a) $-12 \text{ ft}^2/\text{sec}$ (b) $-59.5 \text{ ft}^2/\text{sec}$ (c) -1 rad/sec
 25. 20 ft/sec

27. (a) $\frac{dh}{dt} = 11.19 \text{ cm/min}$ (b) $\frac{dr}{dt} = 14.92 \text{ cm/min}$
 29. (a) $\frac{-1}{24\pi} \text{ m/min}$ (b) $r = \sqrt{26y - y^2} \text{ m}$
 (c) $\frac{dr}{dt} = -\frac{5}{288\pi} \text{ m/min}$
 31. 1 ft/min , $40\pi \text{ ft}^2/\text{min}$ 33. 11 ft/sec
 35. Increasing at $466/1681 \text{ L/min}^2$
 37. -5 m/sec 39. -1500 ft/sec
 41. $\frac{5}{72\pi} \text{ in./min}$, $\frac{10}{3} \text{ in}^2/\text{min}$
 43. (a) $-32/\sqrt{13} \approx -8.875 \text{ ft/sec}$
 (b) $d\theta_1/dt = 8/65 \text{ rad/sec}$, $d\theta_2/dt = -8/65 \text{ rad/sec}$
 (c) $d\theta_1/dt = 1/6 \text{ rad/sec}$, $d\theta_2/dt = -1/6 \text{ rad/sec}$
 45. -5.5 deg/min 47. $12\pi \text{ km/min}$

SECTION 3.11, pp. 209–211

1. $L(x) = 10x - 13$ 3. $L(x) = 2$ 5. $L(x) = x - \pi$
 7. $2x$ 9. $-x - 5$ 11. $\frac{1}{12}x + \frac{4}{3}$ 13. $1 - x$
 15. $f(0) = 1$. Also, $f'(x) = k(1+x)^{k-1}$, so $f'(0) = k$. This means the linearization at $x = 0$ is $L(x) = 1 + kx$.
 17. (a) 1.01 (b) 1.003
 19. $\left(3x^2 - \frac{3}{2\sqrt{x}}\right) dx$ 21. $\frac{2-2x^2}{(1+x^2)^2} dx$
 23. $\frac{1-y}{3\sqrt{y}+x} dx$ 25. $\frac{5}{2\sqrt{x}} \cos(5\sqrt{x}) dx$
 27. $(4x^2) \sec^2\left(\frac{x^3}{3}\right) dx$
 29. $\frac{3}{\sqrt{x}} \left(\csc(1-2\sqrt{x}) \cot(1-2\sqrt{x}) \right) dx$
 31. $\frac{1}{2\sqrt{x}} \cdot e^{\sqrt{x}} dx$ 33. $\frac{2x}{1+x^2} dx$ 35. $\frac{2xe^{x^2}}{1+e^{2x^2}} dx$
 37. $\frac{-1}{\sqrt{e^{-2x}-1}} dx$
 39. (a) 0.41 (b) 0.4 (c) 0.01
 41. (a) 0.231 (b) 0.2 (c) 0.031
 43. (a) $-1/3$ (b) $-2/5$ (c) $1/15$
 45. $dV = 4\pi r_0^2 dr$ 47. $dS = 12x_0 dx$ 49. $dV = 2\pi r_0 h dr$
 51. (a) $0.08\pi \text{ m}^2$ (b) 2% 53. $dV \approx 565.5 \text{ in}^3$
 55. (a) 2% (b) 4% 57. $\frac{1}{3}\%$ 59. 3%
 61. The ratio equals 37.87, so a change in the acceleration of gravity on the moon has about 38 times the effect that a change of the same magnitude has on Earth.
 63. Increase $V \approx 40\%$
 65. (a) i) $b_0 = f(a)$ ii) $b_1 = f'(a)$ iii) $b_2 = \frac{f''(a)}{2}$
 (b) $Q(x) = 1 + x + x^2$ (d) $Q(x) = 1 - (x-1) + (x-1)^2$
 (e) $Q(x) = 1 + \frac{x}{2} - \frac{x^2}{8}$
 (f) The linearization of any differentiable function $u(x)$ at $x = a$ is $L(x) = u(a) + u'(a)(x-a) = b_0 + b_1(x-a)$, where b_0 and b_1 are the coefficients of the constant and linear terms of the quadratic approximation. Thus, the linearization for $f(x)$ at $x = 0$ is $1 + x$; the linearization for $g(x)$ at $x = 1$ is $1 - (x-1)$ or $2 - x$; and the linearization for $h(x)$ at $x = 0$ is $1 + \frac{x}{2}$.

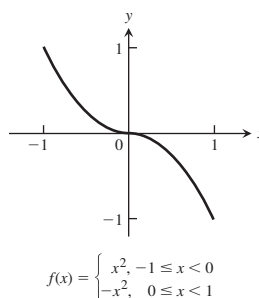
67. (a) $L(x) = x \ln 2 + 1 \approx 0.69x + 1$
(b)



PRACTICE EXERCISES, pp. 212–217

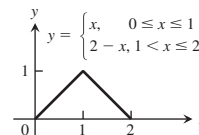
1. $5x^4 - 0.25x + 0.25$ 3. $3x(x - 2)$
5. $2(x + 1)(2x^2 + 4x + 1)$
7. $3(\theta^2 + \sec \theta + 1)^2(2\theta + \sec \theta \tan \theta)$
9. $\frac{1}{2\sqrt{t}(1 + \sqrt{t})^2}$ 11. $2 \sec^2 x \tan x$
13. $8 \cos^3(1 - 2t) \sin(1 - 2t)$ 15. $5(\sec t)(\sec t + \tan t)^5$
17. $\frac{\theta \cos \theta + \sin \theta}{\sqrt{2\theta} \sin \theta}$ 19. $\frac{\cos \sqrt{2\theta}}{\sqrt{2\theta}}$
21. $x \csc\left(\frac{2}{x}\right) + \csc\left(\frac{2}{x}\right) \cot\left(\frac{2}{x}\right)$
23. $\frac{1}{2}x^{1/2} \sec(2x)^2 [16 \tan(2x)^2 - x^{-2}]$
25. $-10x \csc^2(x^2)$ 27. $8x^3 \sin(2x^2) \cos(2x^2) + 2x \sin^2(2x^2)$
29. $\frac{-(t + 1)}{8t^3}$ 31. $\frac{1 - x}{(x + 1)^3}$ 33. $\frac{-1}{2x^2\left(1 + \frac{1}{x}\right)^{1/2}}$
35. $\frac{-2 \sin \theta}{(\cos \theta - 1)^2}$ 37. $3\sqrt{2x + 1}$ 39. $-9 \left[\frac{5x + \cos 2x}{(5x^2 + \sin 2x)^{5/2}} \right]$
41. $-2e^{-x/5}$ 43. xe^{4x} 45. $\frac{2 \sin \theta \cos \theta}{\sin^2 \theta} = 2 \cot \theta$
47. $\frac{2}{(\ln 2)x}$ 49. $-8^{-t}(\ln 8)$ 51. $18x^{2.6}$
53. $(x + 2)^{x+2}(\ln(x + 2) + 1)$ 55. $-\frac{1}{\sqrt{1 - u^2}}$
57. $\frac{-1}{\sqrt{1 - x^2} \cos^{-1} x}$ 59. $\tan^{-1}(t) + \frac{t}{1 + t^2} - \frac{1}{2t}$
61. $\frac{1 - z}{\sqrt{z^2 - 1}} + \sec^{-1} z$ 63. -1 65. $-\frac{y + 2}{x + 3}$
67. $\frac{-3x^2 - 4y + 2}{4x - 4y^{1/3}}$ 69. $-\frac{y}{x}$ 71. $\frac{1}{2y(x + 1)^2}$
73. $-1/2$ 75. y/x 77. $-\frac{2e^{-\tan^{-1} x}}{1 + x^2}$ 79. $\frac{dp}{dq} = \frac{6q - 4p}{3p^2 + 4q}$
81. $\frac{dr}{ds} = (2r - 1)(\tan 2s)$
83. (a) $\frac{d^2y}{dx^2} = \frac{-2xy^3 - 2x^4}{y^5}$ (b) $\frac{d^2y}{dx^2} = \frac{-2xy^2 - 1}{x^4y^3}$
85. (a) 7 (b) -2 (c) 5/12 (d) 1/4 (e) 12 (f) 9/2
(g) 3/4
87. 0 89. $\frac{3\sqrt{2}e^{\sqrt{3/2}}}{4} \cos(e^{\sqrt{3/2}})$ 91. $-\frac{1}{2}$ 93. $\frac{-2}{(2t + 1)^2}$

95. (a)



- (b) Yes (c) Yes

97. (a)



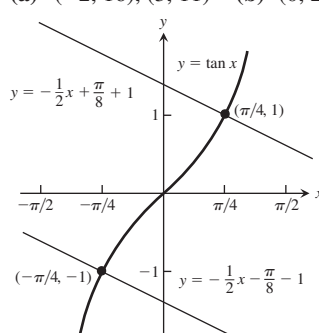
- (b) Yes (c) No

99. $\left(\frac{5}{2}, \frac{9}{4}\right)$ and $\left(\frac{3}{2}, -\frac{1}{4}\right)$

101. $(-1, 27)$ and $(2, 0)$

103. (a) $(-2, 16)$, $(3, 11)$ (b) $(0, 20)$, $(1, 7)$

- 105.



107. $\frac{1}{4}$

109. 4

111. Tangent: $y = -\frac{1}{4}x + \frac{9}{4}$, normal: $y = 4x - 2$

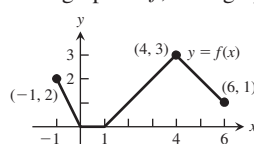
113. Tangent: $y = 2x - 4$, normal: $y = -\frac{1}{2}x + \frac{7}{2}$

115. Tangent: $y = -\frac{5}{4}x + 6$, normal: $y = \frac{4}{5}x - \frac{11}{5}$

117. $(1, 1)$: $m = -\frac{1}{2}$; $(1, -1)$: m not defined

119. $B =$ graph of f , $A =$ graph of f'

- 121.



123. (a) 0, 0 (b) 1700 rabbits, ≈ 1400 rabbits

125. -1 127. 1/2 129. 4 131. 1

133. To make g continuous at the origin, define $g(0) = 1$.

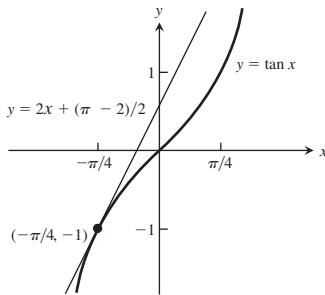
135. $\frac{2(x^2 + 1)}{\sqrt{\cos 2x}} \left[\frac{2x}{x^2 + 1} + \tan 2x \right]$

137. $5 \left[\frac{(t + 1)(t - 1)}{(t - 2)(t + 3)} \right]^5 \left[\frac{1}{t + 1} + \frac{1}{t - 1} - \frac{1}{t - 2} - \frac{1}{t + 3} \right]$

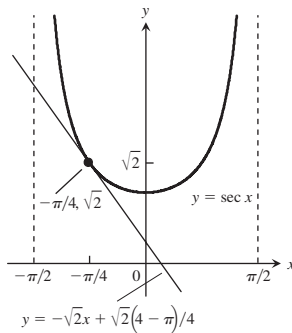
139. $\frac{1}{\sqrt{\theta}} (\sin \theta)^{\sqrt{\theta}} \left(\frac{\ln \sin \theta}{2} + \theta \cot \theta \right)$

141. (a) $\frac{dS}{dt} = (4\pi r + 2\pi h)\frac{dr}{dt}$
 (b) $\frac{dS}{dt} = 2\pi r\frac{dh}{dt}$
 (c) $\frac{dS}{dt} = (4\pi r + 2\pi h)\frac{dr}{dt} + 2\pi r\frac{dh}{dt}$
 (d) $\frac{dr}{dt} = -\frac{r}{2r + h}\frac{dh}{dt}$
 143. $-40 \text{ m}^2/\text{sec}$ 145. $0.02 \text{ ohm}/\text{sec}$ 147. $2 \text{ m}/\text{sec}$

149. (a) $r = \frac{2}{5}h$ (b) $-\frac{125}{144\pi} \text{ ft}/\text{min}$
 151. (a) $\frac{3}{5} \text{ km}/\text{sec}$ or $600 \text{ m}/\text{sec}$ (b) $\frac{18}{\pi} \text{ rpm}$
 153. (a) $L(x) = 2x + \frac{\pi - 2}{2}$



(b) $L(x) = -\sqrt{2}x + \frac{\sqrt{2}(4 - \pi)}{4}$

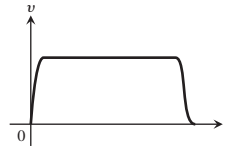


155. $L(x) = 1.5x + 0.5$ 157. $dS = \frac{\pi r h_0}{\sqrt{r^2 + h_0^2}} dh$
 159. (a) 4% (b) 8% (c) 12%

ADDITIONAL AND ADVANCED EXERCISES, pp. 217–219

1. (a) $\sin 2\theta = 2 \sin \theta \cos \theta$; $2 \cos 2\theta = 2 \sin \theta (-\sin \theta) + \cos \theta (2 \cos \theta)$; $2 \cos 2\theta = -2 \sin^2 \theta + 2 \cos^2 \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
 (b) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$; $-2 \sin 2\theta = 2 \cos \theta (-\sin \theta) - 2 \sin \theta (\cos \theta)$; $\sin 2\theta = \cos \theta \sin \theta + \sin \theta \cos \theta$; $\sin 2\theta = 2 \sin \theta \cos \theta$
 3. (a) $a = 1, b = 0, c = -\frac{1}{2}$ (b) $b = \cos a, c = \sin a$

5. $h = -4, k = \frac{9}{2}, a = \frac{5\sqrt{5}}{2}$
 7. (a) $0.09y$ (b) Increasing at 1% per year
 9. Answers will vary. Here is one possibility.

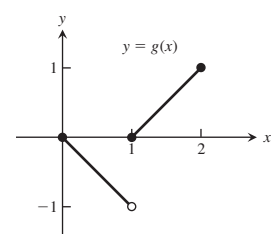
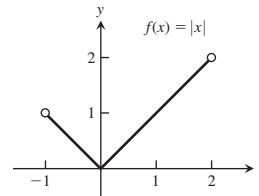


11. (a) 2 sec, 64 ft/sec (b) 12.31 sec, 393.85 ft
 15. $y' = \frac{x^{y-1}y^2 - ye^x(x^y + 1) \ln y}{e^x(x^y + 1) - x^y y \ln x}$
 17. (a) $m = -\frac{b}{\pi}$ (b) $m = -1, b = \pi$
 19. (a) $a = \frac{3}{4}, b = \frac{9}{4}$ 21. f odd $\Rightarrow f'$ is even
 25. h' is defined but not continuous at $x = 0$; k' is defined and continuous at $x = 0$.
 27. $\frac{-7}{75} \text{ rad}/\text{sec}$
 31. (a) 0.8156 ft (b) 0.00613 sec
 (c) It will lose about 8.83 min/day.

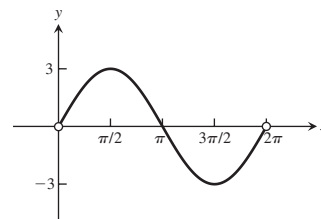
Chapter 4

SECTION 4.1, pp. 227–229

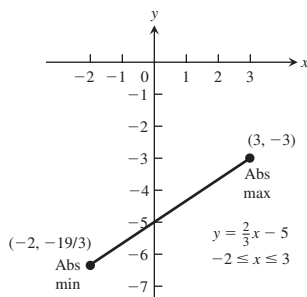
1. Absolute minimum at $x = c_2$; absolute maximum at $x = b$
 3. Absolute maximum at $x = c$; no absolute minimum
 5. Absolute minimum at $x = a$; absolute maximum at $x = c$
 7. No absolute minimum; no absolute maximum
 9. Absolute maximum at $(0, 5)$ 11. (c) 13. (d)
 15. Absolute minimum at $x = 0$; no absolute maximum 17. Absolute maximum at $x = 2$; no absolute minimum



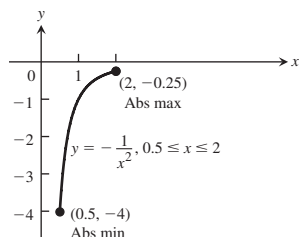
19. Absolute maximum at $x = \pi/2$; absolute minimum at $x = 3\pi/2$



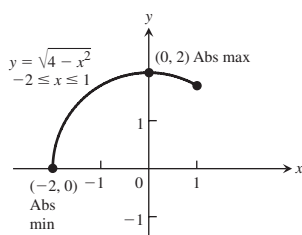
21. Absolute maximum: -3 ;
absolute minimum: $-19/3$



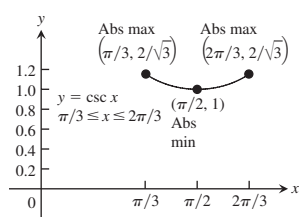
25. Absolute maximum: -0.25 ;
absolute minimum: -4



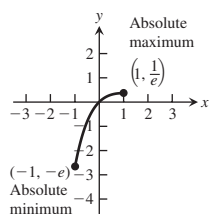
29. Absolute maximum: 2 ;
absolute minimum: 0



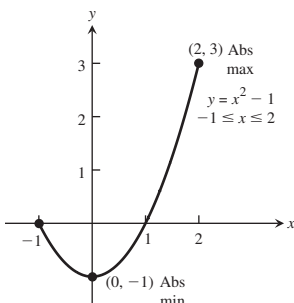
33. Absolute maximum: $2/\sqrt{3}$;
absolute minimum: 1



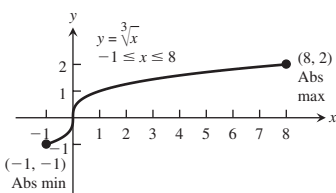
37. Absolute maximum is $1/e$ at $x = 1$; absolute minimum is $-e$ at $x = -1$.



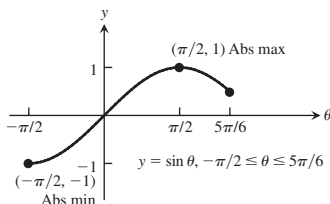
23. Absolute maximum: 3 ;
absolute minimum: -1



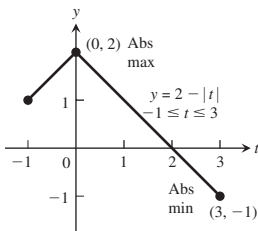
27. Absolute maximum: 2 ;
absolute minimum: -1



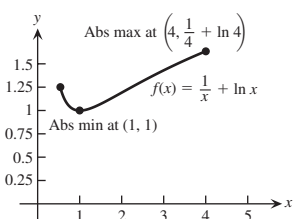
31. Absolute maximum: 1 ;
absolute minimum: -1



35. Absolute maximum: 2 ;
absolute minimum: -1



39. Absolute maximum value is $(1/4) + \ln 4$ at $x = 4$; absolute minimum value is 1 at $x = 1$; local maximum at $(1/2, 2 - \ln 2)$.



41. Increasing on $(0, 8)$, decreasing on $(-1, 0)$; absolute maximum: 16 at $x = 8$; absolute minimum: 0 at $x = 0$

43. Increasing on $(-32, 1)$; absolute maximum: 1 at $\theta = 1$; absolute minimum: -8 at $\theta = -32$

45. $x = 3$

47. $x = 1, x = 4$

49. $x = 1$

51. $x = 0$ and $x = 4$

53. $x = 0$ and $x = 1$

55. $x = 2$ and $x = -4$

Critical point or endpoint	Derivative	Extremum	Value
$x = -\frac{4}{5}$	0	Local max	$\frac{12}{25}10^{1/3} \approx 1.034$
$x = 0$	Undefined	Local min	0

Critical point or endpoint	Derivative	Extremum	Value
$x = -2$	Undefined	Local max	0
$x = -\sqrt{2}$	0	Minimum	-2
$x = \sqrt{2}$	0	Maximum	2
$x = 2$	Undefined	Local min	0

Critical point or endpoint	Derivative	Extremum	Value
$x = 1$	Undefined	Minimum	2

Critical point or endpoint	Derivative	Extremum	Value
$x = -1$	0	Maximum	5
$x = 1$	Undefined	Local min	1
$x = 3$	0	Maximum	5

65. (a) No

- (b) The derivative is defined and nonzero for $x \neq 2$. Also, $f(2) = 0$ and $f(x) > 0$ for all $x \neq 2$.

- (c) No, because $(-\infty, \infty)$ is not a closed interval.

- (d) The answers are the same as parts (a) and (b), with 2 replaced by a .

67. y is increasing on $(-\infty, \infty)$ and so has no extrema.

69. y is decreasing on $(-\infty, \infty)$ and so has no extrema.

71. Yes

73. g assumes a local maximum at $-c$.

75. (a) Maximum value is 144 at $x = 2$.

- (b) The largest volume of the box is 144 cubic units, and it occurs when $x = 2$.

77. $\frac{v_0^2}{2g} + s_0$

79. Maximum value is 11 at $x = 5$; minimum value is 5 on the interval $[-3, 2]$; local maximum at $(-5, 9)$.

81. Maximum value is 5 on the interval $[3, \infty)$; minimum value is -5 on the interval $(-\infty, -2]$.

SECTION 4.2, pp. 235-237

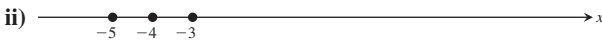
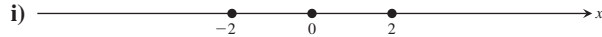
1. $1/2$ 3. 1 5. $\pm \sqrt{1 - \frac{4}{\pi^2}} \approx \pm 0.771$

7. $\frac{1}{3}(1 + \sqrt{7}) \approx 1.22, \frac{1}{3}(1 - \sqrt{7}) \approx -0.549$

9. Does not; f is not differentiable at the interior domain point $x = 0$.

11. Does 13. Does not; f is not differentiable at $x = -1$.

17. (a)



29. Yes 31. (a) 4 (b) 3 (c) 3

33. (a) $\frac{x^2}{2} + C$ (b) $\frac{x^3}{3} + C$ (c) $\frac{x^4}{4} + C$

35. (a) $\frac{1}{x} + C$ (b) $x + \frac{1}{x} + C$ (c) $5x - \frac{1}{x} + C$

37. (a) $-\frac{1}{2}\cos 2t + C$ (b) $2\sin \frac{t}{2} + C$

(c) $-\frac{1}{2}\cos 2t + 2\sin \frac{t}{2} + C$

39. $f(x) = x^2 - x$ 41. $f(x) = 1 + \frac{e^{2x}}{2}$

43. $s = 4.9t^2 + 5t + 10$ 45. $s = \frac{1 - \cos(\pi t)}{\pi}$

47. $s = e^t + 19t + 4$ 49. $s = \sin(2t) - 3$

51. If $T(t)$ is the temperature of the thermometer at time t , then $T(0) = -19^\circ\text{C}$ and $T(14) = 100^\circ\text{C}$. From the Mean Value Theorem, there exists a $0 < t_0 < 14$ such that $\frac{T(14) - T(0)}{14 - 0} = 8.5^\circ\text{C/sec} = T'(t_0)$, the rate at which the

temperature was changing at $t = t_0$ as measured by the rising mercury on the thermometer.

53. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must have been going that speed at least once during the trip.

57. The conclusion of the Mean Value Theorem yields

$$\frac{\frac{1}{b} - \frac{1}{a}}{\frac{b}{a} - 1} = -\frac{1}{c^2} \Rightarrow c^2 \left(\frac{a-b}{ab} \right) = a-b \Rightarrow c = \sqrt{ab}.$$

61. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem. Now suppose that $f(x)$ is zero twice between a and b . Then, by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \neq 0$ on this interval.

Therefore, $f(x)$ is zero once and only once between a and b .

71. $1.09999 \leq f(0.1) \leq 1.1$

SECTION 4.3, pp. 241-242

1. (a) 0, 1

(b) Increasing on $(-\infty, 0)$ and $(1, \infty)$; decreasing on $(0, 1)$

(c) Local maximum at $x = 0$; local minimum at $x = 1$

3. (a) -2, 1

(b) Increasing on $(-2, 1)$ and $(1, \infty)$; decreasing on $(-\infty, -2)$

(c) No local maximum; local minimum at $x = -2$

5. (a) Critical point at $x = 1$

(b) Decreasing on $(-\infty, 1)$, increasing on $(1, \infty)$

(c) Local (and absolute) minimum at $x = 1$

7. (a) 0, 1

(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$; decreasing on $(-2, 0)$ and $(0, 1)$

(c) Local minimum at $x = 1$

9. (a) -2, 2

(b) Increasing on $(-\infty, -2)$ and $(2, \infty)$; decreasing on $(-2, 0)$ and $(0, 2)$

(c) Local maximum at $x = -2$; local minimum at $x = 2$

11. (a) -2, 0

(b) Increasing on $(-\infty, -2)$ and $(0, \infty)$; decreasing on $(-2, 0)$

(c) Local maximum at $x = -2$; local minimum at $x = 0$

13. (a) $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

(b) Increasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$; decreasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$, and $\left(\frac{4\pi}{3}, 2\pi\right)$

(c) Local maximum at $x = 0$ and $x = \frac{4\pi}{3}$; local minimum at $x = \frac{2\pi}{3}$ and $x = 2\pi$

15. (a) Increasing on $(-2, 0)$ and $(2, 4)$; decreasing on $(-4, -2)$ and $(0, 2)$

(b) Absolute maximum at $(-4, 2)$; local maximum at $(0, 1)$ and $(4, -1)$; absolute minimum at $(2, -3)$; local minimum at $(-2, 0)$

17. (a) Increasing on $(-4, -1)$, $(1/2, 2)$, and $(2, 4)$; decreasing on $(-1, 1/2)$

(b) Absolute maximum at $(4, 3)$; local maximum at $(-1, 2)$ and $(2, 1)$; no absolute minimum; local minimum at $(-4, -1)$ and $(1/2, -1)$

19. (a) Increasing on $(-\infty, -1.5)$; decreasing on $(-1.5, \infty)$

(b) Local maximum: 5.25 at $t = -1.5$; absolute maximum: 5.25 at $t = -1.5$

21. (a) Decreasing on $(-\infty, 0)$; increasing on $(0, 4/3)$; decreasing on $(4/3, \infty)$

(b) Local minimum at $x = 0$ (0, 0); local maximum at $x = 4/3$ (4/3, 32/27); no absolute extrema

23. (a) Decreasing on $(-\infty, 0)$; increasing on $(0, 1/2)$; decreasing on $(1/2, \infty)$

(b) Local minimum at $\theta = 0$ (0, 0); local maximum at $\theta = 1/2$ (1/2, 1/4); no absolute extrema

25. (a) Increasing on $(-\infty, \infty)$; never decreasing

(b) No local extrema; no absolute extrema

27. (a) Increasing on $(-2, 0)$ and $(2, \infty)$; decreasing on $(-\infty, -2)$ and $(0, 2)$

(b) Local maximum: 16 at $x = 0$; local minimum: 0 at $x = \pm 2$; no absolute maximum; absolute minimum: 0 at $x = \pm 2$

29. (a) Increasing on $(-\infty, -1)$; decreasing on $(-1, 0)$; increasing on $(0, 1)$; decreasing on $(1, \infty)$

(b) Local maximum: 0.5 at $x = \pm 1$; local minimum: 0 at $x = 0$; absolute maximum: 1/2 at $x = \pm 1$; no absolute minimum

31. (a) Increasing on $(10, \infty)$; decreasing on $(1, 10)$

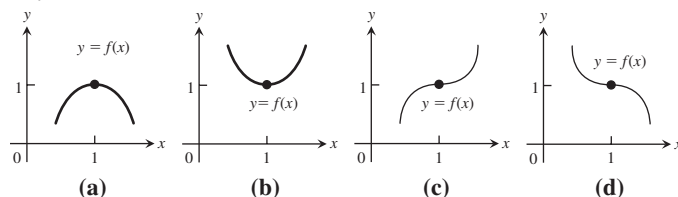
(b) Local maximum: 1 at $x = 1$; local minimum: -8 at $x = 10$; absolute minimum: -8 at $x = 10$

33. (a) Decreasing on $(-2\sqrt{2}, -2)$; increasing on $(-2, 2)$; decreasing on $(2, 2\sqrt{2})$

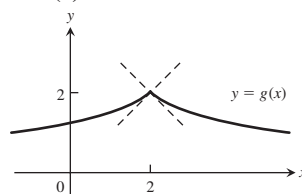
(b) Local minima: $g(-2) = -4, g(2\sqrt{2}) = 0$; local maxima: $g(-2\sqrt{2}) = 0, g(2) = 4$; absolute maximum: 4 at $x = 2$; absolute minimum: -4 at $x = -2$

35. (a) Increasing on $(-\infty, 1)$; decreasing when $1 < x < 2$; decreasing when $2 < x < 3$; discontinuous at $x = 2$; increasing on $(3, \infty)$
 (b) Local minimum at $x = 3$ (3, 6); local maximum at $x = 1$ (1, 2); no absolute extrema
37. (a) Increasing on $(-2, 0)$ and $(0, \infty)$; decreasing on $(-\infty, -2)$
 (b) Local minimum: $-6\sqrt[3]{2}$ at $x = -2$; no absolute maximum; absolute minimum: $-6\sqrt[3]{2}$ at $x = -2$
39. (a) Increasing on $(-\infty, -2/\sqrt{7})$ and $(2/\sqrt{7}, \infty)$; decreasing on $(-2/\sqrt{7}, 0)$ and $(0, 2/\sqrt{7})$
 (b) Local maximum: $24\sqrt[3]{2}/7^{7/6} \approx 3.12$ at $x = -2/\sqrt{7}$; local minimum: $-24\sqrt[3]{2}/7^{7/6} \approx -3.12$ at $x = 2/\sqrt{7}$; no absolute extrema
41. (a) Increasing on $((1/3) \ln(1/2), \infty)$, decreasing on $(-\infty, (1/3) \ln(1/2))$
 (b) Local minimum is $\frac{3}{2^{2/3}}$ at $x = (1/3) \ln(1/2)$; no local maximum; absolute minimum is $\frac{3}{2^{2/3}}$ at $x = (1/3) \ln(1/2)$; no absolute maximum
43. (a) Increasing on (e^{-1}, ∞) , decreasing on $(0, e^{-1})$
 (b) A local minimum is $-e^{-1}$ at $x = e^{-1}$, no local maximum; absolute minimum is $-e^{-1}$ at $x = e^{-1}$, no absolute maximum
45. (a) Increasing on $(0, e^{-2})$ and $(1, \infty)$; decreasing on $(e^{-2}, 1)$
 (b) local maximum is $4/e^2$ at $x = e^{-2}$; absolute minimum is 0 at $x = 1$; no absolute maximum
47. (a) Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
 (b) Absolute maximum: 1 at $x = 1$; no absolute minimum
49. (a) Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
 (b) No absolute maximum; absolute minimum: 0 at $x = 2$
51. (a) Local maxima: -9 at $t = -3$ and 16 at $t = 2$; local minimum: -16 at $t = -2$
 (b) Absolute maximum: 16 at $t = 2$; no absolute minimum
53. (a) Local minimum: 0 at $x = 0$
 (b) No absolute maximum; absolute minimum: 0 at $x = 0$
55. (a) Local maximum: 5 at $x = 0$; local minimum: 0 at $x = -5$ and $x = 5$
 (b) Absolute maximum: 5 at $x = 0$; absolute minimum: 0 at $x = -5$ and $x = 5$
57. (a) Local maximum: 2 at $x = 0$;
 local minimum: $\frac{\sqrt{3}}{4\sqrt{3}-6}$ at $x = 2 - \sqrt{3}$
 (b) No absolute maximum; an absolute minimum at $x = 2 - \sqrt{3}$
59. (a) Local maximum: 1 at $x = \pi/4$;
 local maximum: 0 at $x = \pi$;
 local minimum: 0 at $x = 0$;
 local minimum: -1 at $x = 3\pi/4$
61. Local maximum: 2 at $x = \pi/6$;
 local maximum: $\sqrt{3}$ at $x = 2\pi$;
 local minimum: -2 at $x = 7\pi/6$;
 local minimum: $\sqrt{3}$ at $x = 0$
63. (a) Local minimum: $(\pi/3) - \sqrt{3}$ at $x = 2\pi/3$;
 local maximum: 0 at $x = 0$;
 local maximum: π at $x = 2\pi$
65. (a) Local minimum: 0 at $x = \pi/4$
67. Local minimum at $x = 1$; no local maximum
69. Local maximum: 3 at $\theta = 0$;
 local minimum: -3 at $\theta = 2\pi$

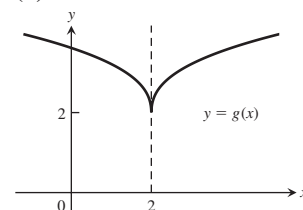
71.



73. (a)



(b)


77. $a = -2, b = 4$

79. (a) Absolute minimum occurs at $x = \pi/3$ with $f(\pi/3) = -\ln 2$, and the absolute maximum occurs at $x = 0$ with $f(0) = 0$.
 (b) Absolute minimum occurs at $x = 1/2$ and $x = 2$ with $f(1/2) = f(2) = \cos(\ln 2)$, and the absolute maximum occurs at $x = 1$ with $f(1) = 1$.

81. Minimum of $2 - 2 \ln 2 \approx 0.613706$ at $x = \ln 2$; maximum of 1 at $x = 0$

83. Absolute maximum value of $1/2e$ assumed at $x = 1/\sqrt{e}$

87. Increasing; $\frac{df^{-1}}{dx} = \frac{1}{9}x^{-2/3}$

89. Decreasing; $\frac{df^{-1}}{dx} = -\frac{1}{3}x^{-2/3}$

SECTION 4.4, pp. 251–255

1. Local maximum: $3/2$ at $x = -1$; local minimum: -3 at $x = 2$;
 point of inflection at $(1/2, -3/4)$; rising on $(-\infty, -1)$ and $(2, \infty)$; falling on $(-1, 2)$; concave up on $(1/2, \infty)$; concave down on $(-\infty, 1/2)$

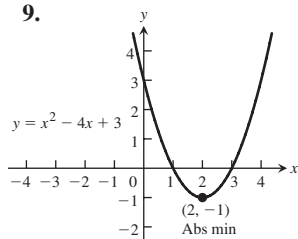
3. Local maximum: $3/4$ at $x = 0$; local minimum: 0 at $x = \pm 1$;
 points of inflection at $(-\sqrt{3}, \frac{3\sqrt{3}}{4})$ and $(\sqrt{3}, \frac{3\sqrt{3}}{4})$;
 rising on $(-1, 0)$ and $(1, \infty)$; falling on $(-\infty, -1)$ and $(0, 1)$;
 concave up on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$; concave down on $(-\sqrt{3}, \sqrt{3})$

5. Local maxima: $-\frac{2\pi}{3} + \frac{\sqrt{3}}{2}$ at $x = -2\pi/3, \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ at

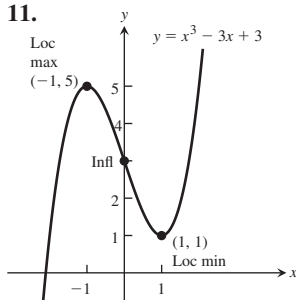
 $x = \pi/3$; local minima: $-\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = -\pi/3, \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = 2\pi/3$; points of inflection at $(-\pi/2, -\pi/2)$, $(0, 0)$, and $(\pi/2, \pi/2)$; rising on $(-\pi/3, \pi/3)$; falling on $(-2\pi/3, -\pi/3)$ and $(\pi/3, 2\pi/3)$; concave up on $(-\pi/2, 0)$ and $(\pi/2, 2\pi/3)$; concave down on $(-2\pi/3, -\pi/2)$ and $(0, \pi/2)$

7. Local maxima: 1 at $x = -\pi/2$ and $x = \pi/2$, 0 at $x = -2\pi$ and $x = 2\pi$; local minima: -1 at $x = -3\pi/2$ and $x = 3\pi/2$, 0 at $x = 0$; points of inflection at $(-\pi, 0)$ and $(\pi, 0)$; rising on $(-3\pi/2, -\pi/2)$, $(0, \pi/2)$, and $(3\pi/2, 2\pi)$; falling on $(-2\pi, -3\pi/2)$, $(-\pi/2, 0)$, and $(\pi/2, 3\pi/2)$; concave up on $(-2\pi, -\pi)$ and $(\pi, 2\pi)$; concave down on $(-\pi, 0)$ and $(0, \pi)$

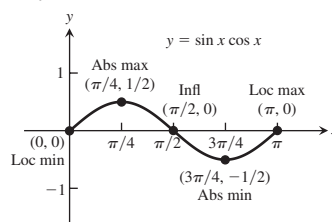
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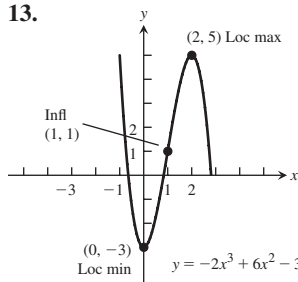
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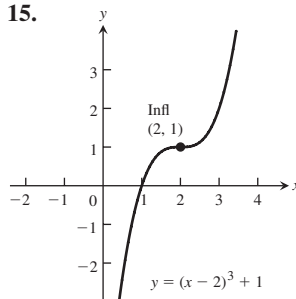
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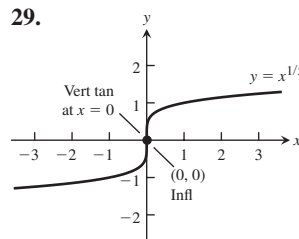
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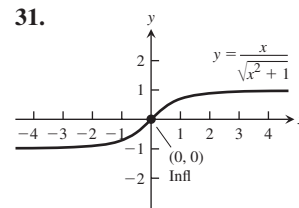
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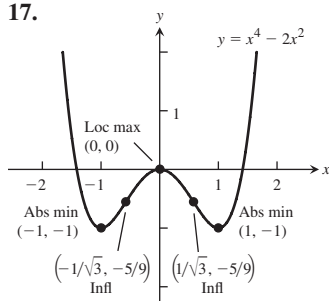
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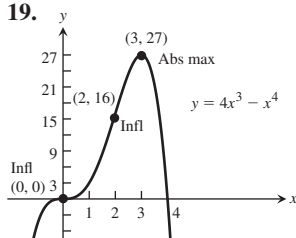
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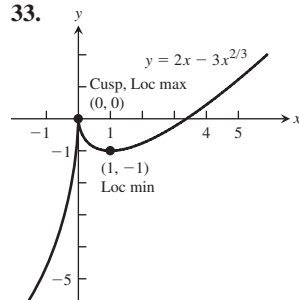
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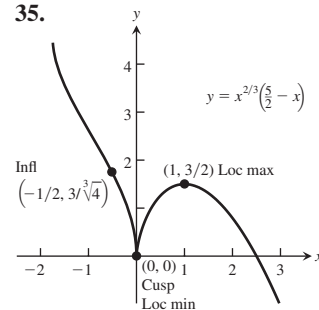
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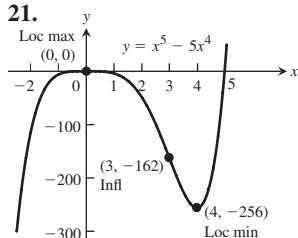
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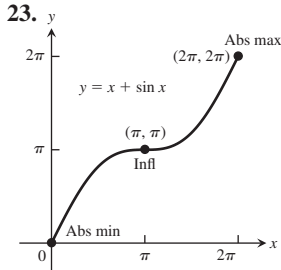
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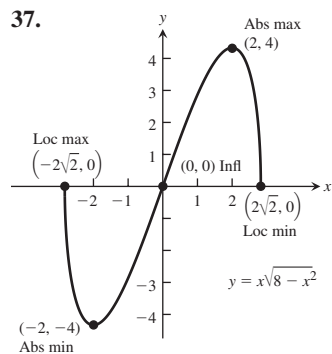
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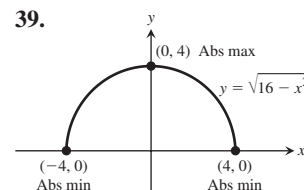
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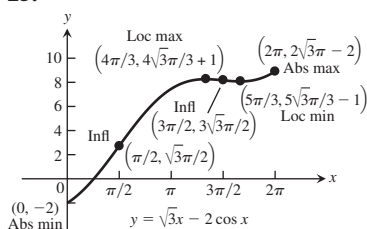
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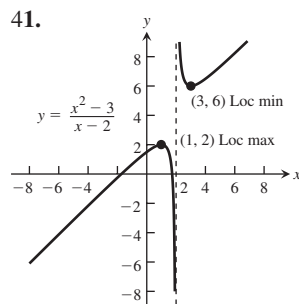
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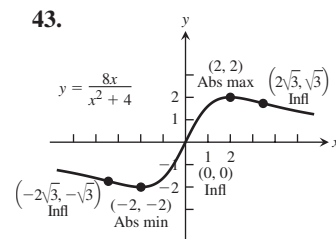
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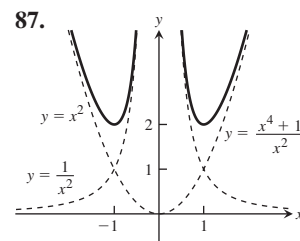
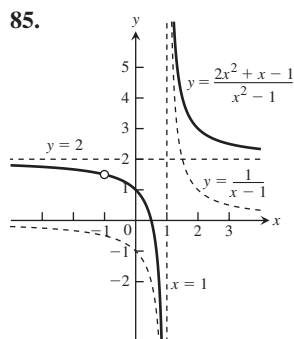
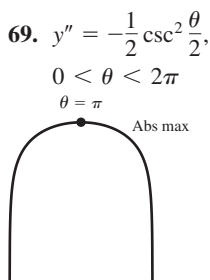
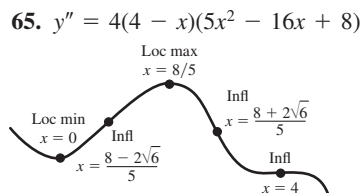
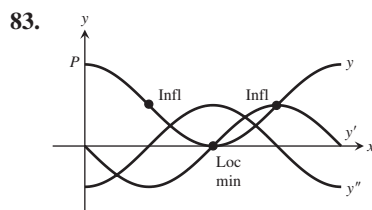
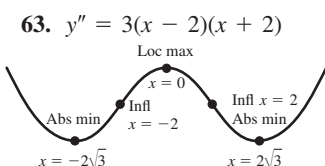
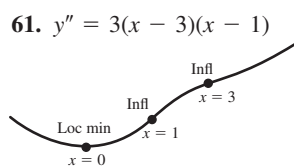
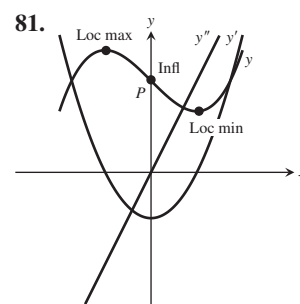
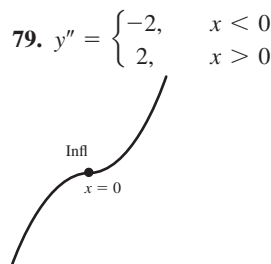
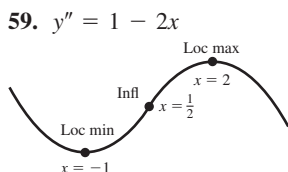
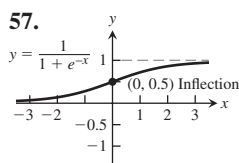
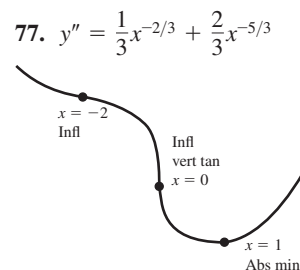
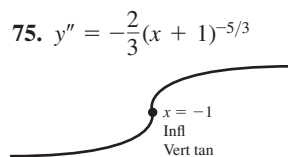
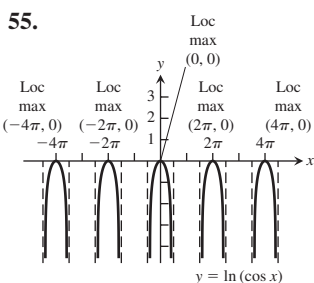
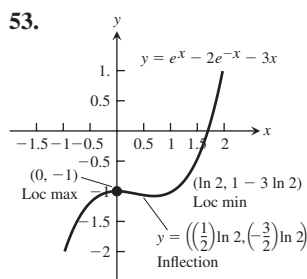
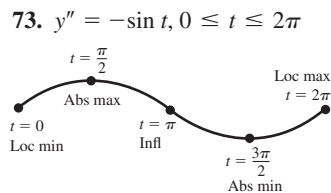
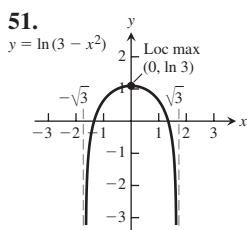
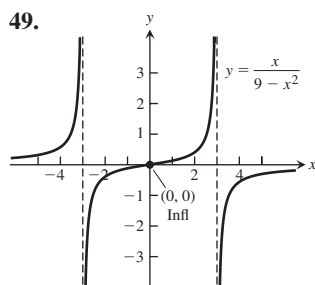
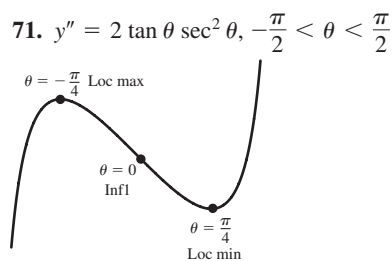
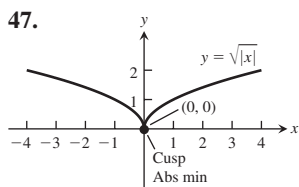
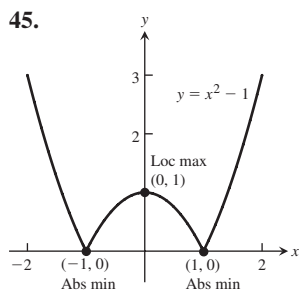


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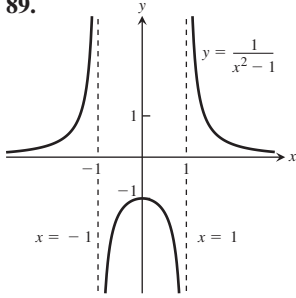


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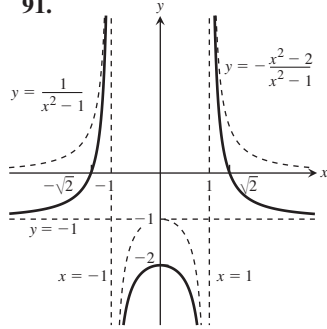




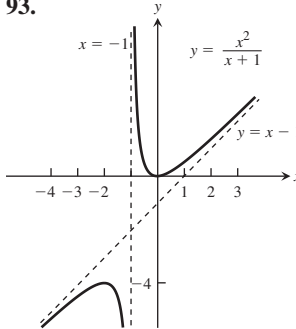
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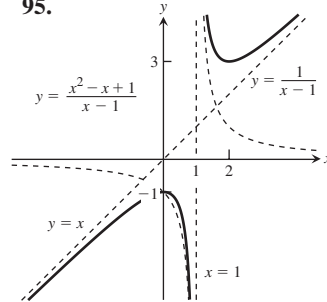
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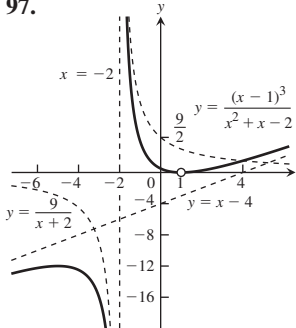
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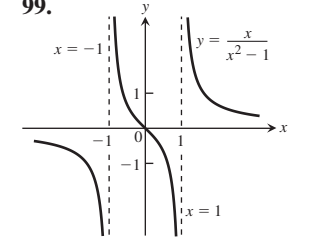
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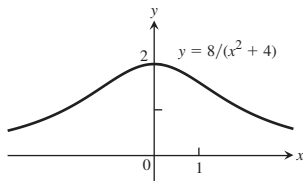
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99.



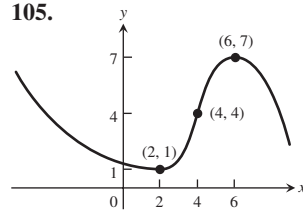
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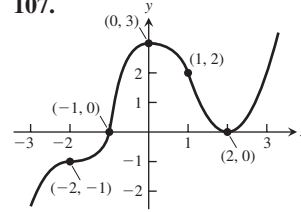
103.

Point	y'	y''
P	-	+
Q	+	0
R	+	-
S	0	-
T	-	-

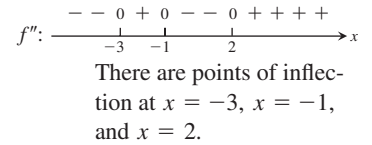
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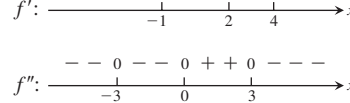
107.



109.



111.



There are points of inflection at $x = 0$ and $x = 3$.

113. (a) Towards origin: $0 \leq t < 2$ and $6 \leq t \leq 10$; away from origin: $2 \leq t \leq 6$ and $10 \leq t \leq 15$

(b) $t = 2, t = 6, t = 10$

(c) $t = 5, t = 7, t = 13$

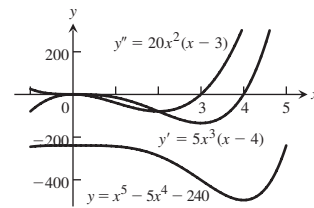
(d) Positive: $5 \leq t \leq 7, 13 \leq t \leq 15$; negative: $0 \leq t \leq 5, 7 \leq t \leq 13$

115. ≈ 60 thousand units

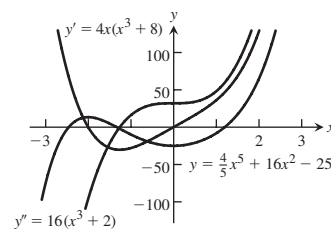
117. Local minimum at $x = 2$; inflection points at $x = 1$ and $x = 5/3$

119. $-1, 2$ 121. $b = -3$ 127. $a = 1, b = 3, c = 9$

129. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = 3$, local maximum at $x = 0$, local minimum at $x = 4$.



131. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = -\sqrt[3]{2}$; local maximum at $x = -2$; local minimum at $x = 0$.



SECTION 4.5, pp. 262-263

1. $-1/4$ 3. $5/7$ 5. $1/2$ 7. $1/4$ 9. $-23/7$

11. $5/7$ 13. 0 15. -16 17. -2 19. $1/4$

21. 2 23. 3 25. -1 27. $\ln 3$ 29. $\frac{1}{\ln 2}$ 31. $\ln 2$

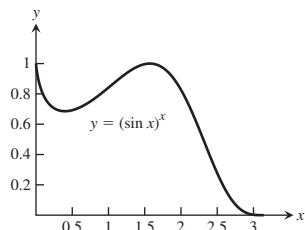
33. 1 35. $1/2$ 37. $\ln 2$ 39. $-\infty$ 41. $-1/2$

43. -1 45. 1 47. 0 49. 2 51. $1/e$ 53. 1
55. $1/e$ 57. $e^{1/2}$ 59. 1 61. e^3 63. 0 65. 1
67. 3 69. 1 71. 0 73. ∞ 75. (b) is correct.

77. (d) is correct. 79. $c = \frac{27}{10}$ 81. (b) $-\frac{1}{2}$ 83. -1

87. (a) $y = 1$ (b) $y = 0, y = \frac{3}{2}$

89. (a) We should assign the value 1 to $f(x) = (\sin x)^x$ to make it continuous at $x = 0$.

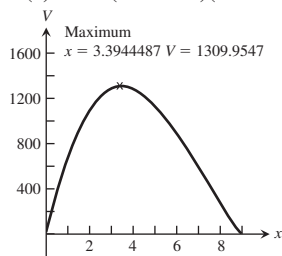


- (c) The maximum value of $f(x)$ is close to 1 near the point $x \approx 1.55$ (see the graph in part (a)).

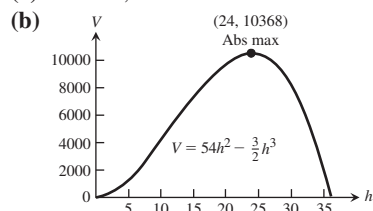
SECTION 4.6, pp. 269–276

1. 16 in., 4 in. by 4 in.
3. (a) $(x, 1 - x)$ (b) $A(x) = 2x(1 - x)$
(c) $\frac{1}{2}$ square units, 1 by $\frac{1}{2}$
5. $\frac{14}{3} \times \frac{35}{3} \times \frac{5}{3}$ in., $\frac{2450}{27}$ in.³
7. 80,000 m²; 400 m by 200 m
9. (a) The optimum dimensions of the tank are 10 ft on the base edges and 5 ft deep.
(b) Minimizing the surface area of the tank minimizes its weight for a given wall thickness. The thickness of the steel walls would likely be determined by other considerations such as structural requirements.
11. 9×18 in. 13. $\frac{\pi}{2}$ 15. $h:r = 8:\pi$

17. (a) $V(x) = 2x(24 - 2x)(18 - 2x)$ (b) Domain: (0, 9)



- (c) Maximum volume ≈ 1309.95 in³ when $x \approx 3.39$ in.
(d) $V'(x) = 24x^2 - 336x + 864$, so the critical point is at $x = 7 - \sqrt{13}$, which confirms the result in part (c).
(e) $x = 2$ in. or $x = 5$ in.
19. ≈ 2418.40 cm³
21. (a) $h = 24, w = 18$



23. If r is the radius of the hemisphere, h the height of the cylinder, and V the volume, then $r = \left(\frac{3V}{8\pi}\right)^{1/3}$ and $h = \left(\frac{3V}{\pi}\right)^{1/3}$.

25. (b) $x = \frac{51}{8}$ (c) $L \approx 11$ in.

27. Radius = $\sqrt{2}$ m, height = 1 m, volume = $\frac{2\pi}{3}$ m³

29. 1 31. $\frac{9b}{9 + \sqrt{3\pi}}$ m, triangle; $\frac{b\sqrt{3\pi}}{9 + \sqrt{3\pi}}$ m, circle

33. $\frac{3}{2} \times 2$ 35. (a) 16 (b) -1

37. $r = \frac{2\sqrt{2}}{3}$ $h = \frac{4}{3}$

39. (a) $4 + \ln 2$ (b) $\left(\frac{1}{2}\right)(4 + \ln 2)$ 41. Area 8 when $a = 2$

43. (a) $v(0) = 96$ ft/sec (b) 256 ft at $t = 3$ sec
(c) Velocity when $s = 0$ is $v(7) = -128$ ft/sec.

45. ≈ 46.87 ft 47. (a) $6 \times 6\sqrt{3}$ in.

49. (a) $4\sqrt{3} \times 4\sqrt{6}$ in.

51. (a) $10\pi \approx 31.42$ cm/sec; when $t = 0.5$ sec, 1.5 sec, 2.5 sec, 3.5 sec; $s = 0$, acceleration is 0.

- (b) 10 cm from rest position; speed is 0.

53. (a) $s = ((12 - 12t)^2 + 64t^2)^{1/2}$

- (b) -12 knots, 8 knots

- (c) No

- (d) $4\sqrt{13}$. This limit is the square root of the sums of the squares of the individual speeds.

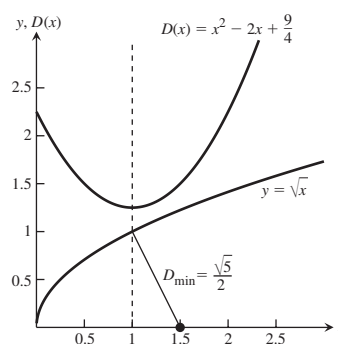
55. $x = \frac{a}{2}, v = \frac{ka^2}{4}$ 57. $\frac{c}{2} + 50$

59. (a) $\sqrt{\frac{2km}{h}}$ (b) $\sqrt{\frac{2km}{h}}$ 63. $4 \times 4 \times 3$ ft, \$288

65. $M = \frac{C}{2}$ 71. (a) $y = -1$

73. (a) The minimum distance is $\frac{\sqrt{5}}{2}$.

- (b) The minimum distance is from the point $(3/2, 0)$ to the point $(1, 1)$ on the graph of $y = \sqrt{x}$, and this occurs at the value $x = 1$, where $D(x)$, the distance squared, has its minimum value.

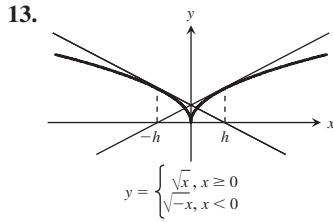


SECTION 4.7, pp. 279–281

1. $x_2 = -\frac{5}{3}, \frac{13}{21}$ 3. $x_2 = -\frac{51}{31}, \frac{5763}{4945}$ 5. $x_2 = \frac{2387}{2000}$

7. x_2 is approximately 2.20794 9. x_2 is approximately 0.68394

11. x_1 , and all later approximations will equal x_0 .



15. The points of intersection of $y = x^3$ and $y = 3x + 1$ or $y = x^3 - 3x$ and $y = 1$ have the same x -values as the roots of part (i) or the solutions of part (iv). 17. 1.165561185

19. (a) Two (b) 0.35003501505249 and -1.0261731615301

21. ± 1.3065629648764 , ± 0.5411961001462 23. $x \approx 0.45$

25. 0.8192 27. 0, 0.53485 29. The root is 1.17951.

31. (a) For $x_0 = -2$ or $x_0 = -0.8$, $x_i \rightarrow -1$ as i gets large.

(b) For $x_0 = -0.5$ or $x_0 = 0.25$, $x_i \rightarrow 0$ as i gets large.

(c) For $x_0 = 0.8$ or $x_0 = 2$, $x_i \rightarrow 1$ as i gets large.

(d) For $x_0 = -\sqrt{21}/7$ or $x_0 = \sqrt{21}/7$, Newton's method does not converge. The values of x_i alternate between $-\sqrt{21}/7$ and $\sqrt{21}/7$ as i increases.

33. Answers will vary with machine speed.

SECTION 4.8, pp. 287-291

1. (a) x^2 (b) $\frac{x^3}{3}$ (c) $\frac{x^3}{3} - x^2 + x$

3. (a) x^{-3} (b) $-\frac{1}{3}x^{-3}$ (c) $-\frac{1}{3}x^{-3} + x^2 + 3x$

5. (a) $-\frac{1}{x}$ (b) $-\frac{5}{x}$ (c) $2x + \frac{5}{x}$

7. (a) $\sqrt{x^3}$ (b) $\sqrt{x}$ (c) $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x}$

9. (a) $x^{2/3}$ (b) $x^{1/3}$ (c) $x^{-1/3}$

11. (a) $\ln x$ (b) $7 \ln x$ (c) $x - 5 \ln x$

13. (a) $\cos(\pi x)$ (b) $-3 \cos x$ (c) $-\frac{1}{\pi} \cos(\pi x) + \cos(3x)$

15. (a) $\tan x$ (b) $2 \tan\left(\frac{x}{3}\right)$ (c) $-\frac{2}{3} \tan\left(\frac{3x}{2}\right)$

17. (a) $-\csc x$ (b) $\frac{1}{5} \csc(5x)$ (c) $2 \csc\left(\frac{\pi x}{2}\right)$

19. (a) $\frac{1}{3}e^{3x}$ (b) $-e^{-x}$ (c) $2e^{x/2}$

21. (a) $\frac{1}{\ln 3} 3^x$ (b) $\frac{-1}{\ln 2} 2^{-x}$ (c) $\frac{1}{\ln(5/3)} \left(\frac{5}{3}\right)^x$

23. (a) $2 \sin^{-1} x$ (b) $\frac{1}{2} \tan^{-1} x$ (c) $\frac{1}{2} \tan^{-1} 2x$

25. $\frac{x^2}{2} + x + C$ 27. $t^3 + \frac{t^2}{4} + C$ 29. $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$

31. $-\frac{1}{x} - \frac{x^3}{3} - \frac{x}{3} + C$ 33. $\frac{3}{2}x^{2/3} + C$

35. $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$ 37. $4y^2 - \frac{8}{3}y^{3/4} + C$

39. $x^2 + \frac{2}{x} + C$ 41. $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$ 43. $-2 \sin t + C$

45. $-21 \cos \frac{\theta}{3} + C$ 47. $3 \cot x + C$ 49. $-\frac{1}{2} \csc \theta + C$

51. $\frac{1}{3}e^{3x} - 5e^{-x} + C$ 53. $-e^{-x} + \frac{4^x}{\ln 4} + C$

55. $4 \sec x - 2 \tan x + C$ 57. $-\frac{1}{2} \cos 2x + \cot x + C$

59. $\frac{t}{2} + \frac{\sin 4t}{8} + C$ 61. $\ln|x| - 5 \tan^{-1} x + C$

63. $\frac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1} + C$ 65. $\tan \theta + C$ 67. $-\cot x - x + C$

69. $-\cos \theta + \theta + C$

83. (a) Wrong: $\frac{d}{dx} \left(\frac{x^2}{2} \sin x + C \right) = \frac{2x}{2} \sin x + \frac{x^2}{2} \cos x = x \sin x + \frac{x^2}{2} \cos x$

(b) Wrong: $\frac{d}{dx} (-x \cos x + C) = -\cos x + x \sin x$

(c) Right: $\frac{d}{dx} (-x \cos x + \sin x + C) = -\cos x + x \sin x + \cos x = x \sin x$

85. (a) Wrong: $\frac{d}{dx} \left(\frac{(2x+1)^3}{3} + C \right) = \frac{3(2x+1)^2(2)}{3} = 2(2x+1)^2$

(b) Wrong: $\frac{d}{dx} ((2x+1)^3 + C) = 3(2x+1)^2(2) = 6(2x+1)^2$

(c) Right: $\frac{d}{dx} ((2x+1)^3 + C) = 6(2x+1)^2$

87. Right 89. (b) 91. $y = x^2 - 7x + 10$

93. $y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$ 95. $y = 9x^{1/3} + 4$

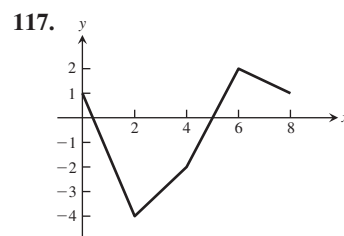
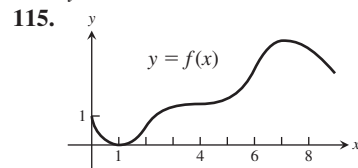
97. $s = t + \sin t + 4$ 99. $r = \cos(\pi \theta) - 1$

101. $v = \frac{1}{2} \sec t + \frac{1}{2}$ 103. $v = 3 \sec^{-1} t - \pi$

105. $y = x^2 - x^3 + 4x + 1$ 107. $r = \frac{1}{t} + 2t - 2$

109. $y = x^3 - 4x^2 + 5$ 111. $y = -\sin t + \cos t + t^3 - 1$

113. $y = 2x^{3/2} - 50$



119. $y = x - x^{4/3} + \frac{1}{2}$ 121. $y = -\sin x - \cos x - 2$

123. (a) (i) 33.2 units, (ii) 33.2 units, (iii) 33.2 units

(b) True

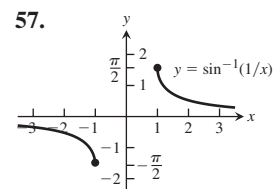
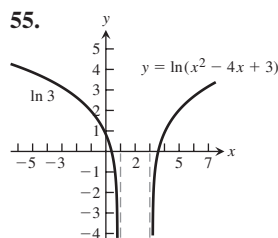
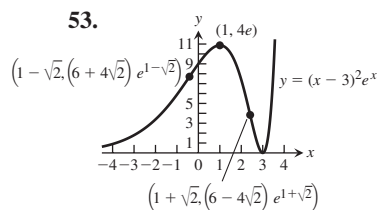
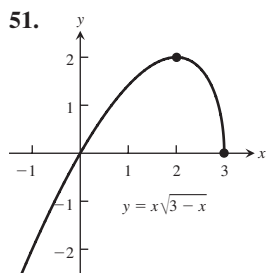
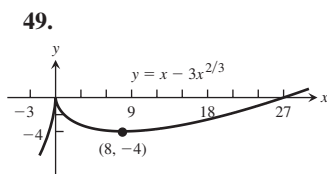
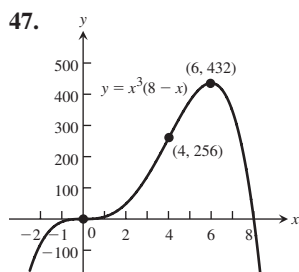
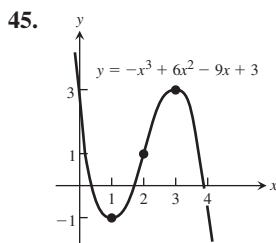
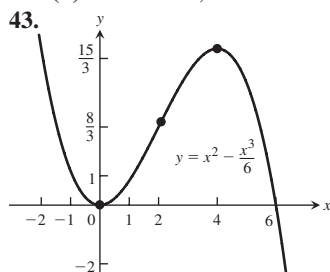
125. $t = 88/k$, $k = 16$

127. (a) $v = 10t^{3/2} - 6t^{1/2}$ (b) $s = 4t^{5/2} - 4t^{3/2}$

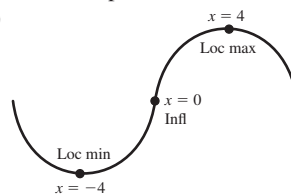
131. (a) $-\sqrt{x} + C$ (b) $x + C$ (c) $\sqrt{x} + C$
 (d) $-x + C$ (e) $x - \sqrt{x} + C$ (f) $-x - \sqrt{x} + C$

PRACTICE EXERCISES, pp. 292-296

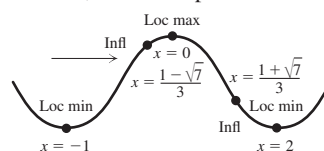
1. Minimum value is 1 at $x = 2$.
 3. Local maximum at $(-2, 17)$; local minimum at $(\frac{4}{3}, -\frac{41}{27})$
 5. Minimum value is 0 at $x = -1$ and $x = 1$.
 7. There is a local minimum at $(0, 1)$.
 9. Maximum value is $\frac{1}{2}$ at $x = 1$; minimum value is $-\frac{1}{2}$ at $x = -1$.
 11. The minimum value is 2 at $x = 0$.
 13. The minimum value is $-\frac{1}{e}$ at $x = \frac{1}{e}$.
 15. The maximum value is $\frac{\pi}{2}$ at $x = 0$; an absolute minimum value is 0 at $x = 1$ and $x = -1$.
 17. No 19. No minimum; absolute maximum: $f(1) = 16$; critical points: $x = 1$ and $11/3$
 21. Absolute minimum: $g(0) = 1$; no absolute maximum; critical point: $x = 0$
 23. Absolute minimum: $2 - 2 \ln 2$ at $x = 2$; absolute maximum: 1 at $x = 1$
 25. Yes, except at $x = 0$ 27. No 31. (b) one
 33. (b) 0.8555 99677 2
 39. Global minimum value of $\frac{1}{2}$ at $x = 2$
 41. (a) $t = 0, 6, 12$ (b) $t = 3, 9$ (c) $6 < t < 12$
 (d) $0 < t < 6, 12 < t < 14$



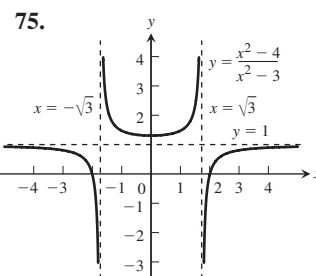
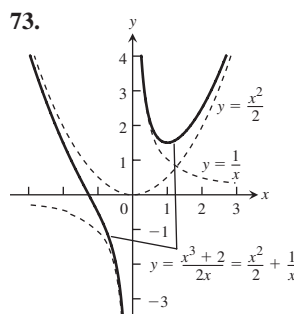
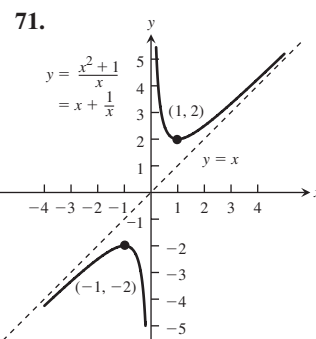
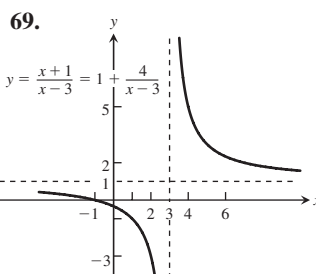
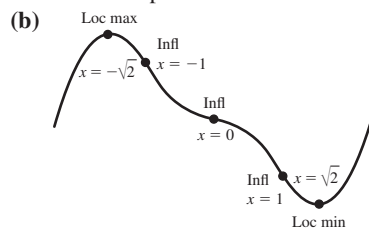
59. (a) Local maximum at $x = 4$, local minimum at $x = -4$, inflection point at $x = 0$
 (b)



61. (a) Local maximum at $x = 0$, local minima at $x = -1$ and $x = 2$, inflection points at $x = (1 \pm \sqrt{7})/3$



63. (a) Local maximum at $x = -\sqrt{2}$, local minimum at $x = \sqrt{2}$, inflection points at $x = \pm 1$ and 0
 (b)

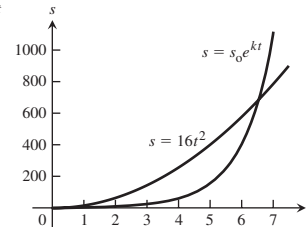


77. 5 79. 0 81. 1 83. $3/7$ 85. 0 87. 1
 89. $\ln 10$ 91. $\ln 2$ 93. 5 95. $-\infty$ 97. 1 99. e^{bk}
 101. $-\infty$ 103. (a) 0, 36 (b) 18, 18 105. 54 square units
 107. height = 2, radius = $\sqrt{2}$
 109. $x = 5 - \sqrt{5}$ hundred ≈ 276 tires,
 $y = 2(5 - \sqrt{5})$ hundred ≈ 553 tires
 111. Dimensions: base is 6 in. by 12 in., height = 2 in.;
 maximum volume = 144 in^3
 113. $x_5 = 2.1958 \ 23345$ 115. $\frac{x^4}{4} + \frac{5}{2}x^2 - 7x + C$
 117. $2t^{3/2} - \frac{4}{t} + C$ 119. $-\frac{1}{r+5} + C$
 121. $(\theta^2 + 1)^{3/2} + C$ 123. $\frac{1}{3}(1 + x^4)^{3/4} + C$
 125. $10 \tan \frac{s}{10} + C$ 127. $-\frac{1}{\sqrt{2}} \csc \sqrt{2}\theta + C$
 129. $\frac{1}{2}x - \sin \frac{x}{2} + C$ 131. $3 \ln x - \frac{x^2}{2} + C$
 133. $\frac{1}{2}e^t + e^{-t} + C$ 135. $\frac{\theta^{2-\pi}}{2-\pi} + C$
 137. $\frac{3}{2} \sec^{-1}|x| + C$ 139. $y = x - \frac{1}{x} - 1$
 141. $r = 4t^{5/2} + 4t^{3/2} - 8t$
 143. Yes, $\sin^{-1}(x)$ and $-\cos^{-1}(x)$ differ by the constant $\pi/2$.
 145. $1/\sqrt{2}$ units long by $1/\sqrt{e}$ units high, $A = 1/\sqrt{2e} \approx 0.43 \text{ units}^2$
 147. Absolute maximum = 0 at $x = e/2$, absolute minimum = -0.5 at $x = 0.5$
 149. $x = \pm 1$ are the critical points; $y = 1$ is a horizontal asymptote in both directions; absolute minimum value of the function is $e^{-\sqrt{2}/2}$ at $x = -1$, and absolute maximum value is $e^{\sqrt{2}/2}$ at $x = 1$.
 151. (a) Absolute maximum of $2/e$ at $x = e^2$, inflection point $(e^{8/3}, (8/3)e^{-4/3})$, concave up on $(e^{8/3}, \infty)$, concave down on $(0, e^{8/3})$
 (b) Absolute maximum of 1 at $x = 0$, inflection points $(\pm 1/\sqrt{2}, 1/\sqrt{e})$, concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$, concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$
 (c) Absolute maximum of 1 at $x = 0$, inflection point $(1, 2/e)$, concave up on $(1, \infty)$, concave down on $(-\infty, 1)$

ADDITIONAL AND ADVANCED EXERCISES, pp. 296–299

1. The function is constant on the interval.
 3. The extreme points will not be at the end of an open interval.
 5. (a) A local minimum at $x = -1$, points of inflection at $x = 0$ and $x = 2$
 (b) A local maximum at $x = 0$ and local minima at $x = -1$ and $x = 2$, points of inflection at $x = \frac{1 \pm \sqrt{7}}{3}$
 9. No 11. $a = 1, b = 0, c = 1$ 13. Yes
 15. Drill the hole at $y = h/2$.
 17. $r = \frac{RH}{2(H-R)}$ for $H > 2R$, $r = R$ if $H \leq 2R$
 19. $\frac{12}{5}$ and 5

21. (a) $\frac{10}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{2}$ (d) 0 (e) $-\frac{1}{2}$ (f) 1
 (g) $\frac{1}{2}$ (h) 3
 23. (a) $\frac{c-b}{2e}$ (b) $\frac{c+b}{2}$ (c) $\frac{b^2 - 2bc + c^2 + 4ae}{4e}$
 (d) $\frac{c+b+t}{2}$
 25. $m_0 = 1 - \frac{1}{q}, m_1 = \frac{1}{q}$
 27. $s = ce^{kt}$



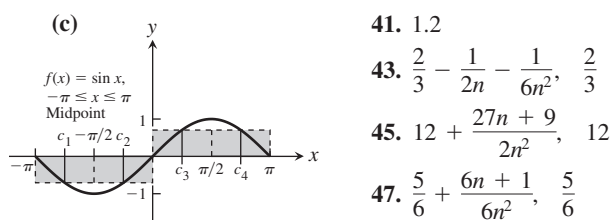
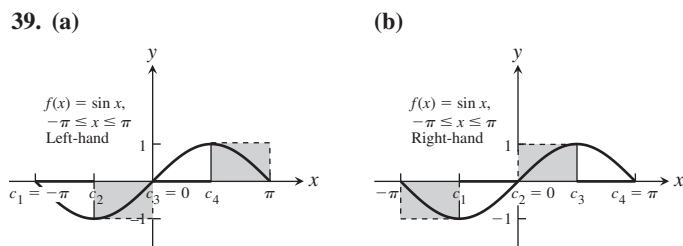
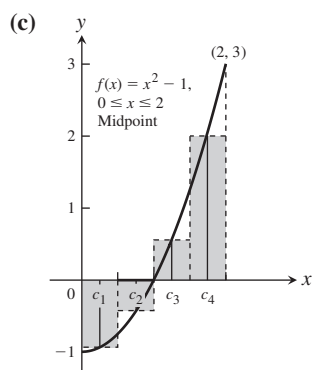
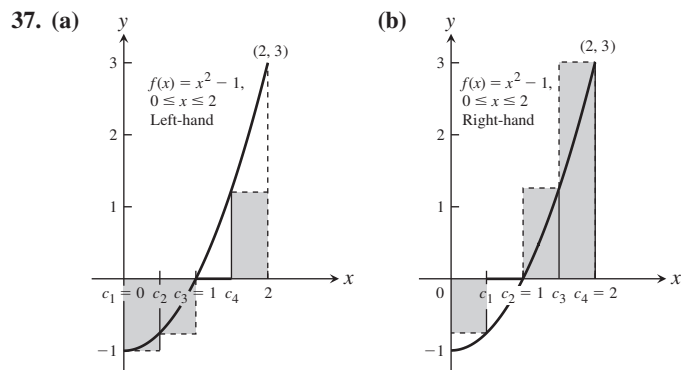
29. (a) $k = -38.72$ (b) 25 ft
 31. Yes, $y = x + C$ 33. $v_0 = \frac{2\sqrt{2}}{3}b^{3/4}$ 39. 3

Chapter 5**SECTION 5.1, pp. 308–310**

1. (a) 0.125 (b) 0.21875 (c) 0.625 (d) 0.46875
 3. (a) 1.066667 (b) 1.283333 (c) 2.666667 (d) 2.083333
 5. 0.3125, 0.328125 7. 1.5, 1.574603
 9. (a) 245 cm (b) 245 cm 11. (a) 3490 ft (b) 3840 ft
 13. (a) 74.65 ft/sec (b) 45.28 ft/sec (c) 146.59 ft
 15. $\frac{31}{16}$ 17. 1
 19. (a) Upper = 758 gal, lower = 543 gal
 (b) Upper = 2363 gal, lower = 1693 gal
 (c) ≈ 31.4 h, ≈ 32.4 h
 21. (a) 2 (b) $2\sqrt{2} \approx 2.828$ (c) $8 \sin\left(\frac{\pi}{8}\right) \approx 3.061$
 (d) Each area is less than the area of the circle, π . As n increases, the polygon area approaches π .

SECTION 5.2, pp. 316–317

1. $\frac{6(1)}{1+1} + \frac{6(2)}{2+1} = 7$
 3. $\cos(1)\pi + \cos(2)\pi + \cos(3)\pi + \cos(4)\pi = 0$
 5. $\sin \pi - \sin \frac{\pi}{2} + \sin \frac{\pi}{3} = \frac{\sqrt{3}-2}{2}$
 7. All of them 9. b
 11. $\sum_{k=1}^6 k$ 13. $\sum_{k=1}^4 \frac{1}{2^k}$ 15. $\sum_{k=1}^5 (-1)^{k+1} \frac{1}{k}$
 17. (a) -15 (b) 1 (c) 1 (d) -11 (e) 16
 19. (a) 55 (b) 385 (c) 3025 21. -56 23. -73
 25. 240 27. 3376 29. (a) 21 (b) 3500 (c) 2620
 31. (a) $4n$ (b) cn (c) $(n^2 - n)/2$ 33. 2600 35. $-2\sqrt{3}$



49. $\frac{1}{2} + \frac{1}{n} + \frac{1}{2n^2}, \frac{1}{2}$

SECTION 5.3, pp. 326–329

1. $\int_0^2 x^2 dx$ 3. $\int_{-7}^5 (x^2 - 3x) dx$ 5. $\int_2^3 \frac{1}{1-x} dx$
7. $\int_{-\pi/4}^0 \sec x dx$
9. (a) 0 (b) -8 (c) -12 (d) 10 (e) -2 (f) 16
11. (a) 5 (b) $5\sqrt{3}$ (c) -5 (d) -5
13. (a) 4 (b) -4 15. Area = 21 square units
17. Area = $9\pi/2$ square units 19. Area = 2.5 square units
21. Area = 3 square units 23. $b^2/4$ 25. $b^2 - a^2$
27. (a) 2π (b) π 29. $1/2$ 31. $3\pi^2/2$ 33. $7/3$
35. $1/24$ 37. $3a^2/2$ 39. $b/3$ 41. -14
43. -2 45. $-7/4$ 47. 7 49. 0

51. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 3x^2 dx = b^3$$

53. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 2x dx = b^2$$

55. $\text{av}(f) = 0$ 57. $\text{av}(f) = -2$ 59. $\text{av}(f) = 1$
61. (a) $\text{av}(g) = -1/2$ (b) $\text{av}(g) = 1$ (c) $\text{av}(g) = 1/4$
63. $c(b-a)$ 65. $b^3/3 - a^3/3$ 67. 9
69. $b^4/4 - a^4/4$ 71. $a = 0$ and $b = 1$ maximize the integral.
73. Upper bound = 1, lower bound = $1/2$
75. For example, $\int_0^1 \sin(x^2) dx \leq \int_0^1 dx = 1$

77. $\int_a^b f(x) dx \geq \int_a^b 0 dx = 0$ 79. Upper bound = $1/2$

SECTION 5.4, pp. 339–341

1. $-10/3$ 3. $124/125$ 5. $753/16$ 7. 1 9. $2\sqrt{3}$
11. 0 13. $-\pi/4$ 15. $1 - \frac{\pi}{4}$ 17. $\frac{2 - \sqrt{2}}{4}$ 19. $-8/3$
21. $-3/4$ 23. $\sqrt{2} - \sqrt[4]{8} + 1$ 25. -1 27. 16
29. $7/3$ 31. $2\pi/3$ 33. $\frac{1}{\pi}(4^\pi - 2^\pi)$ 35. $\frac{1}{2}(e - 1)$
37. $\sqrt{26} - \sqrt{5}$ 39. $(\cos \sqrt{x}) \left(\frac{1}{2\sqrt{x}} \right)$ 41. $4t^5$
43. $3x^2 e^{-x^3}$ 45. $\sqrt{1+x^2}$ 47. $-\frac{1}{2}x^{-1/2} \sin x$ 49. 0
51. 1 53. $2xe^{(1/2)x^2}$ 55. 1 57. $28/3$ 59. $1/2$
61. π 63. $\frac{\sqrt{2}\pi}{2}$
65. d, since $y' = \frac{1}{x}$ and $y(\pi) = \int_\pi^\pi \frac{1}{t} dt - 3 = -3$
67. b, since $y' = \sec x$ and $y(0) = \int_0^0 \sec t dt + 4 = 4$
69. $y = \int_2^x \sec t dt + 3$ 71. $(1 + \ln t)^2$
73. $\frac{2}{3}bh$ 75. \$9.00

77. (a) $T(0) = 70^\circ\text{F}$, $T(16) = 76^\circ\text{F}$, $T(25) = 85^\circ\text{F}$
(b) $\text{av}(T) = 75^\circ\text{F}$
79. $2x - 2$ 81. $-3x + 5$
83. (a) True. Since f is continuous, g is differentiable by Part 1 of the Fundamental Theorem of Calculus.
(b) True: g is continuous because it is differentiable.
(c) True, since $g'(1) = f(1) = 0$.
(d) False, since $g''(1) = f'(1) > 0$.
(e) True, since $g'(1) = 0$ and $g''(1) = f'(1) > 0$.
(f) False: $g''(x) = f'(x) > 0$, so g'' never changes sign.
(g) True, since $g'(1) = f(1) = 0$ and $g'(x) = f(x)$ is an increasing function of x (because $f'(x) > 0$).
85. (a) $v = \frac{ds}{dt} = \frac{d}{dt} \int_0^t f(x) dx = f(t) \Rightarrow v(5) = f(5) = 2 \text{ m/sec}$
(b) $a = df/dt$ is negative, since the slope of the tangent line at $t = 5$ is negative.

- (c) $s = \int_0^3 f(x) dx = \frac{1}{2}(3)(3) = \frac{9}{2}$ m, since the integral is the area of the triangle formed by $y = f(x)$, the x -axis, and $x = 3$.
- (d) $t = 6$, since after $t = 6$ to $t = 9$, the region lies below the x -axis.
- (e) At $t = 4$ and $t = 7$, since there are horizontal tangents there.
- (f) Toward the origin between $t = 6$ and $t = 9$, since the velocity is negative on this interval. Away from the origin between $t = 0$ and $t = 6$, since the velocity is positive there.
- (g) Right or positive side, because the integral of f from 0 to 9 is positive, there being more area above the x -axis than below.

SECTION 5.5, pp. 348–349

1. $\frac{1}{6}(2x + 4)^6 + C$ 3. $-\frac{1}{3}(x^2 + 5)^{-3} + C$
5. $\frac{1}{10}(3x^2 + 4x)^5 + C$ 7. $-\frac{1}{3}\cos 3x + C$
9. $\frac{1}{2}\sec 2t + C$ 11. $-6(1 - r^3)^{1/2} + C$
13. $\frac{1}{3}(x^{3/2} - 1) - \frac{1}{6}\sin(2x^{3/2} - 2) + C$
15. (a) $-\frac{1}{4}(\cot^2 2\theta) + C$ (b) $-\frac{1}{4}(\csc^2 2\theta) + C$
17. $-\frac{1}{3}(3 - 2s)^{3/2} + C$ 19. $-\frac{2}{5}(1 - \theta^2)^{5/4} + C$
21. $(-2/(1 + \sqrt{x})) + C$ 23. $\frac{1}{3}\tan(3x + 2) + C$
25. $\frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$ 27. $\left(\frac{r^3}{18} - 1\right)^6 + C$
29. $-\frac{2}{3}\cos(x^{3/2} + 1) + C$ 31. $\frac{1}{2\cos(2t + 1)} + C$
33. $-\sin\left(\frac{1}{t} - 1\right) + C$ 35. $-\frac{\sin^2(1/\theta)}{2} + C$
37. $\frac{2}{3}(1 + x)^{3/2} - 2(1 + x)^{1/2} + C$ 39. $\frac{2}{3}\left(2 - \frac{1}{x}\right)^{3/2} + C$
41. $\frac{2}{27}\left(1 - \frac{3}{x^3}\right)^{3/2} + C$ 43. $\frac{1}{12}(x - 1)^{12} + \frac{1}{11}(x - 1)^{11} + C$
45. $-\frac{1}{8}(1 - x)^8 + \frac{4}{7}(1 - x)^7 - \frac{2}{3}(1 - x)^6 + C$
47. $\frac{1}{5}(x^2 + 1)^{5/2} - \frac{1}{3}(x^2 + 1)^{3/2} + C$ 49. $\frac{-1}{4(x^2 - 4)^2} + C$
51. $e^{\sin x} + C$ 53. $2\tan(e^{\sqrt{x}} + 1) + C$ 55. $\ln|\ln x| + C$
57. $z - \ln(1 + e^z) + C$ 59. $\frac{5}{6}\tan^{-1}\left(\frac{2r}{3}\right) + C$
61. $e^{\sin^{-1} x} + C$ 63. $\frac{1}{3}(\sin^{-1} x)^3 + C$ 65. $\ln|\tan^{-1} y| + C$
67. (a) $-\frac{6}{2 + \tan^3 x} + C$ (b) $-\frac{6}{2 + \tan^3 x} + C$
 (c) $-\frac{6}{2 + \tan^3 x} + C$
69. $\frac{1}{6}\sin\sqrt{3(2r - 1)^2 + 6} + C$ 73. $s = \frac{1}{2}(3t^2 - 1)^4 - 5$
75. $s = 4t - 2\sin\left(2t + \frac{\pi}{6}\right) + 9$

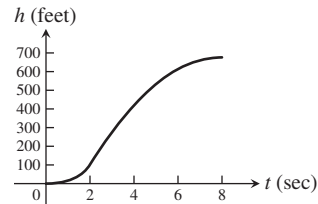
77. $s = \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1$ 79. 6 m

SECTION 5.6, pp. 355–359

1. (a) $14/3$ (b) $2/3$ 3. (a) $1/2$ (b) $-1/2$
5. (a) $15/16$ (b) 0 7. (a) 0 (b) $1/8$ 9. (a) 4 (b) 0
11. (a) $506/375$ (b) $86,744/375$ 13. (a) 0 (b) 0
15. $2\sqrt{3}$ 17. $3/4$ 19. $3^{5/2} - 1$ 21. 3 23. $\pi/3$
25. e 27. $\ln 3$ 29. $(\ln 2)^2$ 31. $\frac{1}{\ln 4}$ 33. $\ln 2$
35. $\ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{2}$ 37. π 39. $\pi/12$ 41. $2\pi/3$
43. $\sqrt{3} - 1$ 45. $-\pi/12$ 47. $\pi^2/32$ 49. $16/3$
51. $2^{5/2}$ 53. $\pi/2$ 55. $128/15$ 57. $4/3$ 59. $5/6$
61. $38/3$ 63. $49/6$ 65. $32/3$ 67. $48/5$ 69. $8/3$
71. 8 73. $5/3$ (There are three intersection points.) 75. 18
77. $243/8$ 79. $8/3$ 81. 2 83. $104/15$ 85. $56/15$
87. 4 89. $\frac{4}{3} - \frac{4}{\pi}$ 91. $\pi/2$ 93. 2 95. $1/2$
97. 1 99. $\ln 16$ 101. 2 103. $2\ln 5$
105. (a) $(\pm\sqrt{c}, c)$ (b) $c = 4^{2/3}$ (c) $c = 4^{2/3}$
107. $11/3$ 109. $3/4$ 111. Neither 113. $F(6) - F(2)$
115. (a) -3 (b) 3 117. $I = a/2$

PRACTICE EXERCISES, pp. 360–363

1. (a) About 680 ft (b) h (feet)



3. (a) $-1/2$ (b) 31 (c) 13 (d) 0
5. $\int_1^5 (2x - 1)^{-1/2} dx = 2$ 7. $\int_{-\pi}^0 \cos \frac{x}{2} dx = 2$
9. (a) 4 (b) 2 (c) -2 (d) -2π (e) $8/5$
11. $8/3$ 13. 62 15. 1 17. $1/6$ 19. 18
21. $9/8$ 23. $\frac{\pi^2}{32} + \frac{\sqrt{2}}{2} - 1$ 25. 4 27. $\frac{8\sqrt{2} - 7}{6}$
29. Min: -4, max: 0, area: $27/4$ 31. $6/5$ 33. 1
37. $y = \int_5^x \left(\frac{\sin t}{t}\right) dt - 3$ 39. $y = \sin^{-1} x$
41. $y = \sec^{-1} x + \frac{2\pi}{3}, x > 1$ 43. $f(x) = e^{x^2/2 - 1/2}$
45. $-4(\cos x)^{1/2} + C$ 47. $\theta^2 + \theta + \sin(2\theta + 1) + C$
49. $\frac{t^3}{3} + \frac{4}{t} + C$ 51. $-\frac{1}{3}\cos(2t^{3/2}) + C$
53. $\tan(e^x - 7) + C$ 55. $e^{\tan x} + C$ 57. $\frac{-\ln 7}{3}$
59. $\ln(9/25)$ 61. $-\frac{1}{2}(\ln x)^{-2} + C$ 63. $\frac{1}{2\ln 3}(3^{x^2}) + C$
65. $\frac{3}{2}\sin^{-1} 2(r - 1) + C$ 67. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x - 1}{\sqrt{2}}\right) + C$
69. $\frac{1}{4}\sec^{-1}\left|\frac{2x - 1}{2}\right| + C$ 71. $e^{\sin^{-1} \sqrt{x}} + C$

73. $2\sqrt{\tan^{-1}y} + C$ 75. $\frac{1}{4(\sin 2\theta + \cos 2\theta)^2} + C$ 77. 16

79. 2 81. 1 83. 8 85. $27\sqrt{3}/160$ 87. $\pi/2$
89. $\sqrt{3}$ 91. $6\sqrt{3} - 2\pi$ 93. -1 95. 2 97. 1
99. $15/16 + \ln 2$ 101. $e - 1$ 103. $1/6$ 105. $9/14$

107. $\frac{9\ln 2}{4}$ 109. π 111. $\pi/\sqrt{3}$ 113. $\pi/6$

115. $\pi/12$ 117. (a) b (b) b

121. (a) $\frac{d}{dx}(x \ln x - x + C) = x \cdot \frac{1}{x} + \ln x - 1 + 0 = \ln x$
(b) $\frac{1}{e - 1}$

123. 25°F 125. $\sqrt{2 + \cos^3 x}$ 127. $\frac{-6}{3 + x^4}$

129. $\frac{dy}{dx} = \frac{-2}{x} e^{\cos(2 \ln x)}$ 131. $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2} \sqrt{1 - 2(\sin^{-1} x)^2}}$

133. Yes 135. $-\sqrt{1 + x^2}$

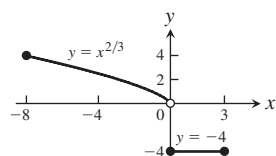
137. Cost $\approx \$10,899$ using a lower sum estimate

ADDITIONAL AND ADVANCED EXERCISES, pp. 364-367

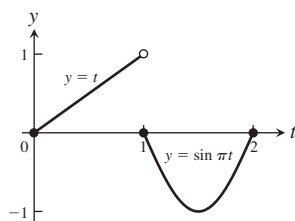
1. (a) Yes (b) No 5. (a) $1/4$ (b) $\sqrt[3]{12}$

7. $f(x) = \frac{x}{\sqrt{x^2 + 1}}$ 9. $y = x^3 + 2x - 4$

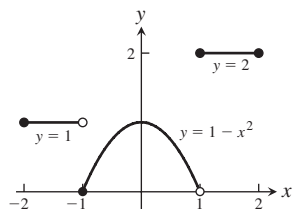
11. $36/5$



13. $\frac{1}{2} - \frac{2}{\pi}$



15. $13/3$



21. $\ln 2$ 23. $1/6$

25. $\int_0^1 f(x) dx$ 27. (b) πr^2

29. (a) 0 (b) -1
(c) $-\pi$ (d) $x = 1$
(e) $y = 2x + 2 - \pi$
(f) $x = -1, x = 2$
(g) $[-2\pi, 0]$

17. $1/2$ 19. $\pi/2$

31. $2/x$ 33. $\frac{\sin 4y}{\sqrt{y}} - \frac{\sin y}{2\sqrt{y}}$ 35. $2x \ln |x| - x \ln \frac{|x|}{\sqrt{2}}$

37. $(\sin x)/x$ 39. $x = 1$ 41. $\frac{1}{\ln 2}, \frac{1}{2 \ln 2}, 2:1$

43. $2/17$

Chapter 6

SECTION 6.1, pp. 375-379

1. 16 3. $16/3$ 5. (a) $2\sqrt{3}$ (b) 8 7. (a) 60 (b) 36
9. 8π 11. 10 13. (a) $s^2 h$ (b) $s^2 h$ 15. $8/3$

17. $\frac{2\pi}{3}$ 19. $4 - \pi$ 21. $\frac{32\pi}{5}$ 23. 36π 25. π

27. $\frac{\pi}{2} \left(1 - \frac{1}{e^2}\right)$ 29. $\frac{\pi}{2} \ln 4$ 31. $\pi \left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3}\right)$

33. 2π 35. 2π 37. $4\pi \ln 4$ 39. $\pi^2 - 2\pi$ 41. $\frac{2\pi}{3}$

43. $\frac{117\pi}{5}$ 45. $\pi(\pi - 2)$ 47. $\frac{4\pi}{3}$ 49. 8π 51. $\frac{7\pi}{6}$

53. (a) 8π (b) $\frac{32\pi}{5}$ (c) $\frac{8\pi}{3}$ (d) $\frac{224\pi}{15}$

55. (a) $\frac{16\pi}{15}$ (b) $\frac{56\pi}{15}$ (c) $\frac{64\pi}{15}$ 57. $V = 2a^2 b \pi^2$

59. (a) $V = \frac{\pi h^2(3a - h)}{3}$ (b) $\frac{1}{120\pi} \text{ m/sec}$

63. $V = 3308 \text{ cm}^3$ 65. $\frac{4 - b + a}{2}$

SECTION 6.2, pp. 384-387

1. 6π 3. 2π 5. $14\pi/3$ 7. 8π 9. $5\pi/6$

11. $\frac{7\pi}{15}$ 13. (b) 4π 15. $\frac{16\pi}{15}(3\sqrt{2} + 5)$

17. $\frac{8\pi}{3}$ 19. $\frac{4\pi}{3}$ 21. $\frac{16\pi}{3}$

23. (a) 16π (b) 32π (c) 28π
(d) 24π (e) 60π (f) 48π

25. (a) $\frac{27\pi}{2}$ (b) $\frac{27\pi}{2}$ (c) $\frac{72\pi}{5}$ (d) $\frac{108\pi}{5}$

27. (a) $\frac{6\pi}{5}$ (b) $\frac{4\pi}{5}$ (c) 2π (d) 2π

29. (a) About the x -axis: $V = \frac{2\pi}{15}$; about the y -axis: $V = \frac{\pi}{6}$

(b) About the x -axis: $V = \frac{2\pi}{15}$; about the y -axis: $V = \frac{\pi}{6}$

31. (a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) 2π (d) $\frac{2\pi}{3}$

33. (a) $\frac{4\pi}{15}$ (b) $\frac{7\pi}{30}$ 35. (a) $\frac{24\pi}{5}$ (b) $\frac{48\pi}{5}$

37. (a) $\frac{9\pi}{16}$ (b) $\frac{9\pi}{16}$

39. Disk: 2 integrals; washer: 2 integrals; shell: 1 integral

41. (a) $\frac{256\pi}{3}$ (b) $\frac{244\pi}{3}$

47. $\pi \left(1 - \frac{1}{e}\right)$ 49. 2

SECTION 6.3, pp. 391-393

1. 12 3. $\frac{53}{6}$ 5. $\frac{123}{32}$ 7. $\frac{99}{8}$ 9. $\ln 2 + \frac{3}{8}$

11. $\frac{53}{6}$ 13. $2^{3/2} - (5/4)^{3/2}$ 15. 2

17. (a) $\int_{-1}^2 \sqrt{1 + 4x^2} dx$ (c) ≈ 6.13

19. (a) $\int_0^\pi \sqrt{1 + \cos^2 y} dy$ (c) ≈ 3.82

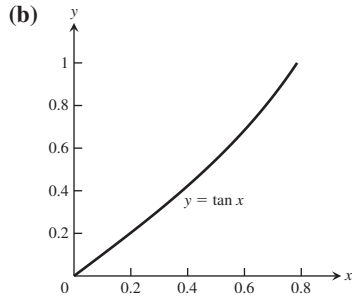
21. (a) $\int_{-1}^3 \sqrt{1 + (y + 1)^2} dy$ (c) ≈ 9.29

23. (a) $\int_0^{\pi/6} \sec x dx$ (c) ≈ 0.55

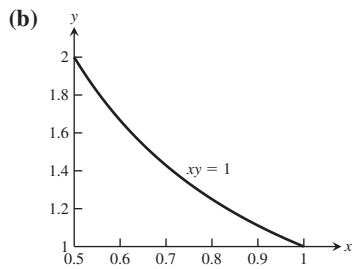
25. (a) $y = \sqrt{x}$ from (1, 1) to (4, 2)
 (b) Only one. We know the derivative of the function and the value of the function at one value of x .
 27. 1 29. Yes, $f(x) = \pm x + C$ where C is any real number.
 37. $\int_0^x \sqrt{1+9t} dt, \frac{2}{27}(10^{3/2} - 1)$

SECTION 6.4, pp. 396–398

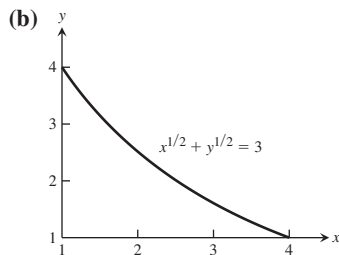
1. (a) $2\pi \int_0^{\pi/4} (\tan x) \sqrt{1 + \sec^4 x} dx$ (c) $S \approx 3.84$



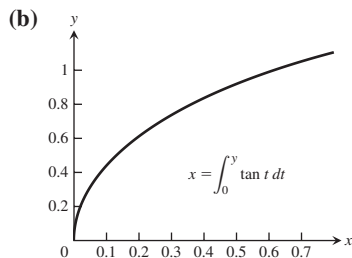
3. (a) $2\pi \int_1^2 \frac{1}{y} \sqrt{1+y^{-4}} dy$ (c) $S \approx 5.02$



5. (a) $2\pi \int_1^4 (3 - x^{1/2})^2 \sqrt{1 + (1 - 3x^{-1/2})^2} dx$ (c) $S \approx 63.37$



7. (a) $2\pi \int_0^{\pi/3} \left(\int_0^y \tan t dt \right) \sec y dy$ (c) $s \approx 2.08$



9. $4\pi\sqrt{5}$ 11. $3\pi\sqrt{5}$ 13. $98\pi/81$ 15. 2π

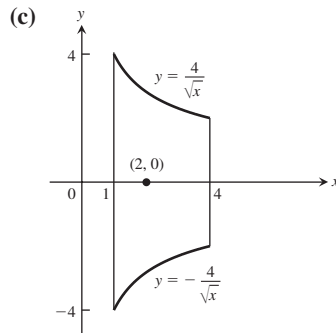
17. $\pi(\sqrt{8} - 1)/9$ 19. $35\pi\sqrt{5}/3$ 21. $\pi\left(\frac{15}{16} + \ln 2\right)$
 23. $253\pi/20$ 27. Order 226.2 liters of each color.

SECTION 6.5, pp. 404–408

1. 116 J 3. 400 N/m 5. 4 cm, 0.08 J
 7. (a) 7238 lb/in. (b) 905 in.-lb, 2714 in.-lb
 9. 780 J 11. 72,900 ft-lb 13. 490 J
 15. (a) 1,497,600 ft-lb (b) 1 hr, 40 min
 (d) At 62.26 lb/ft³: a) 1,494,240 ft-lb b) 1 hr, 40 min
 At 62.59 lb/ft³: a) 1,502,160 ft-lb b) 1 hr, 40.1 min
 17. 37,306 ft-lb 19. 7,238,299.47 ft-lb 21. 2446.25 ft-lb
 23. 15,073,099.75 J 27. 85.1 ft-lb 29. 151.3 J
 31. 91.32 in.-oz 33. 5.144×10^{10} J 35. 1684.8 lb
 37. (a) 6364.8 lb (b) 5990.4 lb 39. 1164.8 lb 41. 1309 lb
 43. (a) 12,480 lb (b) 8580 lb (c) 9722.3 lb
 45. (a) 93.33 lb (b) 3 ft 47. $\frac{wb}{2}$
 49. No. The tank will overflow because the movable end will have moved only $3\frac{1}{3}$ ft by the time the tank is full.

SECTION 6.6, pp. 418–420

1. $M = 14/3, \bar{x} = 93/35$ 3. $M = \ln 4, \bar{x} = (3 - \ln 4)/(\ln 4)$
 5. $M = 13, \bar{x} = 41/26$ 7. $\bar{x} = 0, \bar{y} = 12/5$
 9. $\bar{x} = 1, \bar{y} = -3/5$ 11. $\bar{x} = 16/105, \bar{y} = 8/15$
 13. $\bar{x} = 0, \bar{y} = \pi/8$ 15. $\bar{x} \approx 1.44, \bar{y} \approx 0.36$
 17. $\bar{x} = \frac{\ln 4}{\pi}, \bar{y} = 0$ 19. $\bar{x} = 7, \bar{y} = \frac{\ln 16}{12}$
 21. $\bar{x} = 5/7, \bar{y} = 10/33. (\bar{x})^4 < \bar{y}$, so the center of mass is outside the region.
 23. $\bar{x} = 3/2, \bar{y} = 1/2$
 25. (a) $\frac{224\pi}{3}$ (b) $\bar{x} = 2, \bar{y} = 0$



29. $\bar{x} = \bar{y} = 1/3$ 31. $\bar{x} = a/3, \bar{y} = b/3$ 33. $13\delta/6$
 35. $\bar{x} = 0, \bar{y} = \frac{a\pi}{4}$ 37. $\bar{x} = 1/2, \bar{y} = 4$
 39. $\bar{x} = 6/5, \bar{y} = 8/7$ 41. $V = 32\pi, S = 32\sqrt{2}\pi$ 45. $4\pi^2$
 47. $\bar{x} = 0, \bar{y} = \frac{2a}{\pi}$ 49. $\bar{x} = 0, \bar{y} = \frac{4b}{3\pi}$
 51. $\sqrt{2}\pi a^3(4 + 3\pi)/6$ 53. $\bar{x} = \frac{a}{3}, \bar{y} = \frac{b}{3}$

PRACTICE EXERCISES, pp. 421–423

1. $\frac{9\pi}{280}$ 3. π^2 5. $\frac{72\pi}{35}$
 7. (a) 2π (b) π (c) $12\pi/5$ (d) $26\pi/5$

9. (a) 8π (b) $1088\pi/15$ (c) $512\pi/15$
 11. $\pi(3\sqrt{3} - \pi)/3$ 13. π 15. $\frac{28\pi}{3}\text{ ft}^3$
 17. $(\pi/3)(a^2 + ab + b^2)h$ 19. $\frac{10}{3}$ 21. $3 + \frac{1}{8}\ln 2$
 23. $\sqrt{2}$ 25. $28\pi\sqrt{2}/3$ 27. 4π 29. 4640 J
 31. $\frac{w}{2}(2ar - a^2)$ 33. $418,208.81\text{ ft}\cdot\text{lb}$
 35. $22,500\pi\text{ ft}\cdot\text{lb}$, 257 sec 37. (a) $128\text{ ft}\cdot\text{lb}$ (b) $219.6\text{ ft}\cdot\text{lb}$
 39. $\bar{x} = 0, \bar{y} = 8/5$ 41. $\bar{x} = 3/2, \bar{y} = 12/5$
 43. $\bar{x} = 9/5, \bar{y} = 11/10$ 45. 332.8 lb 47. 2196.48 lb

ADDITIONAL AND ADVANCED EXERCISES, pp. 423–424

1. $f(x) = \sqrt{\frac{2x-a}{\pi}}$ 3. $f(x) = \sqrt{C^2 - 1}x + a$, where $C \geq 1$
 5. $\frac{\pi}{30\sqrt{2}}$ 7. $28/3$ 9. $\frac{4h\sqrt{3mh}}{3}$
 11. $\bar{x} = 0, \bar{y} = \frac{n}{2n+1}, (0, 1/2)$
 15. (a) $\bar{x} = \bar{y} = 4(a^2 + ab + b^2)/(3\pi(a + b))$
 (b) $(2a/\pi, 2a/\pi)$
 17. $\approx 2329.6\text{ lb}$

Chapter 7

SECTION 7.1, pp. 433–435

1. $\ln\left(\frac{2}{3}\right)$ 3. $\ln|y^2 - 25| + C$ 5. $\ln|6 + 3\tan t| + C$
 7. $\ln(1 + \sqrt{x}) + C$ 9. 1 11. $2(\ln 2)^4$ 13. 2
 15. $2e^{\sqrt{r}} + C$ 17. $-e^{-t^2} + C$ 19. $-e^{1/x} + C$
 21. $\frac{1}{\pi}e^{\sec \pi t} + C$ 23. 1 25. $\ln(1 + e^r) + C$ 27. $\frac{1}{2\ln 2}$
 29. $\frac{1}{\ln 2}$ 31. $\frac{6}{\ln 7}$ 33. 32760 35. $3^{\sqrt{2}+1}$
 37. $\frac{1}{\ln 10}\left(\frac{(\ln x)^2}{2}\right) + C$ 39. $2(\ln 2)^2$ 41. $\frac{3\ln 2}{2}$ 43. $\ln 10$
 45. $(\ln 10)\ln|\ln x| + C$ 47. $y = 1 - \cos(e^t - 2)$
 49. $y = 2(e^{-x} + x) - 1$ 51. $y = x + \ln|x| + 2$
 53. $\pi\ln 16$ 55. $6 + \ln 2$ 57. (b) 0.00469
 71. (a) 1.89279 (b) -0.35621 (c) 0.94575 (d) -2.80735
 (e) 5.29595 (f) 0.97041 (g) -1.03972 (h) -1.61181

SECTION 7.2, pp. 442–444

9. $\frac{2}{3}y^{3/2} - x^{1/2} = C$ 11. $e^y - e^x = C$
 13. $-x + 2\tan\sqrt{y} = C$ 15. $e^{-y} + 2e^{\sqrt{x}} = C$
 17. $y = \sin(x^2 + C)$ 19. $\frac{1}{3}\ln|y^3 - 2| = x^3 + C$
 21. $4\ln(\sqrt{y} + 2) = e^{x^2} + C$
 23. (a) -0.00001 (b) $10,536\text{ years}$ (c) 82%
 25. 54.88 g 27. 59.8 ft 29. 2.8147498×10^{14}
 31. (a) 8 years (b) 32.02 years 33. Yes, $y(20) < 1$
 35. 15.28 years 37. $56,562\text{ years}$
 41. (a) 17.5 min (b) 13.26 min 43. -3°C
 45. About 6693 years 47. 54.62% 49. $\approx 15,683\text{ years}$

SECTION 7.3, pp. 450–453

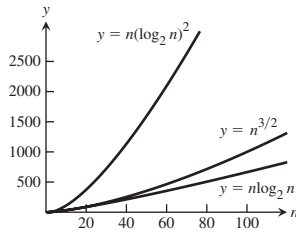
1. $\cosh x = 5/4, \tanh x = -3/5, \coth x = -5/3,$
 $\operatorname{sech} x = 4/5, \operatorname{csch} x = -4/3$
 3. $\sinh x = 8/15, \tanh x = 8/17, \coth x = 17/8, \operatorname{sech} x = 15/17,$
 $\operatorname{csch} x = 15/8$
 5. $x + \frac{1}{x}$ 7. e^{5x} 9. e^{4x} 13. $2\cosh \frac{x}{3}$
 15. $\operatorname{sech}^2 \sqrt{t} + \frac{\tanh \sqrt{t}}{\sqrt{t}}$ 17. $\coth z$
 19. $(\ln \operatorname{sech} \theta)(\operatorname{sech} \theta \tanh \theta)$ 21. $\tanh^3 v$ 23. 2
 25. $\frac{1}{2\sqrt{x(1+x)}}$ 27. $\frac{1}{1+\theta} - \tanh^{-1} \theta$
 29. $\frac{1}{2\sqrt{t}} - \coth^{-1} \sqrt{t}$ 31. $-\operatorname{sech}^{-1} x$ 33. $\frac{\ln 2}{\sqrt{1 + \left(\frac{1}{2}\right)^{2\theta}}}$
 35. $|\sec x|$ 41. $\frac{\cosh 2x}{2} + C$
 43. $12\sinh\left(\frac{x}{2} - \ln 3\right) + C$ 45. $7\ln|e^{x/7} + e^{-x/7}| + C$
 47. $\tanh\left(x - \frac{1}{2}\right) + C$ 49. $-2\operatorname{sech} \sqrt{t} + C$ 51. $\ln \frac{5}{2}$
 53. $\frac{3}{32} + \ln 2$ 55. $e - e^{-1}$ 57. $3/4$ 59. $\frac{3}{8} + \ln \sqrt{2}$
 61. $\ln(2/3)$ 63. $\frac{-\ln 3}{2}$ 65. $\ln 3$
 67. (a) $\sinh^{-1}(\sqrt{3})$ (b) $\ln(\sqrt{3} + 2)$
 69. (a) $\coth^{-1}(2) - \coth^{-1}(5/4)$ (b) $\left(\frac{1}{2}\right)\ln\left(\frac{1}{3}\right)$
 71. (a) $-\operatorname{sech}^{-1}\left(\frac{12}{13}\right) + \operatorname{sech}^{-1}\left(\frac{4}{5}\right)$
 (b) $-\ln\left(\frac{1 + \sqrt{1 - (12/13)^2}}{(12/13)}\right) + \ln\left(\frac{1 + \sqrt{1 - (4/5)^2}}{(4/5)}\right)$
 $= -\ln\left(\frac{3}{2}\right) + \ln(2) = \ln(4/3)$
 73. (a) 0 (b) 0
 77. (a) $\sqrt{\frac{mg}{k}}$ (b) $80\sqrt{5} \approx 178.89\text{ ft/sec}$
 79. 2π 81. $\frac{6}{5}$
- SECTION 7.4, pp. 457–458**
1. (a) Slower (b) Slower (c) Slower (d) Faster
 (e) Slower (f) Slower (g) Same (h) Slower
 3. (a) Same (b) Faster (c) Same (d) Same
 (e) Slower (f) Faster (g) Slower (h) Same
 5. (a) Same (b) Same (c) Same (d) Faster
 (e) Faster (f) Same (g) Slower (h) Faster 7. d, a, c, b
 9. (a) False (b) False (c) True (d) True (e) True
 (f) True (g) False (h) True
 13. When the degree of f is less than or equal to the degree of g .
 15. $1, 1$
 21. (b) $\ln(e^{17000000}) = 17,000,000 < (e^{17 \times 10^6})^{1/10^6}$
 $= e^{17} \approx 24,154,952.75$

(c) $x \approx 3.4306311 \times 10^{15}$

(d) They cross at $x \approx 3.4306311 \times 10^{15}$.

23. (a) The algorithm that takes $O(n \log_2 n)$ steps

(b)



25. It could take one million for a sequential search; at most 20 steps for a binary search.

PRACTICE EXERCISES, pp. 459–460

1. $-\cos e^x + C$ 3. $\ln 8$ 5. $2 \ln 2$

7. $\frac{1}{2}(\ln(x-5))^2 + C$ 9. $3 \ln 7$ 11. $2(\sqrt{2} - 1)$

13. $y = \frac{\ln 2}{\ln(3/2)}$ 15. $y = \ln x - \ln 3$ 17. $y = \frac{1}{1 - e^x}$

19. (a) Same rate (b) Same rate (c) Faster (d) Faster
(e) Same rate (f) Same rate21. (a) True (b) False (c) False (d) True
(e) True (f) True

23. $1/3$ 25. $1/e$ m/sec 27. $\ln 5x - \ln 3x = \ln(5/3)$

29. $1/2$ 31. $y = \left(\tan^{-1} \left(\frac{x+C}{2} \right) \right)^2$

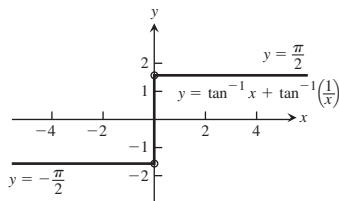
33. $y^2 = \sin^{-1}(2 \tan x + C)$

35. $y = -2 + \ln(2 - e^{-x})$ 37. $y = 4x - 4\sqrt{x} + 1$

39. 19,035 years 41. $\ln(16/9)$

ADDITIONAL AND ADVANCED EXERCISES, p. 460

1. (a) 1 (b) $\pi/2$ (c) π

3. $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$ is a constant and the constant is $\frac{\pi}{2}$ for $x > 0$; it is $-\frac{\pi}{2}$ for $x < 0$.

7. $\bar{x} = \frac{\ln 4}{\pi}$, $\bar{y} = 0$

Chapter 8

SECTION 8.1, pp. 465–466

1. $\ln 5$ 3. $2 \tan x - 2 \sec x - x + C$

5. $\sin^{-1}x + \sqrt{1-x^2} + C$ 7. $e^{-\cot z} + C$

9. $\tan^{-1}(e^z) + C$ 11. π 13. $t + \cot t + \csc t + C$

15. $\sqrt{2}$ 17. $\frac{1}{8} \ln(1 + 4 \ln^2 y) + C$

19. $\ln|1 + \sin \theta| + C$ 21. $2t^2 - t + 2 \tan^{-1}\left(\frac{t}{2}\right) + C$

23. $2(\sqrt{2} - 1) \approx 0.82843$

25. $\sec^{-1}(e^y) + C$

27. $\sin^{-1}(2 \ln x) + C$ 29. $\ln|\sin x| + \ln|\cos x| + C$

31. $7 + \ln 8$ 33. $(\sin^{-1}y - \sqrt{1-y^2})_{-1}^0 = \frac{\pi}{2} - 1$

35. $\sec^{-1}\left|\frac{x-1}{7}\right| + C$ 37. $\frac{\theta^3}{3} - \frac{\theta^2}{2} + \theta + \frac{5}{2} \ln|2\theta - 5| + C$

39. $x - \ln(1 + e^x) + C$ 41. $(1/2)e^{2x} - e^x + \ln(1 + e^x) + C$

43. $2 \arctan(\sqrt{x}) + C$ 45. $2\sqrt{2} - \ln(3 + 2\sqrt{2})$

47. $\ln(2 + \sqrt{3})$ 49. $\bar{x} = 0$, $\bar{y} = \frac{1}{\ln(3 + 2\sqrt{2})}$

51. $xe^{x^3} + C$ 53. $\frac{1}{30}(x^4 + 1)^{3/2}(3x^4 - 2) + C$

SECTION 8.2, pp. 471–474

1. $-2x \cos(x/2) + 4 \sin(x/2) + C$

3. $t^2 \sin t + 2t \cos t - 2 \sin t + C$

5. $\ln 4 - \frac{3}{4}$ 7. $xe^x - e^x + C$

9. $-(x^2 + 2x + 2)e^{-x} + C$

11. $y \tan^{-1}(y) - \ln\sqrt{1+y^2} + C$

13. $x \tan x + \ln|\cos x| + C$

15. $(x^3 - 3x^2 + 6x - 6)e^x + C$ 17. $(x^2 - 7x + 7)e^x + C$

19. $(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)e^x + C$

21. $\frac{1}{2}(-e^\theta \cos \theta + e^\theta \sin \theta) + C$

23. $\frac{e^{2x}}{13}(3 \sin 3x + 2 \cos 3x) + C$

25. $\frac{2}{3}(\sqrt{3s+9}e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}) + C$

27. $\frac{\pi\sqrt{3}}{3} - \ln(2) - \frac{\pi^2}{18}$

29. $\frac{1}{2}[-x \cos(\ln x) + x \sin(\ln x)] + C$

31. $\frac{1}{2} \ln|\sec x^2 + \tan x^2| + C$

33. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$

35. $-\frac{1}{x} \ln x - \frac{1}{x} + C$ 37. $\frac{1}{4}e^{x^4} + C$

39. $\frac{1}{3}x^2(x^2 + 1)^{3/2} - \frac{2}{15}(x^2 + 1)^{5/2} + C$

41. $-\frac{2}{5} \sin 3x \sin 2x - \frac{3}{5} \cos 3x \cos 2x + C$

43. $\frac{2}{9}x^{3/2}(3 \ln x - 2) + C$

45. $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$

47. $\frac{\pi^2 - 4}{8}$ 49. $\frac{5\pi - 3\sqrt{3}}{9}$

51. $\frac{1}{2}(x^2 + 1) \tan^{-1}x - \frac{x}{2} + C$ 53. $xe^{x^2} + C$

55. $(2/3)x^{3/2} \arcsin(\sqrt{x}) + (2/9)x\sqrt{1-x} + (4/9)\sqrt{1-x} + C$

57. (a) π (b) 3π (c) 5π (d) $(2n+1)\pi$

59. $2\pi(1 - \ln 2)$ 61. (a) $\pi(\pi - 2)$ (b) 2π

63. (a) 1 (b) $(e-2)\pi$ (c) $\frac{\pi}{2}(e^2 + 9)$

(d) $\bar{x} = \frac{1}{4}(e^2 + 1)$, $\bar{y} = \frac{1}{2}(e - 2)$

65. $\frac{1}{2\pi}(1 - e^{-2\pi})$ 67. $u = x^n$, $dv = \cos x dx$

69. $u = x^n$, $dv = e^{ax} dx$ 73. $u = x^n$, $dv = (x+1)^{-(1/2)} dx$

77. $x \sin^{-1} x + \cos(\sin^{-1} x) + C$
 79. $x \sec^{-1} x - \ln |x + \sqrt{x^2 - 1}| + C$ **81.** Yes
 83. (a) $x \sinh^{-1} x - \cosh(\sinh^{-1} x) + C$
 (b) $x \sinh^{-1} x - (1 + x^2)^{1/2} + C$

SECTION 8.3, pp. 479–480

1. $\frac{1}{2} \sin 2x + C$ **3.** $-\frac{1}{4} \cos^4 x + C$
 5. $\frac{1}{3} \cos^3 x - \cos x + C$ **7.** $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$
 9. $\sin x - \frac{1}{3} \sin^3 x + C$ **11.** $\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$
 13. $\frac{1}{2} x + \frac{1}{4} \sin 2x + C$ **15.** $16/35$ **17.** 3π
 19. $-4 \sin x \cos^3 x + 2 \cos x \sin x + 2x + C$
 21. $-\cos^4 2\theta + C$ **23.** 4 **25.** 2
 27. $\sqrt{\frac{3}{2} - \frac{2}{3}}$ **29.** $\frac{4}{5} \left(\frac{3}{2}\right)^{5/2} - \frac{18}{35} - \frac{2}{7} \left(\frac{3}{2}\right)^{7/2}$ **31.** $\sqrt{2}$
 33. $\frac{1}{2} \tan^2 x + C$ **35.** $\frac{1}{3} \sec^3 x + C$ **37.** $\frac{1}{3} \tan^3 x + C$
 39. $2\sqrt{3} + \ln(2 + \sqrt{3})$ **41.** $\frac{2}{3} \tan \theta + \frac{1}{3} \sec^2 \theta \tan \theta + C$
 43. $4/3$ **45.** $2 \tan^2 x - 2 \ln(1 + \tan^2 x) + C$
 47. $\frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \ln |\sec x| + C$ **49.** $\frac{4}{3} - \ln \sqrt{3}$
 51. $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$ **53.** π
 55. $\frac{1}{2} \sin x + \frac{1}{14} \sin 7x + C$
 57. $\frac{1}{6} \sin 3\theta - \frac{1}{4} \sin \theta - \frac{1}{20} \sin 5\theta + C$
 59. $-\frac{2}{5} \cos^5 \theta + C$ **61.** $\frac{1}{4} \cos \theta - \frac{1}{20} \cos 5\theta + C$
 63. $\sec x - \ln |\csc x + \cot x| + C$ **65.** $\cos x + \sec x + C$
 67. $\frac{1}{4} x^2 - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + C$ **69.** $\ln(2 + \sqrt{3})$
 71. $\pi^2/2$ **73.** $\bar{x} = \frac{4\pi}{3}, \bar{y} = \frac{8\pi^2 + 3}{12\pi}$ **75.** $(\pi/4)(4 - \pi)$

SECTION 8.4, pp. 484–485

1. $\ln |\sqrt{9 + x^2} + x| + C$ **3.** $\pi/4$ **5.** $\pi/6$
 7. $\frac{25}{2} \sin^{-1}\left(\frac{t}{5}\right) + \frac{t\sqrt{25 - t^2}}{2} + C$
 9. $\frac{1}{2} \ln \left| \frac{2x}{7} + \frac{\sqrt{4x^2 - 49}}{7} \right| + C$
 11. $7 \left[\frac{\sqrt{y^2 - 49}}{7} - \sec^{-1}\left(\frac{y}{7}\right) \right] + C$ **13.** $\frac{\sqrt{x^2 - 1}}{x} + C$
 15. $-\sqrt{9 - x^2} + C$ **17.** $\frac{1}{3}(x^2 + 4)^{3/2} - 4\sqrt{x^2 + 4} + C$
 19. $\frac{-2\sqrt{4 - w^2}}{w} + C$ **21.** $\sin^{-1} x - \sqrt{1 - x^2} + C$
 23. $4\sqrt{3} - \frac{4\pi}{3}$ **25.** $-\frac{x}{\sqrt{x^2 - 1}} + C$
 27. $-\frac{1}{5} \left(\frac{\sqrt{1 - x^2}}{x} \right)^5 + C$ **29.** $2 \tan^{-1} 2x + \frac{4x}{(4x^2 + 1)} + C$
 31. $\frac{1}{2} x^2 + \frac{1}{2} \ln |x^2 - 1| + C$ **33.** $\frac{1}{3} \left(\frac{v}{\sqrt{1 - v^2}} \right)^3 + C$
 35. $\ln 9 - \ln(1 + \sqrt{10})$ **37.** $\pi/6$ **39.** $\sec^{-1} |x| + C$

41. $\sqrt{x^2 - 1} + C$ **43.** $\frac{1}{2} \ln |\sqrt{1 + x^4} + x^2| + C$
 45. $4 \sin^{-1} \frac{\sqrt{x}}{2} + \sqrt{x} \sqrt{4 - x} + C$
 47. $\frac{1}{4} \sin^{-1} \sqrt{x} - \frac{1}{4} \sqrt{x} \sqrt{1 - x} (1 - 2x) + C$
 49. $(9/2) \arcsin\left(\frac{x+1}{3}\right) + (1/2)(x+1)\sqrt{8 - 2x - x^2} + C$
 51. $\sqrt{x^2 + 4x + 3} - \operatorname{arcsec}(x + 2) + C$
 53. $y = 2 \left[\frac{\sqrt{x^2 - 4}}{2} - \sec^{-1}\left(\frac{x}{2}\right) \right]$
 55. $y = \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) - \frac{3\pi}{8}$ **57.** $3\pi/4$
 59. (a) $\frac{1}{12}(\pi + 6\sqrt{3} - 12)$
 (b) $\bar{x} = \frac{3\sqrt{3} - \pi}{4(\pi + 6\sqrt{3} - 12)}, \bar{y} = \frac{\pi^2 + 12\sqrt{3}\pi - 72}{12(\pi + 6\sqrt{3} - 12)}$
 61. (a) $-\frac{1}{3} x^2 (1 - x^2)^{3/2} - \frac{2}{15} (1 - x^2)^{5/2} + C$
 (b) $-\frac{1}{3} (1 - x^2)^{3/2} + \frac{1}{5} (1 - x^2)^{5/2} + C$
 (c) $\frac{1}{5} (1 - x^2)^{5/2} - \frac{1}{3} (1 - x^2)^{3/2} + C$
 63. $\sqrt{3} - \frac{\sqrt{2}}{2} + \frac{1}{2} \ln \left(\frac{2 + \sqrt{3}}{\sqrt{2} + 1} \right)$

SECTION 8.5, pp. 491–493

1. $\frac{2}{x-3} + \frac{3}{x-2}$ **3.** $\frac{1}{x+1} + \frac{3}{(x+1)^2}$
 5. $\frac{-2}{z} + \frac{-1}{z^2} + \frac{2}{z-1}$ **7.** $1 + \frac{17}{t-3} + \frac{-12}{t-2}$
 9. $\frac{1}{2} [\ln |1 + x| - \ln |1 - x|] + C$
 11. $\frac{1}{7} \ln |(x+6)^2(x-1)^5| + C$ **13.** $(\ln 15)/2$
 15. $-\frac{1}{2} \ln |t| + \frac{1}{6} \ln |t+2| + \frac{1}{3} \ln |t-1| + C$ **17.** $3 \ln 2 - 2$
 19. $\frac{1}{4} \ln \left| \frac{x+1}{x-1} \right| - \frac{x}{2(x^2 - 1)} + C$ **21.** $(\pi + 2 \ln 2)/8$
 23. $\tan^{-1} y - \frac{1}{y^2 + 1} + C$
 25. $-(s-1)^{-2} + (s-1)^{-1} + \tan^{-1} s + C$
 27. $\frac{2}{3} \ln |x-1| + \frac{1}{6} \ln |x^2 + x + 1| - \sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + C$
 29. $\frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \tan^{-1} x + C$
 31. $\frac{-1}{\theta^2 + 2\theta + 2} + \ln(\theta^2 + 2\theta + 2) - \tan^{-1}(\theta + 1) + C$
 33. $x^2 + \ln \left| \frac{x-1}{x} \right| + C$
 35. $9x + 2 \ln |x| + \frac{1}{x} + 7 \ln |x-1| + C$
 37. $\frac{y^2}{2} - \ln |y| + \frac{1}{2} \ln(1 + y^2) + C$ **39.** $\ln \left(\frac{e^t + 1}{e^t + 2} \right) + C$
 41. $\frac{1}{5} \ln \left| \frac{\sin y - 2}{\sin y + 3} \right| + C$

$$43. \frac{(\tan^{-1} 2x)^2}{4} - 3 \ln |x - 2| + \frac{6}{x - 2} + C$$

$$45. \ln \left| \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right| + C$$

$$47. 2\sqrt{1+x} + \ln \left| \frac{\sqrt{x+1} - 1}{\sqrt{x+1} + 1} \right| + C$$

$$49. \frac{1}{4} \ln \left| \frac{x^4}{x^4 + 1} \right| + C$$

$$51. \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{2} \cos \theta + 1}{\sqrt{2} \cos \theta - 1} \right| + \frac{1}{2} \ln \left| \frac{1 - \cos \theta}{1 + \cos \theta} \right| + C$$

$$53. 4\sqrt{1+\sqrt{x}} + 2 \ln \left| \frac{\sqrt{1+\sqrt{x}} - 1}{\sqrt{1+\sqrt{x}} + 1} \right| + C$$

$$55. \frac{1}{3}x^3 - 2x^2 + 5x - 10 \ln |x + 2| + C$$

$$57. \frac{1}{\ln 2} \ln(2^x + 2^{-x}) + C \quad 59. \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \arctan x + C$$

$$61. \frac{1}{2} \ln |(\ln x + 1)(\ln x + 3)| + C$$

$$63. \ln |x + \sqrt{x^2 - 1}| + C$$

$$65. \frac{2}{9}x^3(x^3 + 1)^{3/2} - \frac{4}{45}(x^3 + 1)^{5/2} + C$$

$$67. x = \ln |t - 2| - \ln |t - 1| + \ln 2$$

$$69. x = \frac{6t}{t+2} - 1 \quad 71. 3\pi \ln 25$$

$$73. \ln(3) - \frac{1}{2} \quad 75. 1.10$$

$$77. (a) x = \frac{1000e^{4t}}{499 + e^{4t}} \quad (b) 1.55 \text{ days}$$

SECTION 8.6, pp. 497–499

$$1. \frac{2}{\sqrt{3}} \left(\tan^{-1} \sqrt{\frac{x-3}{3}} \right) + C$$

$$3. \sqrt{x-2} \left(\frac{2(x-2)}{3} + 4 \right) + C \quad 5. \frac{(2x-3)^{3/2}(x+1)}{5} + C$$

$$7. \frac{-\sqrt{9-4x}}{x} - \frac{2}{3} \ln \left| \frac{\sqrt{9-4x}-3}{\sqrt{9-4x}+3} \right| + C$$

$$9. \frac{(x+2)(2x-6)\sqrt{4x-x^2}}{6} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) + C$$

$$11. -\frac{1}{\sqrt{7}} \ln \left| \frac{\sqrt{7} + \sqrt{7+x^2}}{x} \right| + C$$

$$13. \sqrt{4-x^2} - 2 \ln \left| \frac{2 + \sqrt{4-x^2}}{x} \right| + C$$

$$15. \frac{e^{2t}}{13} (2 \cos 3t + 3 \sin 3t) + C$$

$$17. \frac{x^2}{2} \cos^{-1} x + \frac{1}{4} \sin^{-1} x - \frac{1}{4} x \sqrt{1-x^2} + C$$

$$19. \frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \ln(1+x^2) + C$$

$$21. -\frac{\cos 5x}{10} - \frac{\cos x}{2} + C$$

$$23. 8 \left[\frac{\sin(7t/2)}{7} - \frac{\sin(9t/2)}{9} \right] + C$$

$$25. 6 \sin(\theta/12) + \frac{6}{7} \sin(7\theta/12) + C$$

$$27. \frac{1}{2} \ln(x^2 + 1) + \frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1} x + C$$

$$29. \left(x - \frac{1}{2} \right) \sin^{-1} \sqrt{x} + \frac{1}{2} \sqrt{x-x^2} + C$$

$$31. \sin^{-1} \sqrt{x} - \sqrt{x-x^2} + C$$

$$33. \sqrt{1-\sin^2 t} - \ln \left| \frac{1 + \sqrt{1-\sin^2 t}}{\sin t} \right| + C$$

$$35. \ln |\ln y + \sqrt{3 + (\ln y)^2}| + C$$

$$37. \ln |x + 1 + \sqrt{x^2 + 2x + 5}| + C$$

$$39. \frac{x+2}{2} \sqrt{5-4x-x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x+2}{3} \right) + C$$

$$41. -\frac{\sin^4 2x \cos 2x}{10} - \frac{2 \sin^2 2x \cos 2x}{15} - \frac{4 \cos 2x}{15} + C$$

$$43. \frac{\sin^3 2\theta \cos^2 2\theta}{10} + \frac{\sin^3 2\theta}{15} + C$$

$$45. \tan^2 2x - 2 \ln |\sec 2x| + C$$

$$47. \frac{(\sec \pi x)(\tan \pi x)}{\pi} + \frac{1}{\pi} \ln |\sec \pi x + \tan \pi x| + C$$

$$49. \frac{-\csc^3 x \cot x}{4} - \frac{3 \csc x \cot x}{8} - \frac{3}{8} \ln |\csc x + \cot x| + C$$

$$51. \frac{1}{2} [\sec(e^t - 1) \tan(e^t - 1) + \ln |\sec(e^t - 1) + \tan(e^t - 1)|] + C$$

$$53. \sqrt{2} + \ln(\sqrt{2} + 1) \quad 55. \pi/3$$

$$57. 2\pi\sqrt{3} + \pi\sqrt{2} \ln(\sqrt{2} + \sqrt{3}) \quad 59. \bar{x} = 4/3, \bar{y} = \ln\sqrt{2}$$

$$61. 7.62 \quad 63. \pi/8 \quad 67. \pi/4$$

SECTION 8.7, pp. 506–508

$$1. \text{ I: (a) 1.5, 0 (b) 1.5, 0 (c) 0\%}$$

$$\text{II: (a) 1.5, 0 (b) 1.5, 0 (c) 0\%}$$

$$3. \text{ I: (a) 2.75, 0.08 (b) 2.67, 0.08 (c) 0.0312 \approx 3\%}$$

$$\text{II: (a) 2.67, 0 (b) 2.67, 0 (c) 0\%}$$

$$5. \text{ I: 6.25, 0.5 (b) 6, 0.25 (c) 0.0417 \approx 4\%}$$

$$\text{II: (a) 6, 0 (b) 6, 0 (c) 0\%}$$

$$7. \text{ I: (a) 0.509, 0.03125 (b) 0.5, 0.009 (c) 0.018 \approx 2\%}$$

$$\text{II: (a) 0.5, 0.002604 (b) 0.5, 0.4794 (c) 0\%}$$

$$9. \text{ I: (a) 1.8961, 0.161 (b) 2, 0.1039 (c) 0.052 \approx 5\%}$$

$$\text{II: (a) 2.0045, 0.0066 (b) 2, 0.00454 (c) 0.2\%}$$

$$11. (a) 1 (b) 2 \quad 13. (a) 116 (b) 2$$

$$15. (a) 283 (b) 2 \quad 17. (a) 71 (b) 10$$

$$19. (a) 76 (b) 12 \quad 21. (a) 82 (b) 8$$

$$23. 15,990 \text{ ft}^3 \quad 25. \approx 10.63 \text{ ft}$$

$$27. (a) \approx 0.00021 (b) \approx 1.37079 (c) \approx 0.015\%$$

$$31. (a) \approx 5.870 (b) |E_T| \leq 0.0032$$

$$33. 21.07 \text{ in.} \quad 35. 14.4 \quad 39. \approx 28.7 \text{ mg}$$

SECTION 8.8, pp. 517–519

$$1. \pi/2 \quad 3. 2 \quad 5. 6 \quad 7. \pi/2 \quad 9. \ln 3 \quad 11. \ln 4$$

$$13. 0 \quad 15. \sqrt{3} \quad 17. \pi \quad 19. \ln \left(1 + \frac{\pi}{2} \right)$$

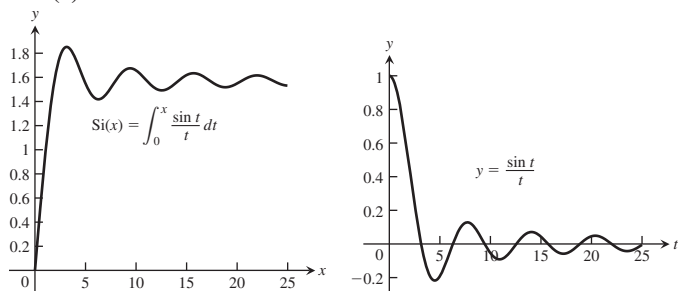
$$21. -1 \quad 23. 1 \quad 25. -1/4 \quad 27. \pi/2 \quad 29. \pi/3$$

$$31. 6 \quad 33. \ln 2 \quad 35. \text{Diverges} \quad 37. \text{Diverges}$$

$$39. \text{Diverges} \quad 41. \text{Diverges} \quad 43. \text{Converges}$$

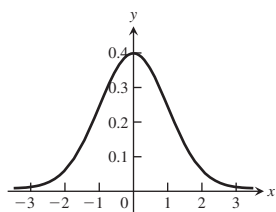
$$45. \text{Converges} \quad 47. \text{Diverges} \quad 49. \text{Converges}$$

51. Converges 53. Diverges 55. Converges
 57. Converges 59. Diverges 61. Converges
 63. Diverges 65. Converges 67. Converges
 69. (a) Converges when $p < 1$ (b) Converges when $p > 1$
 71. 1 73. 2π 75. $\ln 2$
 77. (a) 1 (b) $\pi/3$ (c) Diverges
 79. (a) $\pi/2$ (b) π 81. (b) ≈ 0.88621
 83. (a)



(b) $\pi/2$

85. (a)



(b) $\approx 0.683, \approx 0.954, \approx 0.997$

91. ≈ 0.16462

SECTION 8.9, pp. 530–532

1. No 3. Yes 5. Yes 7. Yes 11. ≈ 0.537
 13. ≈ 0.688 15. ≈ 0.0502 17. $\sqrt{21}$ 19. $\frac{1}{2} \ln 2$
 21. $\frac{1}{\pi}, \frac{1}{\pi} \left(\tan^{-1} 2 - \frac{\pi}{4} \right) \approx 0.10242$
 25. mean $= \frac{8}{3} \approx 2.67$, median $= \sqrt{8} \approx 2.83$
 27. mean $= 2$, median $= \sqrt{2} \approx 1.41$
 29. $P(X < \frac{1}{2}) \approx 0.3935$
 31. (a) ≈ 0.57 , so about 57 in every 100 bulbs will fail.
 (b) ≈ 832 hr
 33. ≈ 60 hydra 35. (a) ≈ 0.393 (b) ≈ 0.135 (c) 0
 (d) The probability that any customer waits longer than 3 minutes is $1 - (0.997521)^{200} \approx 0.391 < 1/2$. So the most likely outcome is that all 200 would be served within 3 minutes.
 37. \$10,256 39. $\approx 323, \approx 262$ 41. ≈ 0.89435
 43. (a) $\approx 16\%$ (b) ≈ 0.23832 45. ≈ 618 females
 47. ≈ 61 adults 49. ≈ 289 shafts
 51. (a) ≈ 0.977 (b) ≈ 0.159 (c) ≈ 0.838
 55. (a) {LLL, LLD, LDL, DLL, LLU, LUL, ULL, LDD, LDU, LUD, LUU, DLD, DLU, ULD, ULU, DDL, DUL, UDL, UUL, DDD, DDU, DUD, UDD, DUU, UDU, UUD, UUU}
 (c) $7/27 \approx 0.26$ (d) $20/27 \approx 0.74$

PRACTICE EXERCISES, pp. 533–535

1. $(x+1)(\ln(x+1)) - (x+1) + C$
 3. $x \tan^{-1}(3x) - \frac{1}{6} \ln(1+9x^2) + C$
 5. $(x+1)^2 e^x - 2(x+1)e^x + 2e^x + C$

7. $\frac{2e^x \sin 2x}{5} + \frac{e^x \cos 2x}{5} + C$
 9. $2 \ln|x-2| - \ln|x-1| + C$
 11. $\ln|x| - \ln|x+1| + \frac{1}{x+1} + C$
 13. $-\frac{1}{3} \ln \left| \frac{\cos \theta - 1}{\cos \theta + 2} \right| + C$
 15. $4 \ln|x| - \frac{1}{2} \ln(x^2 + 1) + 4 \tan^{-1} x + C$
 17. $\frac{1}{16} \ln \left| \frac{(v-2)^5(v+2)}{v^6} \right| + C$
 19. $\frac{1}{2} \tan^{-1} t - \frac{\sqrt{3}}{6} \tan^{-1} \frac{t}{\sqrt{3}} + C$
 21. $\frac{x^2}{2} + \frac{4}{3} \ln|x+2| + \frac{2}{3} \ln|x-1| + C$
 23. $\frac{x^2}{2} - \frac{9}{2} \ln|x+3| + \frac{3}{2} \ln|x+1| + C$
 25. $\frac{1}{3} \ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| + C$ 27. $\ln|1-e^{-x}| + C$
 29. $-\sqrt{16-y^2} + C$ 31. $-\frac{1}{2} \ln|4-x^2| + C$
 33. $\ln \frac{1}{\sqrt{9-x^2}} + C$ 35. $\frac{1}{6} \ln \left| \frac{x+3}{x-3} \right| + C$
 37. $-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$ 39. $\frac{\tan^5 x}{5} + C$
 41. $\frac{\cos \theta}{2} - \frac{\cos 11\theta}{22} + C$ 43. $4\sqrt{1-\cos(t/2)} + C$
 45. At least 16 47. $T = \pi, S = \pi$ 49. 25°F
 51. (a) ≈ 2.42 gal (b) ≈ 24.83 mi/gal
 53. $\pi/2$ 55. 6 57. $\ln 3$ 59. 2 61. $\pi/6$
 63. Diverges 65. Diverges 67. Converges
 69. $\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$ 71. $2 \tan x - x + C$
 73. $x \tan x - \ln|\sec x| + C$ 75. $-\frac{1}{3}(\cos x)^3 + C$
 77. $1 + \frac{1}{2} \ln \left(\frac{2}{1+e^2} \right)$ 79. $2 \ln \left| 1 - \frac{1}{x} \right| + \frac{4x+1}{2x^2} + C$
 81. $\frac{e^{2x}-1}{e^x} + C$ 83. $9/4$ 85. $256/15$
 87. $-\frac{1}{3} \csc^3 x + C$
 89. $\frac{2x^{3/2}}{3} - x + 2\sqrt{x} - 2 \ln(\sqrt{x}+1) + C$
 91. $\frac{1}{2} \sin^{-1}(x-1) + \frac{1}{2}(x-1)\sqrt{2x-x^2} + C$
 93. $-2 \cot x - \ln|\csc x + \cot x| + \csc x + C$
 95. $\frac{1}{12} \ln \left| \frac{3+v}{3-v} \right| + \frac{1}{6} \tan^{-1} \frac{v}{3} + C$
 97. $\frac{\theta \sin(2\theta+1)}{2} + \frac{\cos(2\theta+1)}{4} + C$
 99. $\frac{1}{4} \sec^2 \theta + C$ 101. $2 \left(\frac{(\sqrt{2-x})^3}{3} - 2\sqrt{2-x} \right) + C$
 103. $\tan^{-1}(y-1) + C$
 105. $\frac{1}{4} \ln|z| - \frac{1}{4z} - \frac{1}{4} \left[\frac{1}{2} \ln(z^2+4) + \frac{1}{2} \tan^{-1} \left(\frac{z}{2} \right) \right] + C$

$$107. -\frac{1}{4}\sqrt{9-4t^2} + C \quad 109. \ln\left(\frac{e^t+1}{e^t+2}\right) + C$$

$$111. 1/4 \quad 113. \frac{2}{3}x^{3/2} + C \quad 115. -\frac{1}{5}\tan^{-1}(\cos 5t) + C$$

$$117. 2\sqrt{r} - 2\ln(1 + \sqrt{r}) + C$$

$$119. \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2 + 1) + C$$

$$121. \frac{2}{3}\ln|x+1| + \frac{1}{6}\ln|x^2-x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

$$123. \frac{4}{7}(1+\sqrt{x})^{7/2} - \frac{8}{5}(1+\sqrt{x})^{5/2} + \frac{4}{3}(1+\sqrt{x})^{3/2} + C$$

$$125. 2\ln|\sqrt{x} + \sqrt{1+x}| + C$$

$$127. \ln x - \ln|1 + \ln x| + C$$

$$129. \frac{1}{2}x^{\ln x} + C \quad 131. \frac{1}{2}\ln\left|\frac{1-\sqrt{1-x^4}}{x^2}\right| + C$$

$$133. (b) \frac{\pi}{4} \quad 135. x - \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + C$$

ADDITIONAL AND ADVANCED EXERCISES, pp. 536–538

$$1. x(\sin^{-1}x)^2 + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + C$$

$$3. \frac{x^2 \sin^{-1}x}{2} + \frac{x\sqrt{1-x^2} - \sin^{-1}x}{4} + C$$

$$5. \frac{1}{2}\left(\ln(t - \sqrt{1-t^2}) - \sin^{-1}t\right) + C \quad 7. 0$$

$$9. \ln(4) - 1 \quad 11. 1 \quad 13. 32\pi/35 \quad 15. 2\pi$$

$$17. (a) \pi \quad (b) \pi(2e - 5)$$

$$19. (b) \pi\left(\frac{8(\ln 2)^2}{3} - \frac{16(\ln 2)}{9} + \frac{16}{27}\right)$$

$$21. \left(\frac{e^2+1}{4}, \frac{e-2}{2}\right)$$

$$23. \sqrt{1+e^2} - \ln\left(\frac{\sqrt{1+e^2}}{e} + \frac{1}{e}\right) - \sqrt{2} + \ln(1+\sqrt{2})$$

$$25. \frac{12\pi}{5} \quad 27. a = \frac{1}{2}, -\frac{\ln 2}{4} \quad 29. \frac{1}{2} < p \leq 1$$

$$33. \frac{2}{1 - \tan(x/2)} + C \quad 35. 1 \quad 37. \frac{\sqrt{3}\pi}{9}$$

$$39. \frac{1}{\sqrt{2}}\ln\left|\frac{\tan(t/2) + 1 - \sqrt{2}}{\tan(t/2) + 1 + \sqrt{2}}\right| + C$$

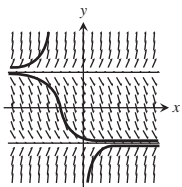
$$41. \ln\left|\frac{1 + \tan(\theta/2)}{1 - \tan(\theta/2)}\right| + C$$

Chapter 9

SECTION 9.1, pp. 546–548

$$1. (d) \quad 3. (a)$$

5.

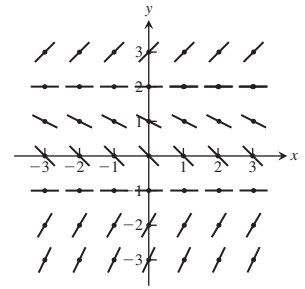


$$7. y' = x - y; y(1) = -1$$

$$9. y' = -(1+y)\sin x; y(0) = 2$$

$$11. y' = 1 + x e^y; y(-2) = 2$$

y	f(y) = $\frac{dy}{dx}$
-3	2
-2	1.5
-1	0
0	-1
1	-0.75
2	0
3	1



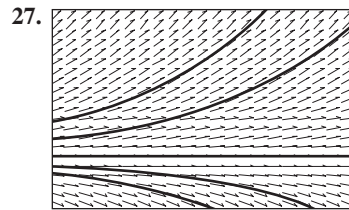
$$15. y(\text{exact}) = -x^2, y_1 = -2, y_2 = -3.3333, y_3 = -5$$

$$17. y(\text{exact}) = 3e^{x(x+2)}, y_1 = 4.2, y_2 = 6.216, y_3 = 9.697$$

$$19. y(\text{exact}) = e^{x^2} + 1, y_1 = 2.0, y_2 = 2.0202, y_3 = 2.0618$$

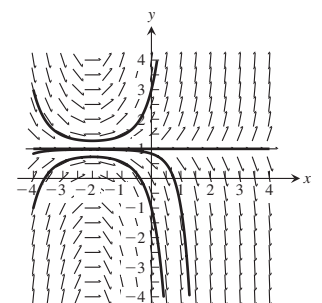
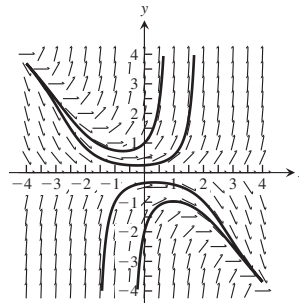
$$21. y \approx 2.48832, \text{ exact value is } e.$$

$$23. y \approx -0.2272, \text{ exact value is } 1/(1 - 2\sqrt{5}) \approx -0.2880.$$



29.

31.



$$39. \text{Euler's method gives } y \approx 3.45835; \text{ the exact solution is } y = 1 + e \approx 3.71828.$$

$$41. y \approx 1.5000; \text{ exact value is } 1.5275.$$

SECTION 9.2, pp. 552–554

$$1. y = \frac{e^x + C}{x}, \quad x > 0 \quad 3. y = \frac{C - \cos x}{x^3}, \quad x > 0$$

$$5. y = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2}, \quad x > 0 \quad 7. y = \frac{1}{2}xe^{x/2} + Ce^{x/2}$$

$$9. y = x(\ln x)^2 + Cx$$

$$11. s = \frac{t^3}{3(t-1)^4} - \frac{t}{(t-1)^4} + \frac{C}{(t-1)^4}$$

$$13. r = (\csc \theta)(\ln|\sec \theta| + C), \quad 0 < \theta < \pi/2$$

$$15. y = \frac{3}{2} - \frac{1}{2}e^{-2t} \quad 17. y = -\frac{1}{\theta}\cos \theta + \frac{\pi}{2\theta}$$

$$19. y = 6e^{x^2} - \frac{e^{x^2}}{x+1}$$

$$21. y = y_0 e^{kt}$$

$$23. (b) \text{ is correct, but (a) is not.} \quad 25. t = \frac{L}{R} \ln 2 \text{ sec}$$

27. (a) $i = \frac{V}{R} - \frac{V}{R}e^{-3} = \frac{V}{R}(1 - e^{-3}) \approx 0.95 \frac{V}{R}$ amp (b) 86%

29. $y = \frac{1}{1 + Ce^{-x}}$

31. $y^3 = 1 + Cx^{-3}$

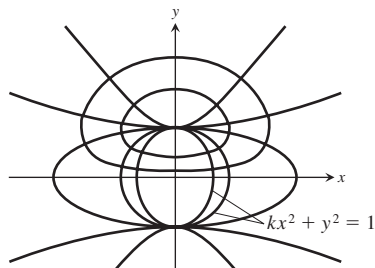
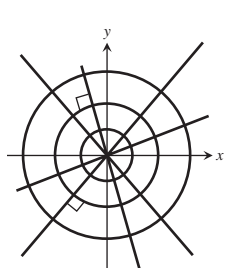
SECTION 9.3, pp. 559–560

1. (a) 168.5 m (b) 41.13 sec

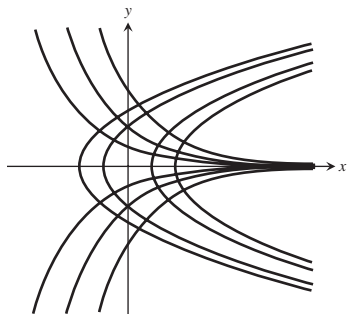
3. $s(t) = 4.91(1 - e^{-(22.36/39.92)t})$

5. $x^2 + y^2 = C$

7. $\ln|y| - \frac{1}{2}y^2 = \frac{1}{2}x^2 + C$



9. $y = \pm \sqrt{2x + C}$



13. (a) 10 lb/min (b) $(100 + t)$ gal (c) $4\left(\frac{y}{100 + t}\right)$ lb/min

(d) $\frac{dy}{dt} = 10 - \frac{4y}{100 + t}$, $y(0) = 50$,

$$y = 2(100 + t) - \frac{150}{\left(1 + \frac{t}{100}\right)^4}$$

(e) Concentration = $\frac{y(25)}{\text{amt. brine in tank}} = \frac{188.6}{125} \approx 1.5$ lb/gal

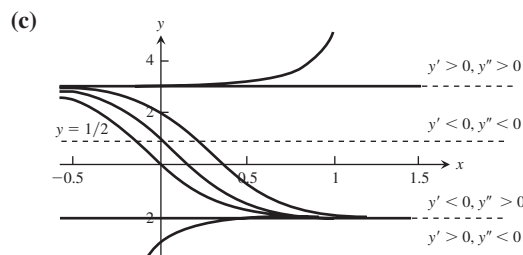
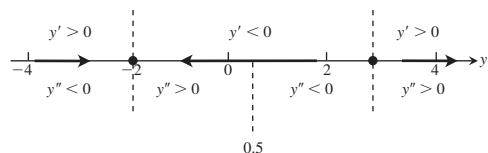
15. $y(27.8) \approx 14.8$ lb, $t \approx 27.8$ min

SECTION 9.4, pp. 566–567

1. $y' = (y + 2)(y - 3)$

(a) $y = -2$ is a stable equilibrium value and $y = 3$ is an unstable equilibrium.

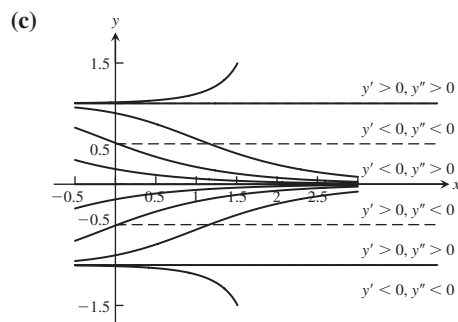
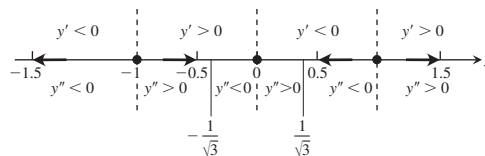
(b) $y'' = 2(y + 2)\left(y - \frac{1}{2}\right)(y - 3)$



3. $y' = y^3 - y = (y + 1)y(y - 1)$

(a) $y = -1$ and $y = 1$ are unstable equilibria and $y = 0$ is a stable equilibrium.

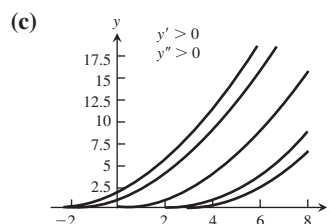
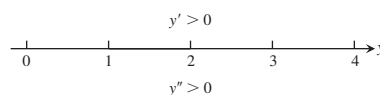
(b) $y'' = (3y^2 - 1)y' = 3(y + 1)\left(y + \frac{1}{\sqrt{3}}\right)y\left(y - \frac{1}{\sqrt{3}}\right)(y - 1)$



5. $y' = \sqrt{y}$, $y > 0$

(a) There are no equilibrium values.

(b) $y'' = \frac{1}{2}$

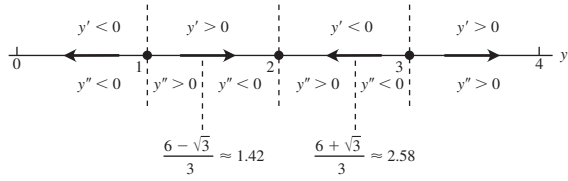


7. $y' = (y - 1)(y - 2)(y - 3)$

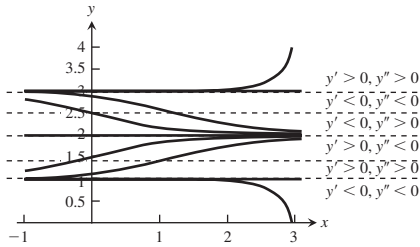
(a) $y = 1$ and $y = 3$ are unstable equilibria and $y = 2$ is a stable equilibrium.

(b) $y'' = (3y^2 - 12y + 11)(y - 1)(y - 2)(y - 3) =$

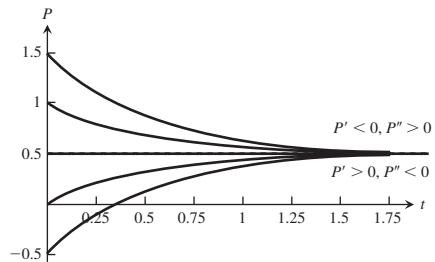
$$3(y - 1)\left(y - \frac{6 - \sqrt{3}}{3}\right)(y - 2)\left(y - \frac{6 + \sqrt{3}}{3}\right)(y - 3)$$



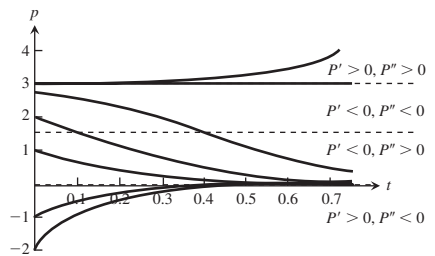
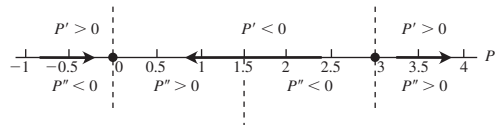
(c)



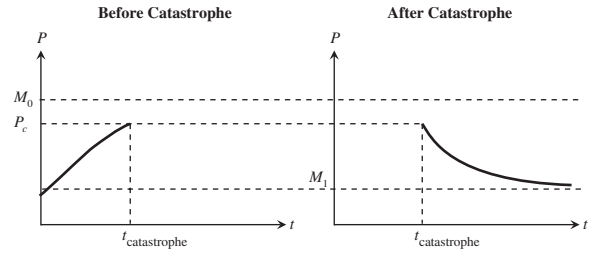
9. $\frac{dP}{dt} = 1 - 2P$ has a stable equilibrium at $P = \frac{1}{2}$;
 $\frac{d^2P}{dt^2} = -2\frac{dP}{dt} = -2(1 - 2P)$.



11. $\frac{dP}{dt} = 2P(P - 3)$ has a stable equilibrium at $P = 0$ and an unstable equilibrium at $P = 3$; $\frac{d^2P}{dt^2} = 2(2P - 3)\frac{dP}{dt} = 4P(2P - 3)(P - 3)$.



13. Before the catastrophe, the population exhibits logistic growth and $P(t)$ increases toward M_0 , the stable equilibrium. After the catastrophe, the population declines logistically and $P(t)$ decreases toward M_1 , the new stable equilibrium.

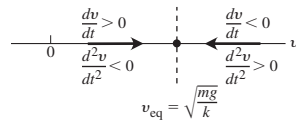


15. $\frac{dv}{dt} = g - \frac{k}{m}v^2$, $g, k, m > 0$ and $v(t) \geq 0$

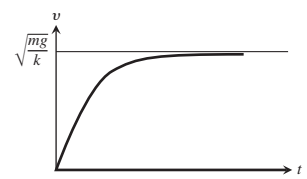
$$\text{Equilibrium: } \frac{dv}{dt} = g - \frac{k}{m}v^2 = 0 \Rightarrow v = \sqrt{\frac{mg}{k}}$$

$$\text{Concavity: } \frac{d^2v}{dt^2} = -2\left(\frac{k}{m}v\right)\frac{dv}{dt} = -2\left(\frac{k}{m}v\right)\left(g - \frac{k}{m}v^2\right)$$

(a)



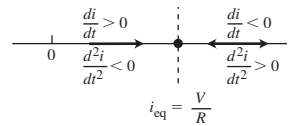
(b)



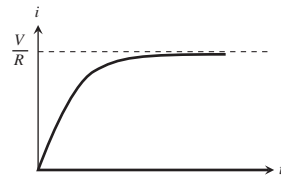
$$(c) \ v_{\text{terminal}} = \sqrt{\frac{160}{0.005}} = 178.9 \text{ ft/sec} = 122 \text{ mph}$$

17. $F = F_p - F_r$; $ma = 50 - 5|v|$; $\frac{dv}{dt} = \frac{1}{m}(50 - 5|v|)$. The maximum velocity occurs when $\frac{dv}{dt} = 0$ or $v = 10 \text{ ft/sec}$.

19. Phase line:



If the switch is closed at $t = 0$, then $i(0) = 0$, and the graph of the solution looks like this:



$$\text{As } t \rightarrow \infty, i(t) \rightarrow i_{\text{steady state}} = \frac{V}{R}.$$

SECTION 9.5, pp. 570–573

- Seasonal variations, nonconformity of the environments, effects of other interactions, unexpected disasters, etc.
- This model assumes that the number of interactions is proportional to the product of x and y :

$$\frac{dx}{dt} = (a - by)x, \quad a < 0,$$

$$\frac{dy}{dt} = m\left(1 - \frac{y}{M}\right)y - nxy = y\left(m - \frac{m}{M}y - nx\right).$$

Rest points are $(0, 0)$, unstable, and $(0, M)$, stable.

5. (a) Logistic growth occurs in the absence of the competitor, and involves a simple interaction between the species: Growth dominates the competition when either population is small, so it is difficult to drive either species to extinction.

- (b) a : per capita growth rate for trout
 m : per capita growth rate for bass
 b : intensity of competition to the trout
 n : intensity of competition to the bass
 k_1 : environmental carrying capacity for the trout
 k_2 : environmental carrying capacity for the bass
 $\frac{a}{b}$: growth versus competition or net growth of trout
 $\frac{m}{n}$: relative survival of bass

(c) $\frac{dx}{dt} = 0$ when $x = 0$ or $y = \frac{a}{b} - \frac{a}{bk_1}x$,

$\frac{dy}{dt} = 0$ when $y = 0$ or $y = k_2 - \frac{k_2n}{m}x$.

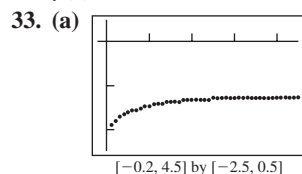
By picking $a/b > k_2$ and $m/n > k_1$, we ensure that an equilibrium point exists inside the first quadrant.

PRACTICE EXERCISES, pp. 573–575

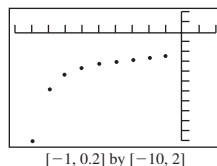
1. $y = -\ln\left(C - \frac{2}{5}(x-2)^{5/2} - \frac{4}{3}(x-2)^{3/2}\right)$
3. $\tan y = -x \sin x - \cos x + C$
5. $(y+1)e^{-y} = -\ln|x| + C$
7. $y = C \frac{x-1}{x}$ 9. $y = \frac{x^2}{4}e^{x/2} + Ce^{x/2}$
11. $y = \frac{x^2 - 2x + C}{2x^2}$ 13. $y = \frac{e^{-x} + C}{1 + e^x}$ 15. $xy + y^3 = C$
17. $\tan y = -\cos x + (1/3)\cos^3 x + C$
19. $e^{x^2}y = (x-1)e^x + C$
21. $\ln|\ln y| = (1/2)x^2 \ln x - (1/4)x^2 + C$
23. $y = \frac{2x^3 + 3x^2 + 6}{6(x+1)^2}$ 25. $y = \frac{1}{3}(1 - 4e^{-x^3})$
27. $y = e^{-x}(3x^3 - 3x^2)$

x	y	x	y
0	0	1.1	1.6241
0.1	0.1000	1.2	1.8319
0.2	0.2095	1.3	2.0513
0.3	0.3285	1.4	2.2832
0.4	0.4568	1.5	2.5285
0.5	0.5946	1.6	2.7884
0.6	0.7418	1.7	3.0643
0.7	0.8986	1.8	3.3579
0.8	1.0649	1.9	3.6709
0.9	1.2411	2.0	4.0057
1.0	1.4273		

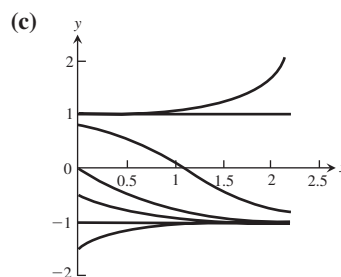
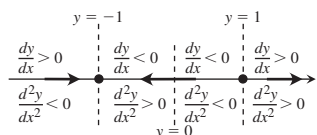
31. $y(3) \approx 0.9131$



- (b) Note that we choose a small interval of x -values because the y -values decrease very rapidly and our calculator cannot handle the calculations for $x \leq -1$. (This occurs because the analytic solution is $y = -2 + \ln(2 - e^{-x})$, which has an asymptote at $x = -\ln 2 \approx -0.69$. Obviously, the Euler approximations are misleading for $x \leq -0.7$.)



35. $y(\text{exact}) = \frac{1}{2}x^2 - \frac{3}{2}$; $y(2) \approx 0.4$; exact value is $\frac{1}{2}$.
37. $y(\text{exact}) = -e^{(x^2-1)/2}$; $y(2) \approx -3.4192$; exact value is $-e^{3/2} \approx -4.4817$.
41. (a) $y = -1$ is stable and $y = 1$ is unstable.
(b) $\frac{d^2y}{dx^2} = 2y \frac{dy}{dx} = 2y(y^2 - 1)$



43. (a) 1 min: 102 gal; 10 min: 120 gal; 60 min: 220 gal
(b) $S(1) \approx 12.55$ lb, $S(10) \approx 32.22$ lb, $S(60) \approx 101.74$ lb

ADDITIONAL AND ADVANCED EXERCISES, p. 575

1. (a) $y = c + (y_0 - c)e^{-k(A/V)t}$
(b) Steady-state solution: $y_\infty = c$
5. $x^2(x^2 + 2y^2) = C$
7. $\ln|x| + e^{-y/x} = C$
9. $\ln|x| - \ln|\sec(y/x - 1) + \tan(y/x - 1)| = C$

Chapter 10

SECTION 10.1, pp. 586–590

1. $a_1 = 0, a_2 = -1/4, a_3 = -2/9, a_4 = -3/16$
3. $a_1 = 1, a_2 = -1/3, a_3 = 1/5, a_4 = -1/7$
5. $a_1 = 1/2, a_2 = 1/2, a_3 = 1/2, a_4 = 1/2$
7. $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \frac{127}{64}, \frac{255}{128}, \frac{511}{256}, \frac{1023}{512}$
9. $2, 1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, -\frac{1}{64}, \frac{1}{128}, \frac{1}{256}$
11. $1, 1, 2, 3, 5, 8, 13, 21, 34, 55$
13. $a_n = (-1)^{n+1}, n \geq 1$

15. $a_n = (-1)^{n+1}(n)^2, n \geq 1$ 17. $a_n = \frac{2^{n-1}}{3(n+2)}, n \geq 1$ 43. $s_n = \frac{\pi}{3} - \cos^{-1}\left(\frac{1}{n+2}\right)$; converges, $-\frac{\pi}{6}$ 45. 1 47. 5
19. $a_n = n^2 - 1, n \geq 1$ 21. $a_n = 4n - 3, n \geq 1$ 49. 1 51. $-\frac{1}{\ln 2}$ 53. Converges, $2 + \sqrt{2}$
23. $a_n = \frac{3n+2}{n!}, n \geq 1$ 25. $a_n = \frac{1 + (-1)^{n+1}}{2}, n \geq 1$ 55. Converges, 1 57. Diverges
27. $a_n = \frac{1}{(n+1)(n+2)}$ 29. $a_n = \sin\left(\frac{\sqrt{n+1}}{1+(n+1)^2}\right)$ 59. Converges, $\frac{e^2}{e^2-1}$
31. Converges, 2 33. Converges, -1 35. Converges, -5 61. Converges, $2/9$ 63. Converges, $3/2$ 65. Diverges
37. Diverges 39. Diverges 41. Converges, $1/2$ 67. Converges, 4 69. Diverges 71. Converges, $\frac{\pi}{\pi-e}$
43. Converges, 0 45. Converges, $\sqrt{2}$ 47. Converges, 1 73. Converges, $-5/6$ 75. Diverges
49. Converges, 0 51. Converges, 0 53. Converges, 0 77. $a = 1, r = -x$; converges to $1/(1+x)$ for $|x| < 1$
55. Converges, 1 57. Converges, e^7 59. Converges, 1 79. $a = 3, r = (x-1)/2$; converges to $6/(3-x)$ for x in $(-1, 3)$
61. Converges, 1 63. Diverges 65. Converges, 4 81. $|x| < \frac{1}{2}, \frac{1}{1-2x}$ 83. $-2 < x < 0, \frac{1}{2+x}$
67. Converges, 0 69. Diverges 71. Converges, e^{-1} 85. $x \neq (2k+1)\frac{\pi}{2}, k$ an integer; $\frac{1}{1-\sin x}$
73. Diverges 75. Converges, 0 77. Diverges 87. (a) $\sum_{n=2}^{\infty} \frac{1}{(n+4)(n+5)}$ (b) $\sum_{n=0}^{\infty} \frac{1}{(n+2)(n+3)}$
79. Converges, $e^{2/3}$ 81. Converges, $x (x > 0)$ 83. Converges, 1 85. Converges, $1/2$
83. Converges, 0 85. Converges, 1 87. Converges, $1/2$ 89. Converges, 1 91. Converges, $\pi/2$ 93. Converges, 0
89. Converges, 1 91. Converges, $\pi/2$ 93. Converges, 0 95. Converges, 0 97. Converges, $1/2$ 99. Converges, 0
101. 8 103. 4 105. 5 107. $1 + \sqrt{2}$ 109. $x_n = 2^{n-2}$
111. (a) $f(x) = x^2 - 2, 1.414213562 \approx \sqrt{2}$ (b) $f(x) = \tan(x) - 1, 0.7853981635 \approx \pi/4$ (c) $f(x) = e^x$, diverges
113. 1
121. Nondecreasing, bounded
123. Not nondecreasing, bounded
125. Converges, nondecreasing sequence theorem
127. Converges, nondecreasing sequence theorem
129. Diverges, definition of divergence
131. Converges
133. Converges
145. (b) $\sqrt{3}$

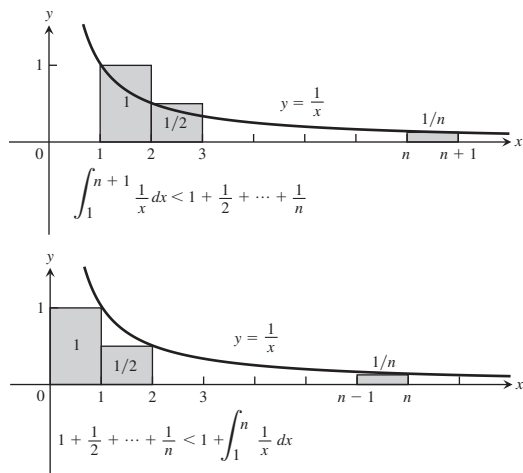
SECTION 10.2, pp. 597-599

1. $s_n = \frac{2(1 - (1/3)^n)}{1 - (1/3)}, 3$ 3. $s_n = \frac{1 - (-1/2)^n}{1 - (-1/2)}, 2/3$
5. $s_n = \frac{1}{2} - \frac{1}{n+2}, \frac{1}{2}$ 7. $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots, \frac{4}{5}$
9. $-\frac{3}{4} + \frac{9}{16} + \frac{57}{64} + \frac{249}{256} + \dots$, diverges.
11. $(5+1) + \left(\frac{5}{2} + \frac{1}{3}\right) + \left(\frac{5}{4} + \frac{1}{9}\right) + \left(\frac{5}{8} + \frac{1}{27}\right) + \dots, \frac{23}{2}$
13. $(1+1) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{4} + \frac{1}{25}\right) + \left(\frac{1}{8} - \frac{1}{125}\right) + \dots, \frac{17}{6}$
15. Converges, $5/3$ 17. Converges, $1/7$
19. Converges, $\frac{e}{e+2}$ 21. Diverges 23. $23/99$
25. $7/9$ 27. $1/15$ 29. $41333/33300$ 31. Diverges
33. Inconclusive 35. Diverges 37. Diverges
39. $s_n = 1 - \frac{1}{n+1}$; converges, 1
41. $s_n = \ln \sqrt{n+1}$; diverges

SECTION 10.3, pp. 604-606

1. Converges 3. Converges 5. Converges 7. Diverges
9. Converges 11. Diverges
13. Converges; geometric series, $r = \frac{1}{10} < 1$
15. Diverges; $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0$
17. Diverges; p -series, $p < 1$
19. Converges; geometric series, $r = \frac{1}{8} < 1$
21. Diverges; Integral Test
23. Converges; geometric series, $r = 2/3 < 1$
25. Diverges; Integral Test
27. Diverges; $\lim_{n \rightarrow \infty} \frac{2^n}{n+1} \neq 0$
29. Diverges; $\lim_{n \rightarrow \infty} (\sqrt{n}/\ln n) \neq 0$
31. Diverges; geometric series, $r = \frac{1}{\ln 2} > 1$
33. Converges; Integral Test
35. Diverges; n th-Term Test
37. Converges; Integral Test
39. Diverges; n th-Term Test
41. Converges; by taking limit of partial sums
43. Converges; Integral Test
45. Converges; Integral Test 47. $a = 1$

49. (a)



(b) ≈ 41.55

51. True 53. $n \geq 251,415$

55. $s_8 = \sum_{n=1}^8 \frac{1}{n^3} \approx 1.195$ 57. 10^{60}

65. (a) $1.20166 \leq S \leq 1.20253$

(b) $S \approx 1.2021$, error < 0.0005

67. $\left(\frac{\pi^2}{6} - 1\right) \approx 0.64493$

SECTION 10.4, pp. 610–611

1. Converges; compare with $\sum (1/n^2)$
3. Diverges; compare with $\sum (1/\sqrt{n})$
5. Converges; compare with $\sum (1/n^{3/2})$
7. Converges; compare with $\sum \sqrt{\frac{n+4n}{n^4+0}} = \sqrt{5} \sum \frac{1}{n^{3/2}}$
9. Converges
11. Diverges; limit comparison with $\sum (1/n)$
13. Diverges; limit comparison with $\sum (1/\sqrt{n})$
15. Diverges
17. Diverges; limit comparison with $\sum (1/\sqrt{n})$
19. Converges; compare with $\sum (1/2^n)$
21. Diverges; n th-Term Test
23. Converges; compare with $\sum (1/n^2)$
25. Converges; $\left(\frac{n}{3n+1}\right)^n < \left(\frac{n}{3n}\right)^n = \left(\frac{1}{3}\right)^n$
27. Diverges; direct comparison with $\sum (1/n)$
29. Diverges; limit comparison with $\sum (1/n)$
31. Diverges; limit comparison with $\sum (1/n)$
33. Converges; compare with $\sum (1/n^{3/2})$
35. Converges; $\frac{1}{n2^n} \leq \frac{1}{2^n}$ 37. Converges; $\frac{1}{3^{n-1}+1} < \frac{1}{3^{n-1}}$
39. Converges; comparison with $\sum (1/5n^2)$
41. Diverges; comparison with $\sum (1/n)$
43. Converges; comparison with $\sum \frac{1}{n(n-1)}$ or limit comparison with $\sum (1/n^2)$
45. Diverges; limit comparison with $\sum (1/n)$
47. Converges; $\frac{\tan^{-1} n}{n^{1.1}} < \frac{\pi/2}{n^{1.1}}$
49. Converges; compare with $\sum (1/n^2)$
51. Diverges; limit comparison with $\sum (1/n)$
53. Converges; limit comparison with $\sum (1/n^2)$
55. Diverges n th-Term Test
67. Converges 69. Converges 71. Converges

SECTION 10.5, pp. 616–617

1. Converges 3. Diverges 5. Converges
7. Converges 9. Converges 11. Diverges
13. Converges 15. Converges
17. Converges; Ratio Test 19. Diverges; Ratio Test
21. Converges; Ratio Test
23. Converges; compare with $\sum (3/(1.25)^n)$
25. Diverges; $\lim_{n \rightarrow \infty} \left(1 - \frac{3}{n}\right)^n = e^{-3} \neq 0$
27. Converges; compare with $\sum (1/n^2)$
29. Diverges; compare with $\sum (1/(2n))$ 31. Diverges; $a_n \not\rightarrow 0$
33. Converges; Ratio Test 35. Converges; Ratio Test
37. Converges; Ratio Test 39. Converges; Root Test
41. Converges; compare with $\sum (1/n^2)$
43. Converges; Ratio Test 45. Diverges; Ratio Test
47. Converges; Ratio Test 49. Diverges; Ratio Test
51. Converges; Ratio Test 53. Converges; Ratio Test
55. Diverges; $a_n = \left(\frac{1}{3}\right)^{(1/n!)} \rightarrow 1$ 57. Converges; Ratio Test
59. Diverges; Root Test 61. Converges; Root Test
63. Converges; Ratio Test
65. (a) Diverges; n th-Term Test
- (b) Diverges; Root Test
- (c) Converges; Root Test
- (d) Converges; Ratio Test
69. Yes

SECTION 10.6, pp. 622–624

1. Converges by Alternating Series Test
3. Converges; Alternating Series Test
5. Converges; Alternating Series Test
7. Diverges; $a_n \not\rightarrow 0$
9. Diverges; $a_n \not\rightarrow 0$
11. Converges; Alternating Series Test
13. Converges by Alternating Series Test
15. Converges absolutely. Series of absolute s is a convergent geometric series.
17. Converges conditionally; $1/\sqrt{n} \rightarrow 0$ but $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.
19. Converges absolutely; compare with $\sum_{n=1}^{\infty} (1/n^2)$.
21. Converges conditionally; $1/(n+3) \rightarrow 0$ but $\sum_{n=1}^{\infty} \frac{1}{n+3}$ diverges (compare with $\sum_{n=1}^{\infty} (1/n)$).
23. Diverges; $\frac{3+n}{5+n} \rightarrow 1$
25. Converges conditionally; $\left(\frac{1}{n^2} + \frac{1}{n}\right) \rightarrow 0$ but $(1+n)/n^2 > 1/n$
27. Converges absolutely; Ratio Test
29. Converges absolutely by Integral Test
31. Diverges; $a_n \not\rightarrow 0$
33. Converges absolutely by Ratio Test
35. Converges absolutely, since $\left|\frac{\cos n\pi}{n\sqrt{n}}\right| = \left|\frac{(-1)^{n+1}}{n^{3/2}}\right| = \frac{1}{n^{3/2}}$ (convergent p -series)
37. Converges absolutely by Root Test
39. Diverges; $a_n \rightarrow \infty$
41. Converges conditionally; $\sqrt{n+1} - \sqrt{n} = 1/(\sqrt{n+1} + \sqrt{n}) \rightarrow 0$, but series of absolute values diverges (compare with $\sum (1/\sqrt{n})$).

43. Diverges, $a_n \rightarrow 1/2 \neq 0$

45. Converges absolutely; $\text{sech } n = \frac{2}{e^n + e^{-n}} = \frac{2e^n}{e^{2n} + 1} < \frac{2e^n}{e^{2n}} = \frac{2}{e^n}$, a term from a convergent geometric series.

47. Converges conditionally; $\sum (-1)^{n+1} \frac{1}{2(n+1)}$ converges by

Alternating Series Test; $\sum \frac{1}{2(n+1)}$ diverges by limit comparison with $\sum (1/n)$.

49. $|\text{Error}| < 0.2$ 51. $|\text{Error}| < 2 \times 10^{-11}$

53. $n \geq 31$ 55. $n \geq 4$ 57. Converges; Root Test

59. Converges; Limit of Partial Sums

61. Converges; Ratio Test 63. Diverges; p -series Test

65. Converges; Root Test 67. Converges; Limit Comparison Test

69. Diverges; Limit of Partial Sums

71. Diverges; Limit Comparison Test

73. Diverges; n th-Term Test 75. Diverges; Limit of Partial Sums

77. Converges; Limit Comparison Test

79. Converges; Limit Comparison Test

81. Converges; Ratio Test

83. 0.54030 85. (a) $a_n \geq a_{n+1}$ (b) $-1/2$

SECTION 10.7, pp. 633–636

1. (a) $1, -1 < x < 1$ (b) $-1 < x < 1$ (c) none

3. (a) $1/4, -1/2 < x < 0$ (b) $-1/2 < x < 0$ (c) none

5. (a) $10, -8 < x < 12$ (b) $-8 < x < 12$ (c) none

7. (a) $1, -1 < x < 1$ (b) $-1 < x < 1$ (c) none

9. (a) $3, -3 \leq x \leq 3$ (b) $-3 \leq x \leq 3$ (c) none

11. (a) ∞ , for all x (b) for all x (c) none

13. (a) $1/2, -1/2 < x < 1/2$ (b) $-1/2 < x < 1/2$ (c) none

15. (a) $1, -1 \leq x < 1$ (b) $-1 < x < 1$ (c) $x = -1$

17. (a) $5, -8 < x < 2$ (b) $-8 < x < 2$ (c) none

19. (a) $3, -3 < x < 3$ (b) $-3 < x < 3$ (c) none

21. (a) $1, -2 < x < 0$ (b) $-2 < x < 0$ (c) none

23. (a) $1, -1 < x < 1$ (b) $-1 < x < 1$ (c) none

25. (a) $0, x = 0$ (b) $x = 0$ (c) none

27. (a) $2, -4 < x \leq 0$ (b) $-4 < x < 0$ (c) $x = 0$

29. (a) $1, -1 \leq x \leq 1$ (b) $-1 \leq x \leq 1$ (c) none

31. (a) $1/4, 1 \leq x \leq 3/2$ (b) $1 \leq x \leq 3/2$ (c) none

33. (a) ∞ , for all x (b) for all x (c) none

35. (a) $1, -1 \leq x < 1$ (b) $-1 < x < 1$ (c) -1

37. 3 39. 8 41. $-1/3 < x < 1/3, 1/(1-3x)$

43. $-1 < x < 3, 4/(3+2x-x^2)$

45. $0 < x < 16, 2/(4-\sqrt{x})$

47. $-\sqrt{2} < x < \sqrt{2}, 3/(2-x^2)$

49. $\frac{2}{x} = \sum_{n=0}^{\infty} 2(-1)^n(x-1)^n, 0 < x < 2$

51. $\sum_{n=0}^{\infty} (-\frac{1}{3})^n(x-5)^n, 2 < x < 8$

53. $1 < x < 5, 2/(x-1), \sum_{n=1}^{\infty} (-\frac{1}{2})^n n(x-3)^{n-1},$

$1 < x < 5, -2/(x-1)^2$

55. (a) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$; converges for all x

(b) Same answer as part (c)

(c) $2x - \frac{2^3 x^3}{3!} + \frac{2^5 x^5}{5!} - \frac{2^7 x^7}{7!} + \frac{2^9 x^9}{9!} - \frac{2^{11} x^{11}}{11!} + \cdots$

57. (a) $\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + \frac{31x^{10}}{14175}, -\frac{\pi}{2} < x < \frac{\pi}{2}$

(b) $1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + \frac{62x^8}{315} + \cdots, -\frac{\pi}{2} < x < \frac{\pi}{2}$

63. (a) T (b) T (c) F (d) T (e) N (f) F (g) N (h) T

SECTION 10.8, pp. 640–641

1. $P_0(x) = 1, P_1(x) = 1 + 2x, P_2(x) = 1 + 2x + 2x^2,$

$P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$

3. $P_0(x) = 0, P_1(x) = x - 1, P_2(x) = (x - 1) - \frac{1}{2}(x - 1)^2,$

$P_3(x) = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3$

5. $P_0(x) = \frac{1}{2}, P_1(x) = \frac{1}{2} - \frac{1}{4}(x - 2),$

$P_2(x) = \frac{1}{2} - \frac{1}{4}(x - 2) + \frac{1}{8}(x - 2)^2,$

$P_3(x) = \frac{1}{2} - \frac{1}{4}(x - 2) + \frac{1}{8}(x - 2)^2 - \frac{1}{16}(x - 2)^3$

7. $P_0(x) = \frac{\sqrt{2}}{2}, P_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right),$

$P_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2,$

$P_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$

$- \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$

9. $P_0(x) = 2, P_1(x) = 2 + \frac{1}{4}(x - 4),$

$P_2(x) = 2 + \frac{1}{4}(x - 4) - \frac{1}{64}(x - 4)^2,$

$P_3(x) = 2 + \frac{1}{4}(x - 4) - \frac{1}{64}(x - 4)^2 + \frac{1}{512}(x - 4)^3$

11. $\sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

13. $\sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!}$ 17. $7 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ 19. $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

21. $x^4 - 2x^3 - 5x + 4$ 23. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{(2n-1)!}$

25. $8 + 10(x - 2) + 6(x - 2)^2 + (x - 2)^3$

27. $21 - 36(x + 2) + 25(x + 2)^2 - 8(x + 2)^3 + (x + 2)^4$

29. $\sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$ 31. $\sum_{n=0}^{\infty} \frac{e^2}{n!} (x-2)^n$

33. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{2^{2n}}{(2n)!} \left(x - \frac{\pi}{4}\right)^{2n}$

35. $-1 - 2x - \frac{5}{2}x^2 - \cdots, -1 < x < 1$

37. $x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 + \cdots, -1 < x < 1$

39. $x^4 + x^6 + \frac{x^8}{2} + \cdots, (-\infty, \infty)$

45. $L(x) = 0, Q(x) = -x^2/2$ 47. $L(x) = 1, Q(x) = 1 + x^2/2$

49. $L(x) = x, Q(x) = x$

SECTION 10.9, pp. 647–648

1. $\sum_{n=0}^{\infty} \frac{(-5x)^n}{n!} = 1 - 5x + \frac{5^2x^2}{2!} - \frac{5^3x^3}{3!} + \dots$
3. $\sum_{n=0}^{\infty} \frac{5(-1)^n(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{5(-1)^{n+1}x^{2n+1}}{(2n+1)!}$
 $= -5x + \frac{5x^3}{3!} - \frac{5x^5}{5!} + \frac{5x^7}{7!} + \dots$
5. $\sum_{n=0}^{\infty} \frac{(-1)^n(5x^2)^{2n}}{(2n)!} = 1 - \frac{25x^4}{2!} + \frac{625x^8}{4!} - \dots$
7. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n} = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \dots$
9. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n x^{3n} = 1 - \frac{3}{4}x^3 + \frac{3^2}{4^2}x^6 - \frac{3^3}{4^3}x^9 + \dots$
11. $\ln 3 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n x^n}{n} = \ln 3 + 2x - 2x^2 + \frac{8}{3}x^3 - \dots$
13. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \dots$
15. $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$
17. $x - \frac{\pi^2 x^3}{2!} + \frac{\pi^4 x^5}{4!} - \frac{\pi^6 x^7}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n+1}}{(2n)!}$
19. $1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{2 \cdot (2n)!} =$
 $1 - \frac{(2x)^2}{2 \cdot 2!} + \frac{(2x)^4}{2 \cdot 4!} - \frac{(2x)^6}{2 \cdot 6!} + \frac{(2x)^8}{2 \cdot 8!} - \dots$
21. $x^2 \sum_{n=0}^{\infty} (2x)^n = x^2 + 2x^3 + 4x^4 + \dots$
23. $\sum_{n=1}^{\infty} nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \dots$
25. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{4n-1}}{2n-1} = x^3 - \frac{x^7}{3} + \frac{x^{11}}{5} - \frac{x^{15}}{7} + \dots$
27. $\sum_{n=0}^{\infty} \left(\frac{1}{n!} + (-1)^n \right) x^n = 2 + \frac{3}{2}x^2 - \frac{5}{6}x^3 + \frac{25}{24}x^4 - \dots$
29. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n+1}}{3n} = \frac{x^3}{3} - \frac{x^5}{6} + \frac{x^7}{9} - \dots$
31. $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \dots$
33. $x^2 - \frac{2}{3}x^4 + \frac{23}{45}x^6 - \frac{44}{105}x^8 + \dots$
35. $1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \dots$ 37. $1 - \frac{x^2}{2} - \frac{x^3}{2} - \frac{x^4}{4} - \dots$
39. $|\text{Error}| \leq \frac{1}{10^4 \cdot 4!} < 4.2 \times 10^{-6}$
41. $|x| < (0.06)^{1/5} < 0.56968$
43. $|\text{Error}| < (10^{-3})^3/6 < 1.67 \times 10^{-10}, \quad -10^{-3} < x < 0$
45. $|\text{Error}| < (3^{0.1})(0.1)^3/6 < 1.87 \times 10^{-4}$
53. (a) $Q(x) = 1 + kx + \frac{k(k-1)}{2}x^2$ (b) $0 \leq x < 100^{-1/3}$

SECTION 10.10, pp. 655–657

1. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ 3. $1 + 3x + 6x^2 + 10x^3$
5. $1 - x + \frac{3x^2}{4} - \frac{x^3}{2}$ 7. $1 - \frac{x^3}{2} + \frac{3x^6}{8} - \frac{5x^9}{16}$
9. $1 + \frac{1}{2x} - \frac{1}{8x^2} + \frac{1}{16x^3}$
11. $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

13. $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$
15. 0.0713362 17. 0.4969536 19. 0.0999445 21. 0.10000
23. $\frac{1}{13 \cdot 6!} \approx 0.00011$ 25. $\frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!}$
27. (a) $\frac{x^2}{2} - \frac{x^4}{12}$
 (b) $\frac{x^2}{2} - \frac{x^4}{3 \cdot 4} + \frac{x^6}{5 \cdot 6} - \frac{x^8}{7 \cdot 8} + \dots + (-1)^{15} \frac{x^{32}}{31 \cdot 32}$
29. $1/2$ 31. $-1/24$ 33. $1/3$ 35. -1 37. 2
39. $3/2$ 41. e 43. $\cos \frac{3}{4}$ 45. $\frac{\sqrt{3}}{2}$ 47. $\frac{x^3}{1-x}$
49. $\frac{x^3}{1+x^2}$ 51. $\frac{-1}{(1+x)^2}$ 55. 500 terms 57. 4 terms
59. (a) $x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112}$, radius of convergence = 1
 (b) $\frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{112}$
61. $1 - 2x + 3x^2 - 4x^3 + \dots$
67. (a) -1 (b) $(1/\sqrt{2})(1+i)$ (c) $-i$
71. $x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + \dots$, for all x

PRACTICE EXERCISES, pp. 658–660

1. Converges to 1 3. Converges to -1 5. Diverges
7. Converges to 0 9. Converges to 1 11. Converges to e^{-5}
13. Converges to 3 15. Converges to $\ln 2$ 17. Diverges
19. $1/6$ 21. $3/2$ 23. $e/(e-1)$ 25. Diverges
27. Converges conditionally 29. Converges conditionally
31. Converges absolutely 33. Converges absolutely
35. Converges absolutely 37. Converges absolutely
39. Converges absolutely 41. Converges absolutely
43. Diverges
45. (a) $3, -7 \leq x < -1$ (b) $-7 < x < -1$ (c) $x = -7$
47. (a) $1/3, 0 \leq x \leq 2/3$ (b) $0 \leq x \leq 2/3$ (c) None
49. (a) ∞ , for all x (b) For all x (c) None
51. (a) $\sqrt{3}, -\sqrt{3} < x < \sqrt{3}$ (b) $-\sqrt{3} < x < \sqrt{3}$ (c) None
53. (a) $e, -e < x < e$ (b) $-e < x < e$ (c) Empty set
55. $\frac{1}{1+x}, \frac{1}{4}, \frac{4}{5}$ 57. $\sin x, \pi, 0$ 59. $e^x, \ln 2, 2$ 61. $\sum_{n=0}^{\infty} 2^n x^n$
63. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$ 65. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{10n/3}}{(2n)!}$ 67. $\sum_{n=0}^{\infty} \frac{((\pi x)/2)^n}{n!}$
69. $2 - \frac{(x+1)}{2 \cdot 1!} + \frac{3(x+1)^2}{2^3 \cdot 2!} + \frac{9(x+1)^3}{2^5 \cdot 3!} + \dots$
71. $\frac{1}{4} - \frac{1}{4^2}(x-3) + \frac{1}{4^3}(x-3)^2 - \frac{1}{4^4}(x-3)^3$
73. 0.4849171431 75. 0.4872223583 77. $7/2$ 79. $1/12$
81. -2 83. $r = -3, s = 9/2$ 85. $2/3$
87. $\ln \left(\frac{n+1}{2n} \right)$; the series converges to $\ln \left(\frac{1}{2} \right)$.
89. (a) ∞ (b) $a = 1, b = 0$ 91. It converges.
99. (a) Converges; Limit Comparison Test
 (b) Converges; Direct Comparison Test
 (c) Diverges; n th-Term Test
101. 2

ADDITIONAL AND ADVANCED EXERCISES, pp. 660–662

1. Converges; Comparison Test
3. Diverges; n th-Term Test
5. Converges; Comparison Test
7. Diverges; n th-Term Test

9. With $a = \pi/3$, $\cos x = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \pi/3) - \frac{1}{4}(x - \pi/3)^2 + \frac{\sqrt{3}}{12}(x - \pi/3)^3 + \cdots$

11. With $a = 0$, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

13. With $a = 22\pi$, $\cos x = 1 - \frac{1}{2}(x - 22\pi)^2 + \frac{1}{4!}(x - 22\pi)^4 - \frac{1}{6!}(x - 22\pi)^6 + \cdots$

15. Converges, limit = b 17. $\pi/2$ 21. $b = \pm \frac{1}{5}$

23. $a = 2$, $L = -7/6$ 27. (b) Yes

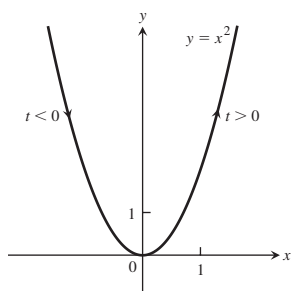
31. (a) $\sum_{n=1}^{\infty} nx^{n-1}$ (b) 6 (c) $\frac{1}{q}$

33. (a) $R_n = C_0 e^{-kt_0} (1 - e^{-nkt_0}) / (1 - e^{-kt_0})$,
 $R = C_0 (e^{-kt_0}) / (1 - e^{-kt_0}) = C_0 / (e^{kt_0} - 1)$
 (b) $R_1 = 1/e \approx 0.368$,
 $R_{10} = R(1 - e^{-10}) \approx R(0.9999546) \approx 0.58195$;
 $R \approx 0.58198$; $0 < (R - R_{10})/R < 0.0001$
 (c) 7

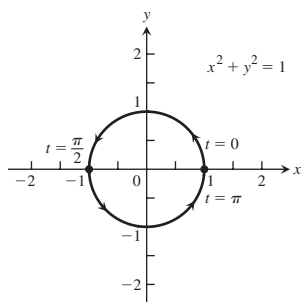
Chapter 11

SECTION 11.1, pp. 669–672

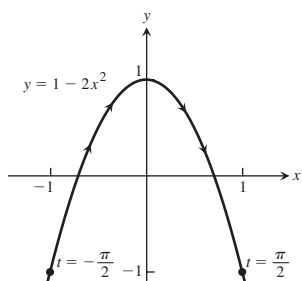
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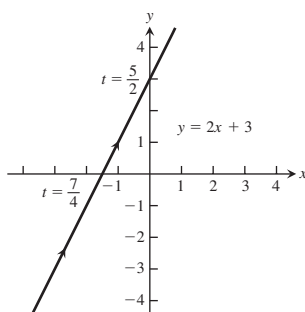
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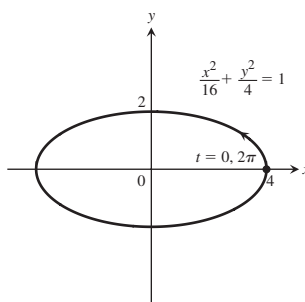
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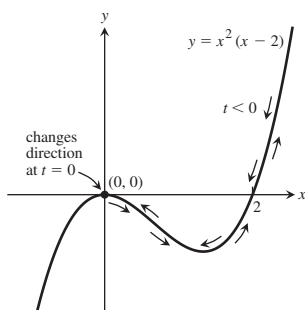
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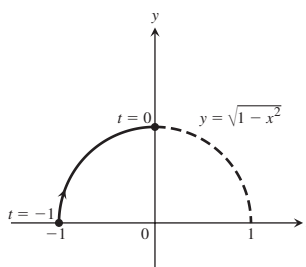
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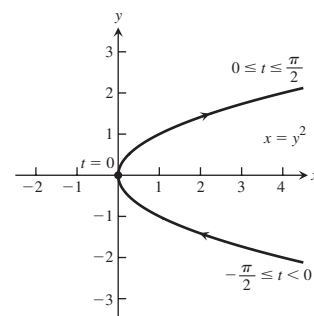
11.



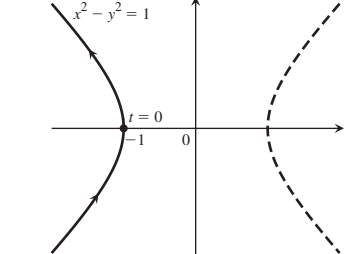
13.



15.



17.

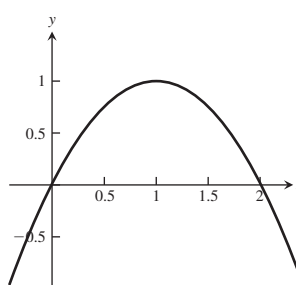


19. D

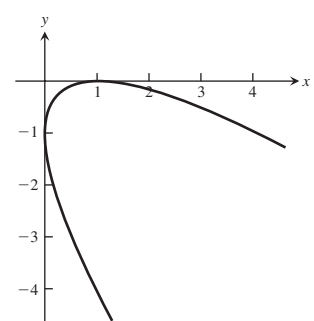
21. E

23. C

25.



27.



29. (a) $x = a \cos t$, $y = -a \sin t$, $0 \leq t \leq 2\pi$

(b) $x = a \cos t$, $y = a \sin t$, $0 \leq t \leq 2\pi$

(c) $x = a \cos t$, $y = -a \sin t$, $0 \leq t \leq 4\pi$

(d) $x = a \cos t$, $y = a \sin t$, $0 \leq t \leq 4\pi$

31. Possible answer: $x = -1 + 5t$, $y = -3 + 4t$, $0 \leq t \leq 1$

33. Possible answer: $x = t^2 + 1$, $y = t$, $t \leq 0$

35. Possible answer: $x = 2 - 3t$, $y = 3 - 4t$, $t \geq 0$

37. Possible answer: $x = 2 \cos t$, $y = 2 |\sin t|$, $0 \leq t \leq 4\pi$

39. Possible answer: $x = \frac{-at}{\sqrt{1+t^2}}$, $y = \frac{a}{\sqrt{1+t^2}}$, $-\infty < t < \infty$

41. Possible answer: $x = \frac{4}{1+2 \tan \theta}$, $y = \frac{4 \tan \theta}{1+2 \tan \theta}$,
 $0 \leq \theta < \pi/2$ and $x = 0$, $y = 2$ if $\theta = \pi/2$

43. Possible answer: $x = 2 - \cos t$, $y = \sin t$, $0 \leq t \leq 2\pi$

45. $x = 2 \cot t$, $y = 2 \sin^2 t$, $0 < t < \pi$

47. $x = a \sin^2 t \tan t$, $y = a \sin^2 t$, $0 \leq t < \pi/2$ 49. (1, 1)

SECTION 11.2, pp. 680–681

1. $y = -x + 2\sqrt{2}$, $\frac{d^2y}{dx^2} = -\sqrt{2}$

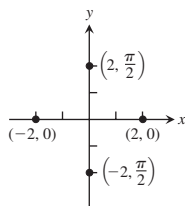
3. $y = -\frac{1}{2}x + 2\sqrt{2}$, $\frac{d^2y}{dx^2} = -\frac{\sqrt{2}}{4}$

5. $y = x + \frac{1}{4}$, $\frac{d^2y}{dx^2} = -2$ 7. $y = 2x - \sqrt{3}$, $\frac{d^2y}{dx^2} = -3\sqrt{3}$

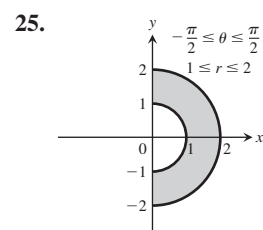
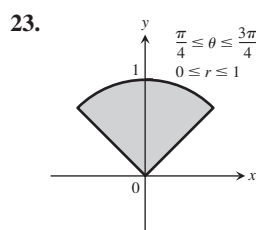
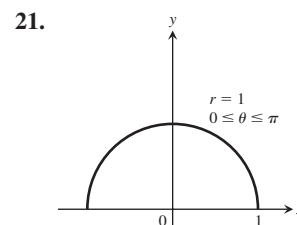
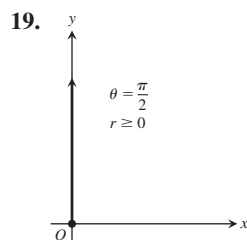
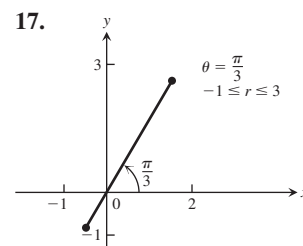
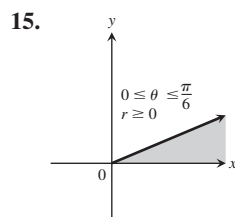
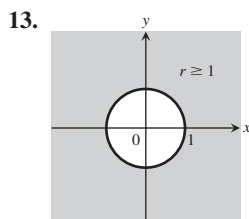
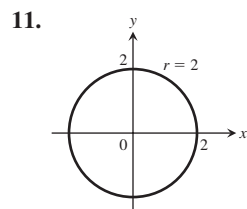
9. $y = x - 4$, $\frac{d^2y}{dx^2} = \frac{1}{2}$
11. $y = \sqrt{3}x - \frac{\pi\sqrt{3}}{3} + 2$, $\frac{d^2y}{dx^2} = -4$
13. $y = 9x - 1$, $\frac{d^2y}{dx^2} = 108$ 15. $-\frac{3}{16}$ 17. -6
19. 1 21. $3a^2\pi$ 23. $|ab|\pi$ 25. 4 27. 12
29. π^2 31. $8\pi^2$ 33. $\frac{52\pi}{3}$ 35. $3\pi\sqrt{5}$
37. $(\bar{x}, \bar{y}) = \left(\frac{12}{\pi} - \frac{24}{\pi^2}, \frac{24}{\pi^2} - 2\right)$
39. $(\bar{x}, \bar{y}) = \left(\frac{1}{3}, \pi - \frac{4}{3}\right)$ 41. (a) π (b) π
43. (a) $x = 1$, $y = 0$, $\frac{dy}{dx} = \frac{1}{2}$ (b) $x = 0$, $y = 3$, $\frac{dy}{dx} = 0$
- (c) $x = \frac{\sqrt{3}-1}{2}$, $y = \frac{3-\sqrt{3}}{2}$, $\frac{dy}{dx} = \frac{2\sqrt{3}-1}{\sqrt{3}-2}$
45. $\left(\frac{\sqrt{2}}{2}, 1\right)$, $y = 2x$ at $t = 0$, $y = -2x$ at $t = \pi$
47. (a) $8a$ (b) $\frac{64\pi}{3}$ 49. $32\pi/15$

SECTION 11.3, pp. 684–685

1. a, e; b, g; c, h; d, f 3.



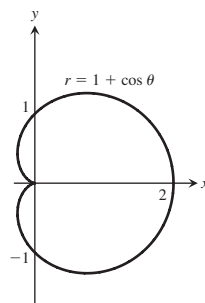
- (a) $\left(2, \frac{\pi}{2} + 2n\pi\right)$ and $\left(-2, \frac{\pi}{2} + (2n+1)\pi\right)$, n an integer
- (b) $(2, 2n\pi)$ and $(-2, (2n+1)\pi)$, n an integer
- (c) $\left(2, \frac{3\pi}{2} + 2n\pi\right)$ and $\left(-2, \frac{3\pi}{2} + (2n+1)\pi\right)$, n an integer
- (d) $(2, (2n+1)\pi)$ and $(-2, 2n\pi)$, n an integer
5. (a) $(3, 0)$ (b) $(-3, 0)$ (c) $(-1, \sqrt{3})$ (d) $(1, \sqrt{3})$
- (e) $(3, 0)$ (f) $(1, \sqrt{3})$ (g) $(-3, 0)$ (h) $(-1, \sqrt{3})$
7. (a) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ (b) $(3, \pi)$ (c) $\left(2, \frac{11\pi}{6}\right)$
- (d) $\left(5, \pi - \tan^{-1} \frac{4}{3}\right)$
9. (a) $\left(-3\sqrt{2}, \frac{5\pi}{4}\right)$ (b) $(-1, 0)$ (c) $\left(-2, \frac{5\pi}{3}\right)$
- (d) $\left(-5, \pi - \tan^{-1} \frac{3}{4}\right)$



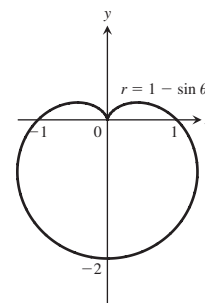
27. $x = 2$, vertical line through $(2, 0)$ 29. $y = 0$, the x -axis
31. $y = 4$, horizontal line through $(0, 4)$
33. $x + y = 1$, line, $m = -1$, $b = 1$
35. $x^2 + y^2 = 1$, circle, $C(0, 0)$, radius 1
37. $y - 2x = 5$, line, $m = 2$, $b = 5$
39. $y^2 = x$, parabola, vertex $(0, 0)$, opens right
41. $y = e^x$, graph of natural exponential function
43. $x + y = \pm 1$, two straight lines of slope -1 , y -intercepts $b = \pm 1$
45. $(x + 2)^2 + y^2 = 4$, circle, $C(-2, 0)$, radius 2
47. $x^2 + (y - 4)^2 = 16$, circle, $C(0, 4)$, radius 4
49. $(x - 1)^2 + (y - 1)^2 = 2$, circle, $C(1, 1)$, radius $\sqrt{2}$
51. $\sqrt{3}y + x = 4$ 53. $r \cos \theta = 7$ 55. $\theta = \pi/4$
57. $r = 2$ or $r = -2$ 59. $4r^2 \cos^2 \theta + 9r^2 \sin^2 \theta = 36$
61. $r \sin^2 \theta = 4 \cos \theta$ 63. $r = 4 \sin \theta$
65. $r^2 = 6r \cos \theta - 2r \sin \theta - 6$
67. $(0, \theta)$, where θ is any angle

SECTION 11.4, pp. 688–689

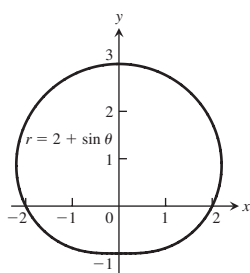
1. x -axis



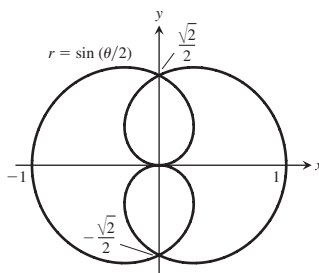
3. y -axis



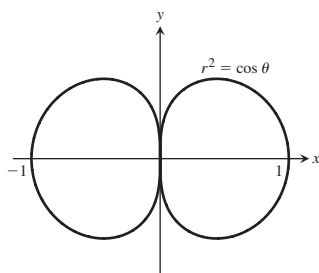
5. y-axis



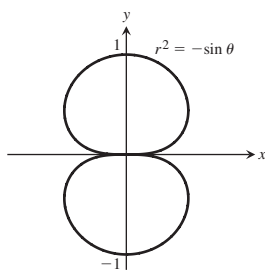
7. x-axis, y-axis, origin



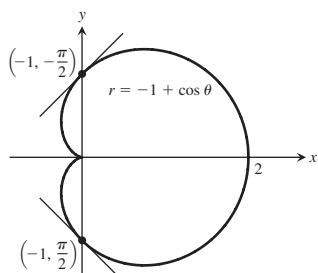
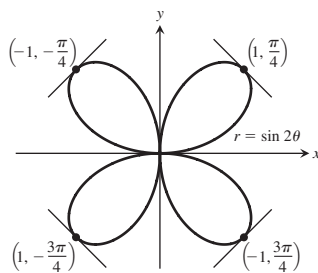
9. x-axis, y-axis, origin



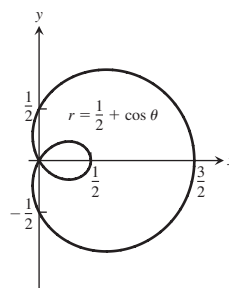
11. y-axis, x-axis, origin



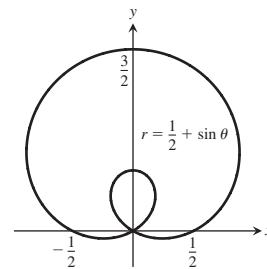
13. x-axis, y-axis, origin

17. The slope at $(-1, \pi/2)$ is -1 , at $(-1, -\pi/2)$ is 1 .19. The slope at $(1, \pi/4)$ is -1 , at $(-1, -\pi/4)$ is 1 , at $(-1, 3\pi/4)$ is 1 , at $(1, -3\pi/4)$ is -1 .21. At $\pi/6$: slope $\sqrt{3}$, concavity 16 (concave up); at $\pi/3$: slope $-\sqrt{3}$, concavity -16 (concave down).23. At 0: slope 0, concavity 2 (concave up); at $\pi/2$: slope $-2/\pi$, concavity $-2(8 + \pi^2)/\pi^3$ (concave down).

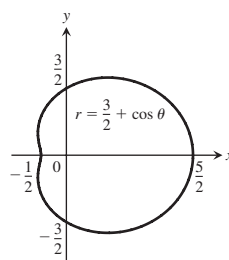
25. (a)



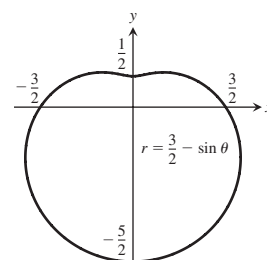
(b)



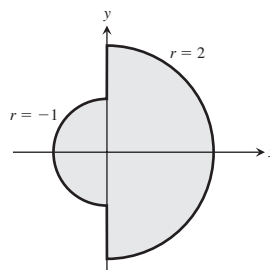
27. (a)



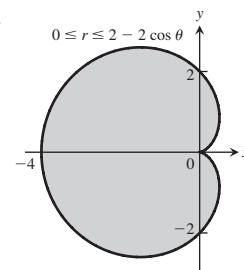
(b)



29.



31.



33. Equation (a)

SECTION 11.5, pp. 693–694

1. $\frac{1}{6}\pi^3$ 3. 18π 5. $\frac{\pi}{8}$ 7. 2 9. $\frac{\pi}{2} - 1$

11. $5\pi - 8$ 13. $3\sqrt{3} - \pi$ 15. $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$

17. $\frac{8\pi}{3} + \sqrt{3}$ 19. (a) $\frac{3}{2} - \frac{\pi}{4}$ 21. $19/3$ 23. 8

25. $3(\sqrt{2} + \ln(1 + \sqrt{2}))$ 27. $\frac{\pi}{8} + \frac{3}{8}$

31. (a) a (b) a (c) $2a/\pi$

SECTION 11.6, pp. 700–702

1. $y^2 = 8x$, $F(2, 0)$, directrix: $x = -2$

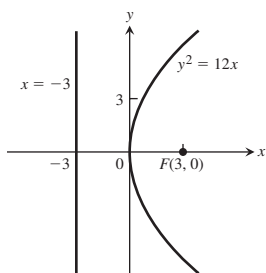
3. $x^2 = -6y$, $F(0, -3/2)$, directrix: $y = 3/2$

5. $\frac{x^2}{4} - \frac{y^2}{9} = 1$, $F(\pm\sqrt{13}, 0)$, $V(\pm 2, 0)$,

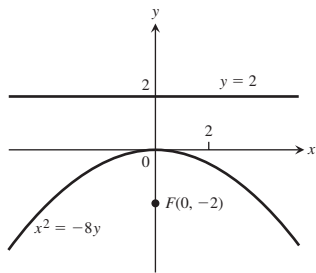
asymptotes: $y = \pm \frac{3}{2}x$

7. $\frac{x^2}{2} + y^2 = 1$, $F(\pm 1, 0)$, $V(\pm\sqrt{2}, 0)$

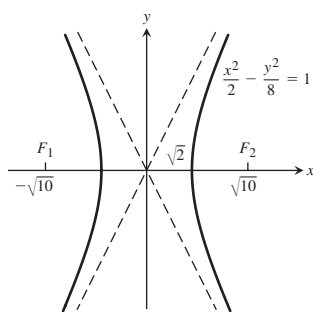
9.



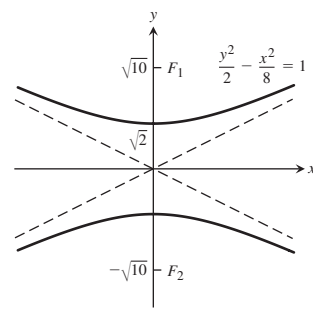
11.



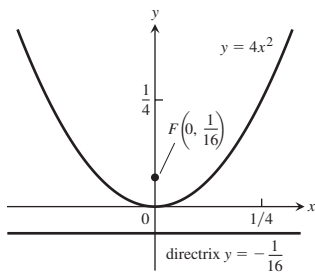
31. Asymptotes: $y = \pm 2x$



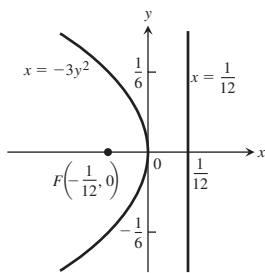
33. Asymptotes: $y = \pm x/2$



13.



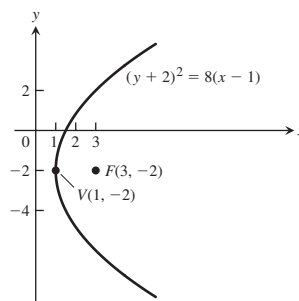
15.



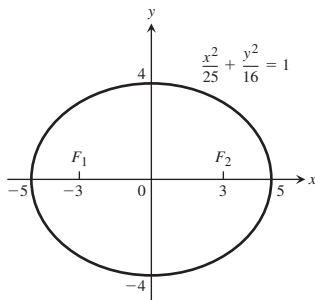
35. $y^2 - x^2 = 1$

37. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

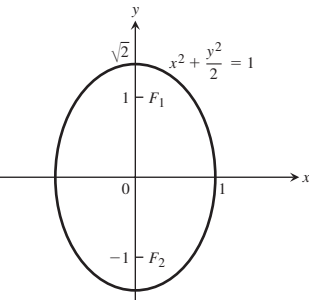
39. (a) Vertex: (1, -2); focus: (3, -2); directrix: $x = -1$
(b)



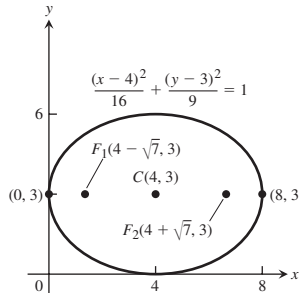
17.



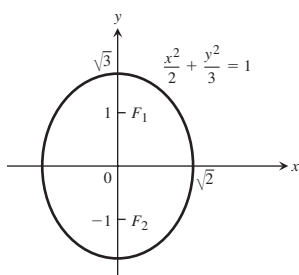
19.



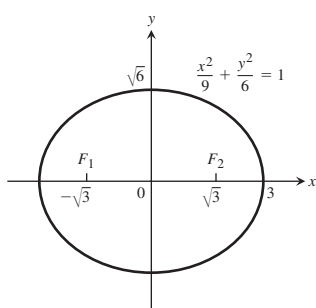
41. (a) Foci: $(4 \pm \sqrt{7}, 3)$; vertices: (8, 3) and (0, 3); center: (4, 3)
(b)



21.

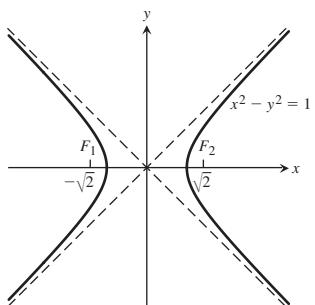


23.

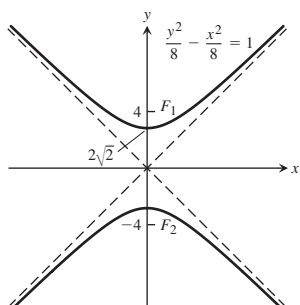


25. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

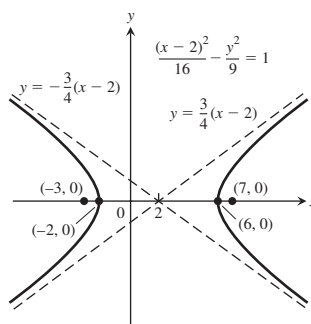
27. Asymptotes: $y = \pm x$



29. Asymptotes: $y = \pm x$



43. (a) Center: (2, 0); foci: (7, 0) and (-3, 0); vertices: (6, 0) and (-2, 0); asymptotes: $y = \pm \frac{3}{4}(x - 2)$
(b)



45. $(y + 3)^2 = 4(x + 2)$, $V(-2, -3)$, $F(-1, -3)$,
directrix: $x = -3$

47. $(x - 1)^2 = 8(y + 7)$, $V(1, -7)$, $F(1, -5)$, directrix: $y = -9$

49. $\frac{(x + 2)^2}{6} + \frac{(y + 1)^2}{9} = 1$, $F(-2, \pm \sqrt{3} - 1)$,
 $V(-2, \pm 3 - 1)$, $C(-2, -1)$

51. $\frac{(x-2)^2}{3} + \frac{(y-3)^2}{2} = 1$, $F(3, 3)$ and $F(1, 3)$,
 $V(\pm\sqrt{3} + 2, 3)$, $C(2, 3)$

53. $\frac{(x-2)^2}{4} - \frac{(y-2)^2}{5} = 1$, $C(2, 2)$, $F(5, 2)$ and $F(-1, 2)$,

$V(4, 2)$ and $V(0, 2)$; asymptotes: $(y-2) = \pm\frac{\sqrt{5}}{2}(x-2)$

55. $(y+1)^2 - (x+1)^2 = 1$, $C(-1, -1)$, $F(-1, \sqrt{2}-1)$
and $F(-1, -\sqrt{2}-1)$, $V(-1, 0)$ and $V(-1, -2)$; asymptotes
 $(y+1) = \pm(x+1)$

57. $C(-2, 0)$, $a = 4$ 59. $V(-1, 1)$, $F(-1, 0)$

61. Ellipse: $\frac{(x+2)^2}{5} + y^2 = 1$, $C(-2, 0)$, $F(0, 0)$ and
 $F(-4, 0)$, $V(\sqrt{5}-2, 0)$ and $V(-\sqrt{5}-2, 0)$

63. Ellipse: $\frac{(x-1)^2}{2} + (y-1)^2 = 1$, $C(1, 1)$, $F(2, 1)$ and
 $F(0, 1)$, $V(\sqrt{2}+1, 1)$ and $V(-\sqrt{2}+1, 1)$

65. Hyperbola: $(x-1)^2 - (y-2)^2 = 1$, $C(1, 2)$,
 $F(1+\sqrt{2}, 2)$ and $F(1-\sqrt{2}, 2)$, $V(2, 2)$ and
 $V(0, 2)$; asymptotes: $(y-2) = \pm(x-1)$

67. Hyperbola: $\frac{(y-3)^2}{6} - \frac{x^2}{3} = 1$, $C(0, 3)$, $F(0, 6)$
and $F(0, 0)$, $V(0, \sqrt{6}+3)$ and $V(0, -\sqrt{6}+3)$;
asymptotes: $y = \sqrt{2}x + 3$ or $y = -\sqrt{2}x + 3$

69. (b) 1:1 73. Length = $2\sqrt{2}$, width = $\sqrt{2}$, area = 4

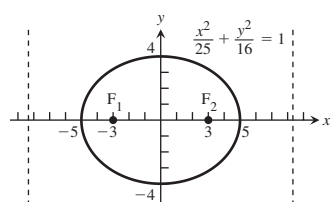
75. 24π

77. $x = 0, y = 0: y = -2x; x = 0, y = 2: y = 2x + 2$;
 $x = 4, y = 0: y = 2x - 8$

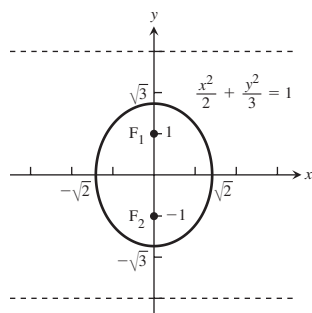
79. $\bar{x} = 0$, $\bar{y} = \frac{16}{3\pi}$

SECTION 11.7, pp. 707-708

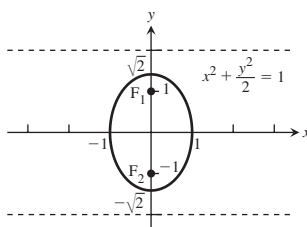
1. $e = \frac{3}{5}$, $F(\pm 3, 0)$;
directrices are $x = \pm \frac{25}{3}$.



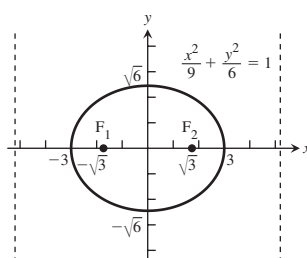
5. $e = \frac{1}{\sqrt{3}}$; $F(0, \pm 1)$;
directrices are $y = \pm 3$.



3. $e = \frac{1}{\sqrt{2}}$; $F(0, \pm 1)$;
directrices are $y = \pm 2$.



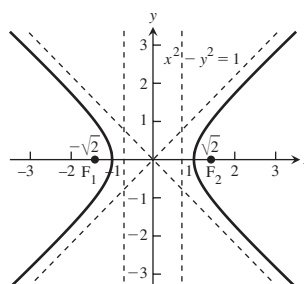
7. $e = \frac{\sqrt{3}}{3}$; $F(\pm\sqrt{3}, 0)$;
directrices are
 $x = \pm 3\sqrt{3}$.



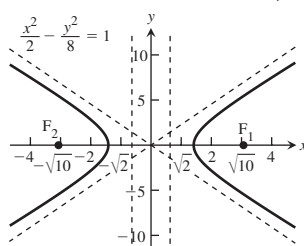
9. $\frac{x^2}{27} + \frac{y^2}{36} = 1$ 11. $\frac{x^2}{4851} + \frac{y^2}{4900} = 1$

13. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 15. $\frac{x^2}{64} + \frac{y^2}{48} = 1$

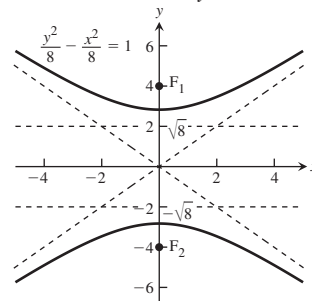
17. $e = \sqrt{2}$; $F(\pm\sqrt{2}, 0)$;
directrices are $x = \pm \frac{1}{\sqrt{2}}$.



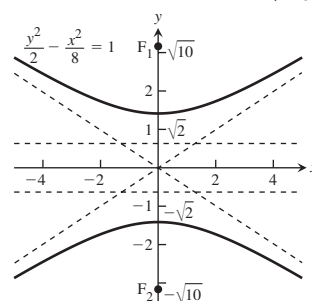
21. $e = \sqrt{5}$; $F(\pm\sqrt{10}, 0)$;
directrices are $x = \pm \frac{2}{\sqrt{10}}$.



19. $e = \sqrt{2}$; $F(0, \pm 4)$;
directrices are $y = \pm 2$.



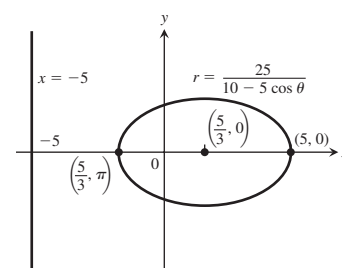
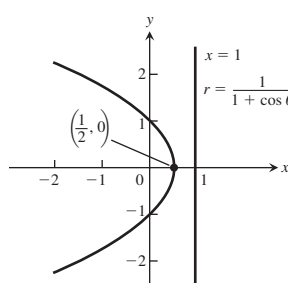
23. $e = \sqrt{5}$; $F(0, \pm\sqrt{10})$;
directrices are $y = \pm \frac{2}{\sqrt{10}}$.



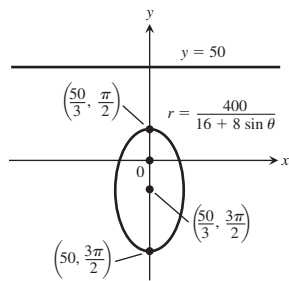
25. $y^2 - \frac{x^2}{8} = 1$ 27. $x^2 - \frac{y^2}{8} = 1$ 29. $r = \frac{2}{1 + \cos \theta}$

31. $r = \frac{30}{1 - 5 \sin \theta}$ 33. $r = \frac{1}{2 + \cos \theta}$ 35. $r = \frac{10}{5 - \sin \theta}$

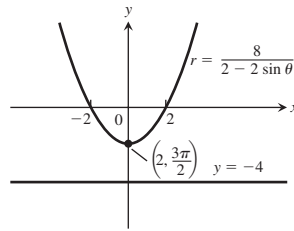
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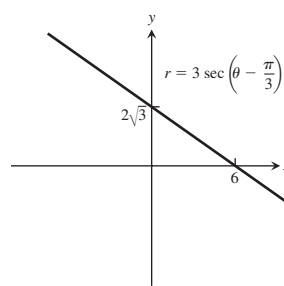
41.



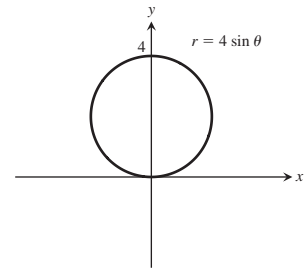
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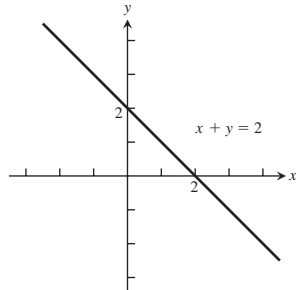
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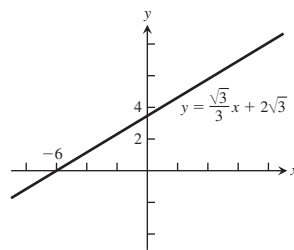
67.



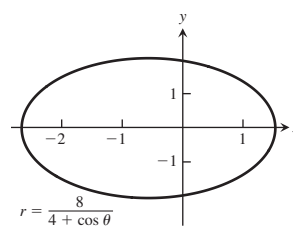
45. $y = 2 - x$



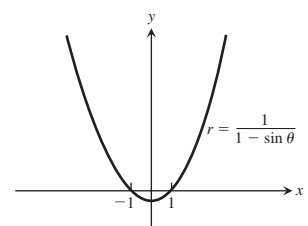
47. $y = \frac{\sqrt{3}}{3}x + 2\sqrt{3}$



69.

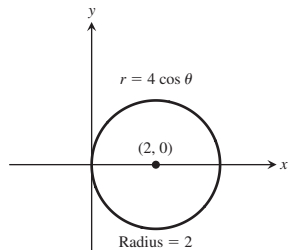


71.

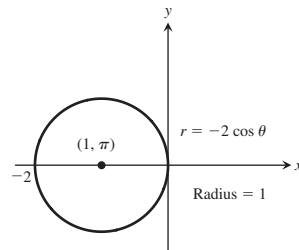


49. $r \cos\left(\theta - \frac{\pi}{4}\right) = 3$ 51. $r \cos\left(\theta + \frac{\pi}{2}\right) = 5$

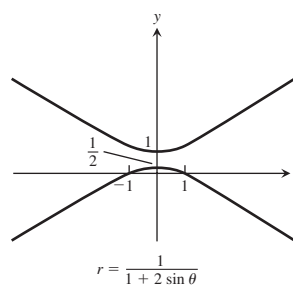
53.



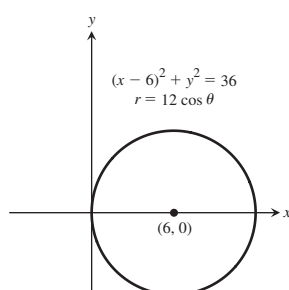
55.



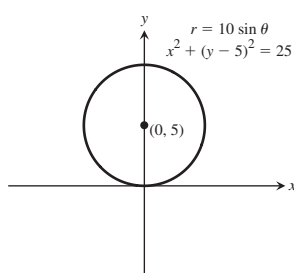
73.



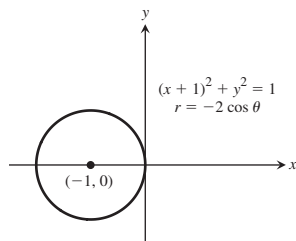
57. $r = 12 \cos \theta$



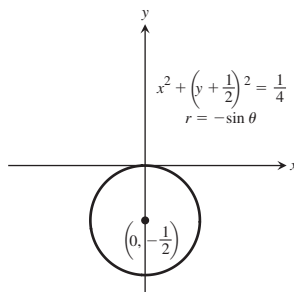
59. $r = 10 \sin \theta$



61. $r = -2 \cos \theta$



63. $r = -\sin \theta$

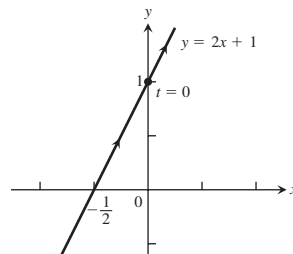


75. (b)

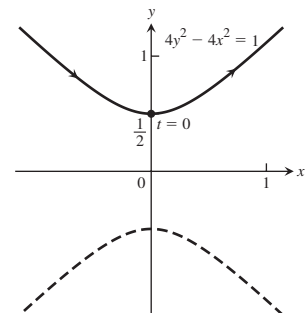
Planet	Perihelion	Aphelion
Mercury	0.3075 AU	0.4667 AU
Venus	0.7184 AU	0.7282 AU
Earth	0.9833 AU	1.0167 AU
Mars	1.3817 AU	1.6663 AU
Jupiter	4.9512 AU	5.4548 AU
Saturn	9.0210 AU	10.0570 AU
Uranus	18.2977 AU	20.0623 AU
Neptune	29.8135 AU	30.3065 AU

PRACTICE EXERCISES, pp. 709–711

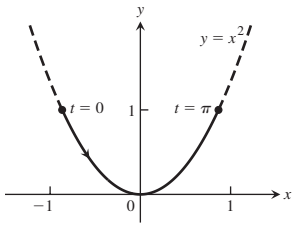
1.



3.



5.



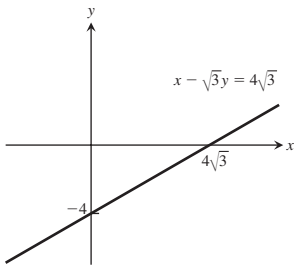
7. $x = 3 \cos t$, $y = 4 \sin t$, $0 \leq t \leq 2\pi$

9. $y = \frac{\sqrt{3}}{2}x + \frac{1}{4}$

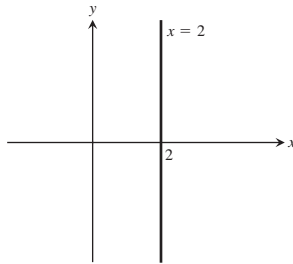
11. (a) $y = \frac{\pm |x|^{3/2}}{8} - 1$ (b) $y = \frac{\pm \sqrt{1-x^2}}{x}$

13. $\frac{10}{3}$ 15. $\frac{285}{8}$ 17. 10 19. $\frac{9\pi}{2}$ 21. $\frac{76\pi}{3}$

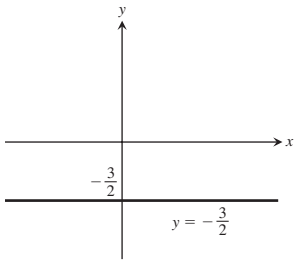
23. $y = \frac{\sqrt{3}}{3}x - 4$



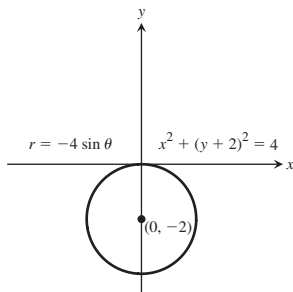
25. $x = 2$



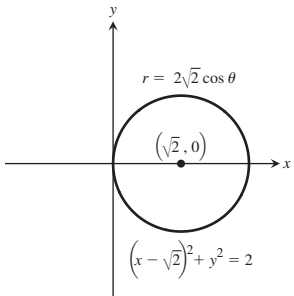
27. $y = -\frac{3}{2}$



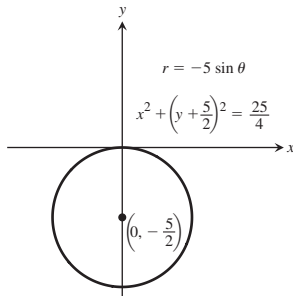
29. $x^2 + (y + 2)^2 = 4$



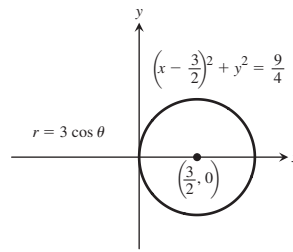
31. $(x - \sqrt{2})^2 + y^2 = 2$



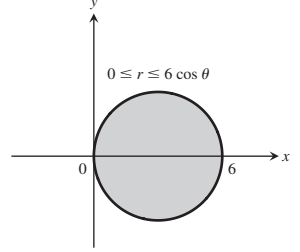
33. $r = -5 \sin \theta$



35. $r = 3 \cos \theta$



37.



39. d

41. l

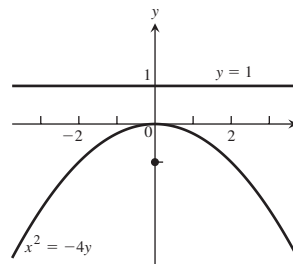
43. k

45. i

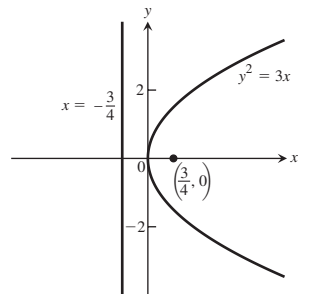
47. $\frac{9}{2}\pi$ 49. $2 + \frac{\pi}{4}$

51. 8 53. $\pi - 3$

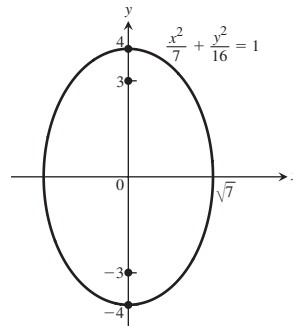
55. Focus is $(0, -1)$,
directrix is $y = 1$.



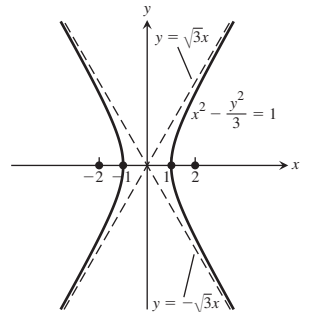
57. Focus is $(\frac{3}{4}, 0)$,
directrix is $x = -\frac{3}{4}$.



59. $e = \frac{3}{4}$



61. $e = 2$; the asymptotes are
 $y = \pm \sqrt{3}x$.



63. $(x - 2)^2 = -12(y - 3)$, $V(2, 3)$, $F(2, 0)$, directrix is $y = 6$.

65. $\frac{(x + 3)^2}{9} + \frac{(y + 5)^2}{25} = 1$, $C(-3, -5)$, $F(-3, -1)$ and
 $F(-3, -9)$, $V(-3, -10)$ and $V(-3, 0)$.

67. $\frac{(y - 2\sqrt{2})^2}{8} - \frac{(x - 2)^2}{2} = 1$, $C(2, 2\sqrt{2})$,
 $F(2, 2\sqrt{2} \pm \sqrt{10})$, $V(2, 4\sqrt{2})$ and $V(2, 0)$, the asymptotes
are $y = 2x - 4 + 2\sqrt{2}$ and $y = -2x + 4 + 2\sqrt{2}$.

69. Hyperbola: $C(2, 0)$, $V(0, 0)$ and $V(4, 0)$, the foci are

$$F(2 \pm \sqrt{5}, 0), \text{ and the asymptotes are } y = \pm \frac{x-2}{2}.$$

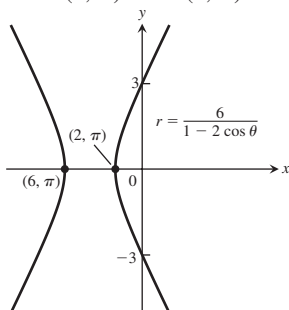
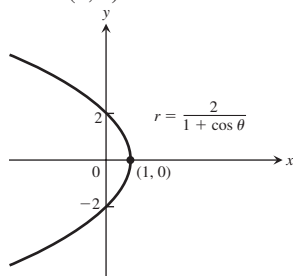
71. Parabola: $V(-3, 1)$, $F(-7, 1)$, and the directrix is $x = 1$.

73. Ellipse: $C(-3, 2)$, $F(-3 \pm \sqrt{7}, 2)$, $V(1, 2)$ and $V(-7, 2)$

75. Circle: $C(1, 1)$ and radius $= \sqrt{2}$

77. $V(1, 0)$

79. $V(2, \pi)$ and $V(6, \pi)$



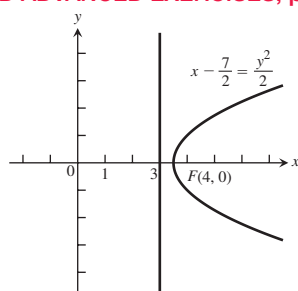
81. $r = \frac{4}{1 + 2 \cos \theta}$

83. $r = \frac{2}{2 + \sin \theta}$

85. (a) 24π (b) 16π

ADDITIONAL AND ADVANCED EXERCISES, pp. 711-713

1. $x - \frac{7}{2} = \frac{y^2}{2}$



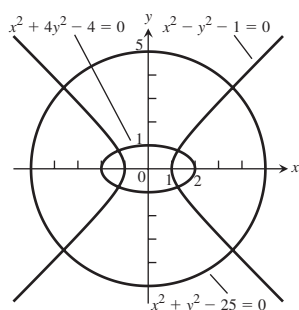
3. $3x^2 + 3y^2 - 8y + 4 = 0$

5. $F(0, \pm 1)$

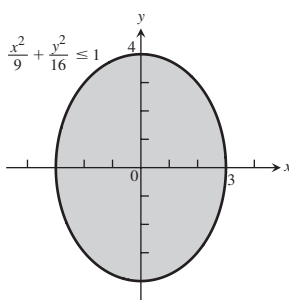
7. (a) $\frac{(y-1)^2}{16} - \frac{x^2}{48} = 1$

(b) $\frac{\left(y + \frac{3}{4}\right)^2}{\left(\frac{25}{16}\right)} - \frac{x^2}{\left(\frac{75}{2}\right)} = 1$

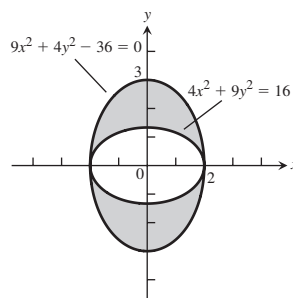
11.



13.



15.



17. (a) $r = e^{2\theta}$ (b) $\frac{\sqrt{5}}{2}(e^{4\pi} - 1)$

19. $r = \frac{4}{1 + 2 \cos \theta}$ 21. $r = \frac{2}{2 + \sin \theta}$

23. $x = (a + b) \cos \theta - b \cos\left(\frac{a+b}{b}\theta\right),$

$$y = (a + b) \sin \theta - b \sin\left(\frac{a+b}{b}\theta\right)$$

27. $\frac{\pi}{2}$

Chapter 12

SECTION 12.1, pp. 717-719

1. The line through the point $(2, 3, 0)$ parallel to the z -axis

3. The x -axis

5. The circle $x^2 + y^2 = 4$ in the xy -plane

7. The circle $x^2 + z^2 = 4$ in the xz -plane

9. The circle $y^2 + z^2 = 1$ in the yz -plane

11. The circle $x^2 + y^2 = 16$ in the xy -plane

13. The ellipse formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = y$

15. The parabola $y = x^2$ in the xy -plane

17. (a) The first quadrant of the xy -plane

(b) The fourth quadrant of the xy -plane

19. (a) The ball of radius 1 centered at the origin

(b) All points more than 1 unit from the origin

21. (a) The ball of radius 2 centered at the origin with the interior of the ball of radius 1 centered at the origin removed

(b) The solid upper hemisphere of radius 1 centered at the origin

23. (a) The region on or inside the parabola $y = x^2$ in the xy -plane and all points above this region

(b) The region on or to the left of the parabola $x = y^2$ in the xy -plane and all points above it that are 2 units or less away from the xy -plane

25. 3 27. 7 29. $2\sqrt{3}$ 31. (a) 2 (b) 3 (c) 4

33. (a) 3 (b) 4 (c) 5

35. (a) $x = 3$ (b) $y = -1$ (c) $z = -2$

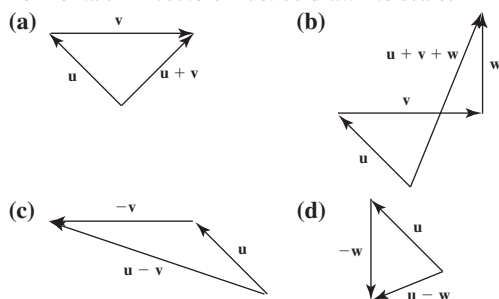
37. (a) $z = 1$ (b) $x = 3$ (c) $y = -1$

39. (a) $x^2 + (y - 2)^2 = 4, z = 0$
 (b) $(y - 2)^2 + z^2 = 4, x = 0$ (c) $x^2 + z^2 = 4, y = 2$
 41. (a) $y = 3, z = -1$ (b) $x = 1, z = -1$ (c) $x = 1, y = 3$
 43. $x^2 + y^2 + z^2 = 25, z = 3$ 45. $0 \leq z \leq 1$ 47. $z \leq 0$
 49. (a) $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 < 1$
 (b) $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 > 1$
 51. $C(-2, 0, 2), a = 2\sqrt{2}$ 53. $C(\sqrt{2}, \sqrt{2}, -\sqrt{2}), a = \sqrt{2}$
 55. $C(-2, 0, 2), a = \sqrt{8}$ 57. $C(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}), a = \frac{5\sqrt{3}}{4}$
 59. $C(2, -3, 5), a = 7$
 61. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 14$
 63. $(x + 1)^2 + (y - \frac{1}{2})^2 + (z + \frac{2}{3})^2 = \frac{16}{81}$
 65. (a) $\sqrt{y^2 + z^2}$ (b) $\sqrt{x^2 + z^2}$ (c) $\sqrt{x^2 + y^2}$
 67. $\sqrt{17} + \sqrt{33} + 6$ 69. $y = 1$
 71. (a) $(0, 3, -3)$ (b) $(0, 5, -5)$
 73. $z = x^2/4 + 1$ 75. (a) $z^2 = x^2$ (b) $y^2 = x^2$

SECTION 12.2, pp. 726–728

1. (a) $\langle 9, -6 \rangle$ (b) $3\sqrt{13}$ 3. (a) $\langle 1, 3 \rangle$ (b) $\sqrt{10}$
 5. (a) $\langle 12, -19 \rangle$ (b) $\sqrt{505}$
 7. (a) $\langle \frac{1}{5}, \frac{14}{5} \rangle$ (b) $\frac{\sqrt{197}}{5}$ 9. $\langle 1, -4 \rangle$
 11. $\langle -2, -3 \rangle$ 13. $\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$ 15. $\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$
 17. $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ 19. $-3\mathbf{i} + 16\mathbf{j}$
 21. $3\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}$

23. The vector $\mathbf{v}$ is horizontal and 1 in. long. The vectors $\mathbf{u}$ and $\mathbf{w}$ are $\frac{11}{16}$ in. long. $\mathbf{w}$ is vertical and $\mathbf{u}$ makes a 45° angle with the horizontal. All vectors must be drawn to scale.



25. $3(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k})$ 27. $5(\mathbf{k})$
 29. $\sqrt{\frac{1}{2}}(\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k})$
 31. (a) $2\mathbf{i}$ (b) $-\sqrt{3}\mathbf{k}$ (c) $\frac{3}{10}\mathbf{j} + \frac{2}{5}\mathbf{k}$ (d) $6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
 33. $\frac{7}{13}(12\mathbf{i} - 5\mathbf{k})$

35. (a) $\frac{3}{5\sqrt{2}}\mathbf{i} + \frac{4}{5\sqrt{2}}\mathbf{j} - \frac{1}{\sqrt{2}}\mathbf{k}$ (b) $(1/2, 3, 5/2)$

37. (a) $-\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}$ (b) $(\frac{5}{2}, \frac{7}{2}, \frac{9}{2})$

39. $A(4, -3, 5)$ 41. $a = \frac{3}{2}, b = \frac{1}{2}$

43. $a = -1, b = 2, c = 1$ 45. $\approx \langle -338.095, 725.046 \rangle$

47. $|\mathbf{F}_1| = \frac{100 \cos 45^\circ}{\sin 75^\circ} \approx 73.205 \text{ N},$

$|\mathbf{F}_2| = \frac{100 \cos 30^\circ}{\sin 75^\circ} \approx 89.658 \text{ N},$

$\mathbf{F}_1 = \langle -|\mathbf{F}_1| \cos 30^\circ, |\mathbf{F}_1| \sin 30^\circ \rangle \approx \langle -63.397, 36.603 \rangle,$

$\mathbf{F}_2 = \langle |\mathbf{F}_2| \cos 45^\circ, |\mathbf{F}_2| \sin 45^\circ \rangle \approx \langle 63.397, 63.397 \rangle$

49. $w = \frac{100 \sin 75^\circ}{\cos 40^\circ} \approx 126.093 \text{ N},$

$|\mathbf{F}_1| = \frac{w \cos 35^\circ}{\sin 75^\circ} \approx 106.933 \text{ N}$

51. (a) $(5 \cos 60^\circ, 5 \sin 60^\circ) = (\frac{5}{2}, \frac{5\sqrt{3}}{2})$

(b) $(5 \cos 60^\circ + 10 \cos 315^\circ, 5 \sin 60^\circ + 10 \sin 315^\circ) = (\frac{5 + 10\sqrt{2}}{2}, \frac{5\sqrt{3} - 10\sqrt{2}}{2})$

53. (a) $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - 3\mathbf{k}$ (b) $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ (c) $(2, 2, 1)$

59. (a) $\langle 0, 0, 0 \rangle$ (b) $\langle 0, 0, 0 \rangle$

SECTION 12.3, pp. 734–736

1. (a) $-25, 5, 5$ (b) -1 (c) -5 (d) $-2\mathbf{i} + 4\mathbf{j} - \sqrt{5}\mathbf{k}$

3. (a) $25, 15, 5$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{1}{9}(10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k})$

5. (a) $2, \sqrt{34}, \sqrt{3}$ (b) $\frac{2}{\sqrt{3}\sqrt{34}}$ (c) $\frac{2}{\sqrt{34}}$

(d) $\frac{1}{17}(5\mathbf{j} - 3\mathbf{k})$

7. (a) $10 + \sqrt{17}, \sqrt{26}, \sqrt{21}$ (b) $\frac{10 + \sqrt{17}}{\sqrt{546}}$ (c) $\frac{10 + \sqrt{17}}{\sqrt{26}}$

(d) $\frac{10 + \sqrt{17}}{26}(5\mathbf{i} + \mathbf{j})$

9. 0.75 rad 11. 1.77 rad

13. Angle at $A = \cos^{-1}(\frac{1}{\sqrt{5}}) \approx 63.435$ degrees, angle at

$B = \cos^{-1}(\frac{3}{5}) \approx 53.130$ degrees, angle at

$C = \cos^{-1}(\frac{1}{\sqrt{5}}) \approx 63.435$ degrees.

17. $\cos^{-1}(\frac{3}{\sqrt{10}}) \approx 0.322$ radian or 18.43 degrees

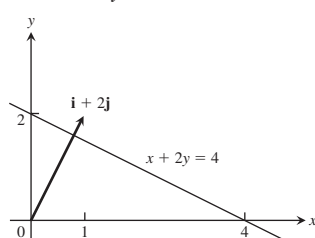
25. Horizontal component: ≈ 1188 ft/sec, vertical component: ≈ 167 ft/sec

27. (a) Since $|\cos \theta| \leq 1$, we have $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| |\cos \theta| \leq |\mathbf{u}| |\mathbf{v}| (1) = |\mathbf{u}| |\mathbf{v}|$.

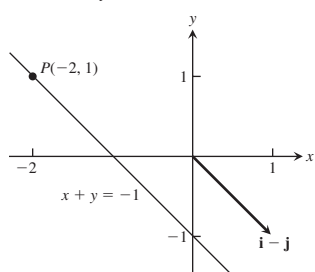
(b) We have equality precisely when $|\cos \theta| = 1$ or when one or both of $\mathbf{u}$ and $\mathbf{v}$ are $\mathbf{0}$. In the case of nonzero vectors, we have equality when $\theta = 0$ or π , that is, when the vectors are parallel.

29. a

35. $x + 2y = 4$



39. $x + y = -1$



43. 5 J 45. 3464 J 47. $\frac{\pi}{4}$ 49. $\frac{\pi}{6}$ 51. 0.14

SECTION 12.4, pp. 741–742

1. $|\mathbf{u} \times \mathbf{v}| = 3$, direction is $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$; $|\mathbf{v} \times \mathbf{u}| = 3$,

direction is $-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

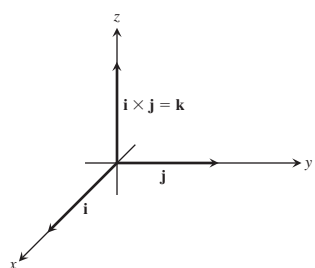
3. $|\mathbf{u} \times \mathbf{v}| = 0$, no direction; $|\mathbf{v} \times \mathbf{u}| = 0$, no direction

5. $|\mathbf{u} \times \mathbf{v}| = 6$, direction is $-\mathbf{k}$; $|\mathbf{v} \times \mathbf{u}| = 6$, direction is $\mathbf{k}$

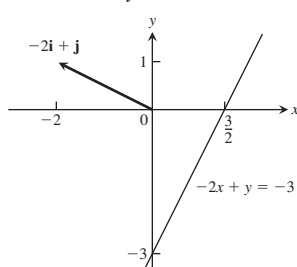
7. $|\mathbf{u} \times \mathbf{v}| = 6\sqrt{5}$, direction is $\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{k}$; $|\mathbf{v} \times \mathbf{u}| = 6\sqrt{5}$,

direction is $-\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{k}$

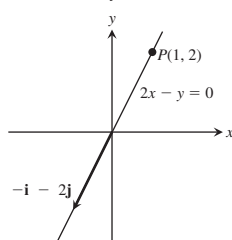
9.



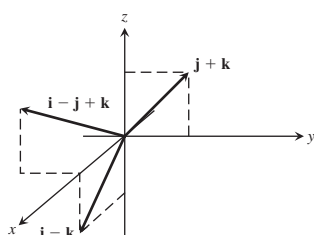
37. $-2x + y = -3$



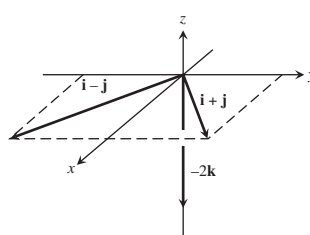
41. $2x - y = 0$



11.



13.



15. (a) $2\sqrt{6}$ (b) $\pm \frac{1}{\sqrt{6}}(2\mathbf{i} + \mathbf{j} + \mathbf{k})$

17. (a) $\frac{\sqrt{2}}{2}$ (b) $\pm \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$

19. 8 21. 7 23. (a) None (b) $\mathbf{u}$ and $\mathbf{w}$

25. $10\sqrt{3}$ ft-lb

27. (a) True (b) Not always true (c) True (d) True
(e) Not always true (f) True (g) True (h) True

29. (a) $\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v}$ (b) $\pm \mathbf{u} \times \mathbf{v}$ (c) $\pm (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

(d) $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$ (e) $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{u} \times \mathbf{w})$ (f) $|\mathbf{u}| \frac{|\mathbf{v}|}{|\mathbf{w}|}$

31. (a) Yes (b) No (c) Yes (d) No

33. No, $\mathbf{v}$ need not equal $\mathbf{w}$. For example, $\mathbf{i} + \mathbf{j} \neq -\mathbf{i} + \mathbf{j}$, but $\mathbf{i} \times (\mathbf{i} + \mathbf{j}) = \mathbf{i} \times \mathbf{i} + \mathbf{i} \times \mathbf{j} = \mathbf{0} + \mathbf{k} = \mathbf{k}$ and $\mathbf{i} \times (-\mathbf{i} + \mathbf{j}) = -\mathbf{i} \times \mathbf{i} + \mathbf{i} \times \mathbf{j} = \mathbf{0} + \mathbf{k} = \mathbf{k}$.

35. 2 37. 13 39. $\sqrt{129}$ 41. $\frac{11}{2}$ 43. $\frac{25}{2}$

45. $\frac{3}{2}$ 47. $\frac{\sqrt{21}}{2}$

49. If $\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j}$ and $\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j}$, then

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

and the triangle's area is

$$\frac{1}{2} |\mathbf{A} \times \mathbf{B}| = \pm \frac{1}{2} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}.$$

The applicable sign is (+) if the acute angle from $\mathbf{A}$ to $\mathbf{B}$ runs counterclockwise in the xy -plane, and (−) if it runs clockwise.

51. 4 53. $44/3$ 55. Coplanar 57. Not coplanar

SECTION 12.5, pp. 749–751

1. $x = 3 + t$, $y = -4 + t$, $z = -1 + t$

3. $x = -2 + 5t$, $y = 5t$, $z = 3 - 5t$

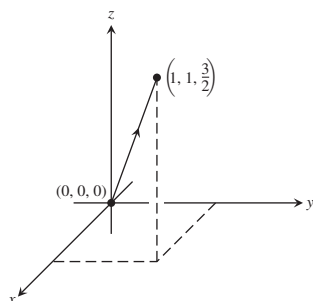
5. $x = 0$, $y = 2t$, $z = t$

7. $x = 1$, $y = 1$, $z = 1 + t$

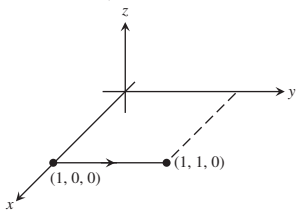
9. $x = t$, $y = -7 + 2t$, $z = 2t$

11. $x = t$, $y = 0$, $z = 0$

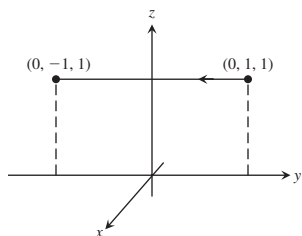
13. $x = t, y = t, z = \frac{3}{2}t$,
 $0 \leq t \leq 1$



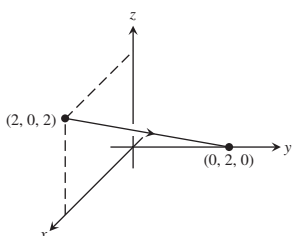
15. $x = 1, y = 1 + t, z = 0, -1 \leq t \leq 0$



17. $x = 0, y = 1 - 2t, z = 1, 0 \leq t \leq 1$



19. $x = 2 - 2t, y = 2t, z = 2 - 2t, 0 \leq t \leq 1$



21. $3x - 2y - z = -3$ 23. $7x - 5y - 4z = 6$
25. $x + 3y + 4z = 34$ 27. $(1, 2, 3), -20x + 12y + z = 7$
29. $y + z = 3$ 31. $x - y + z = 0$ 33. $2\sqrt{30}$ 35. 0

37. $\frac{9\sqrt{42}}{7}$ 39. 3 41. $19/5$ 43. $5/3$ 45. $9/\sqrt{41}$

47. $\pi/4$ 49. $\arccos(-1/6) \approx 1.738$ radians
51. $\arcsin(2/\sqrt{154}) \approx 0.161$ radians 53. 1.38 rad

55. 0.82 rad 57. $(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2})$ 59. $(1, 1, 0)$

61. $x = 1 - t, y = 1 + t, z = -1$

63. $x = 4, y = 3 + 6t, z = 1 + 3t$

65. L_1 intersects L_2 ; L_2 is parallel to L_3 , $\sqrt{5}/3$; L_1 and L_3 are skew, $10\sqrt{2}/3$

67. $x = 2 + 2t, y = -4 - t, z = 7 + 3t; x = -2 - t, y = -2 + (1/2)t, z = 1 - (3/2)t$

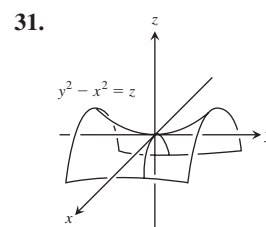
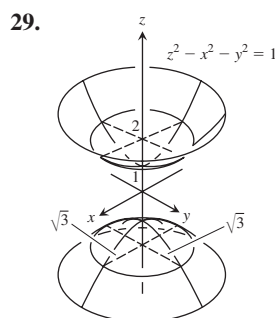
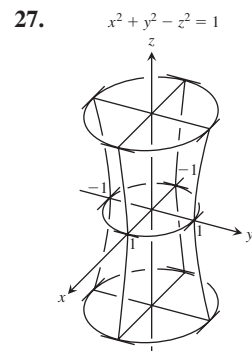
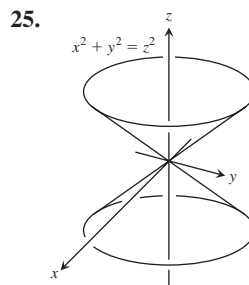
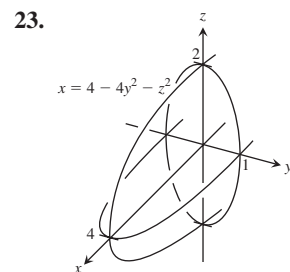
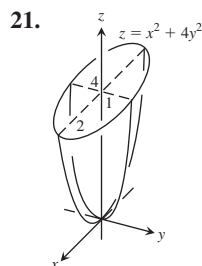
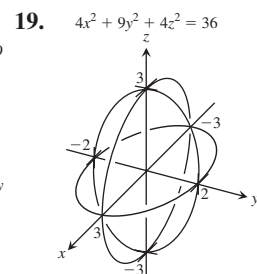
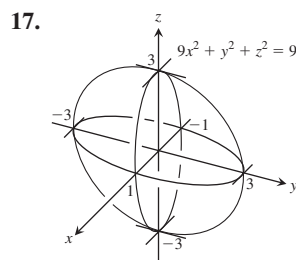
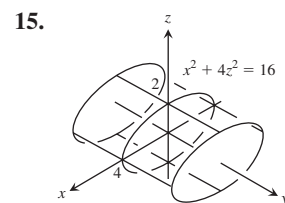
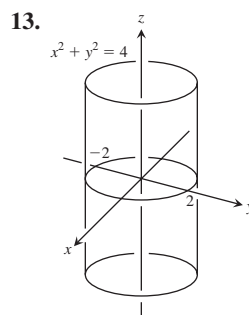
69. $(0, -\frac{1}{2}, -\frac{3}{2}), (-1, 0, -3), (1, -1, 0)$

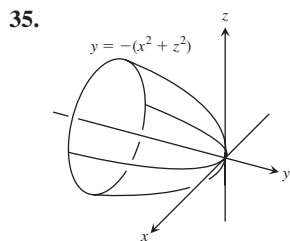
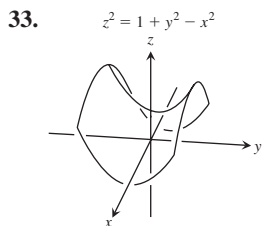
73. Many possible answers. One possibility: $x + y = 3$ and $2y + z = 7$.

75. $(x/a) + (y/b) + (z/c) = 1$ describes all planes *except* those through the origin or parallel to a coordinate axis.

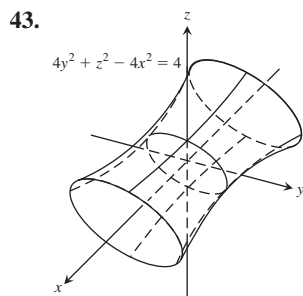
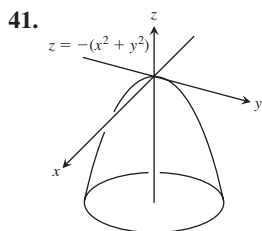
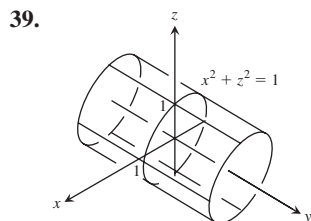
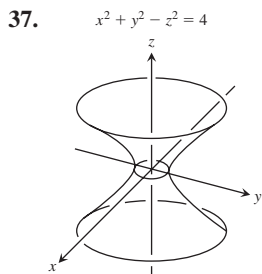
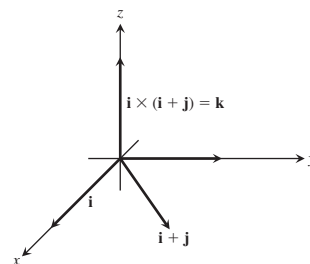
SECTION 12.6, pp. 755–757

1. (d), ellipsoid 3. (a), cylinder 5. (l), hyperbolic paraboloid
7. (b), cylinder 9. (k), hyperbolic paraboloid 11. (h), cone





21. $\mathbf{u} \times \mathbf{v} = \mathbf{k}$

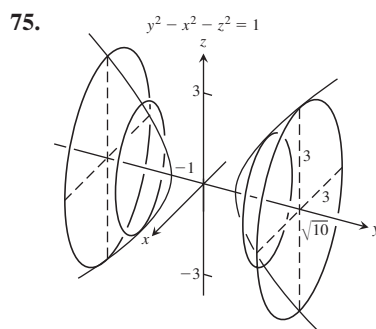
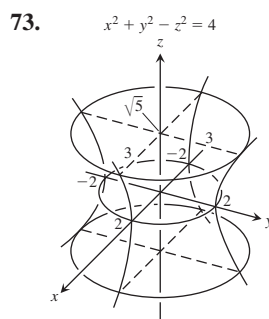
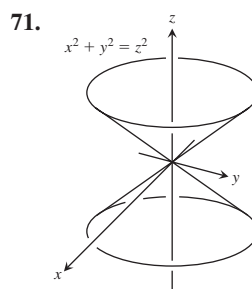
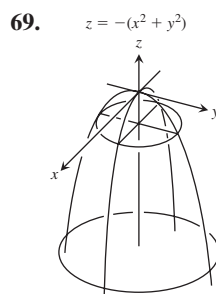
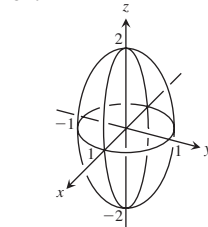
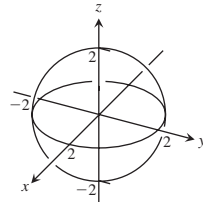


45. (a) $\frac{2\pi(9 - c^2)}{9}$ (b) 8π (c) $\frac{4\pi abc}{3}$

PRACTICE EXERCISES, pp. 757-759

1. (a) $\langle -17, 32 \rangle$ (b) $\sqrt{1313}$
3. (a) $\langle 6, -8 \rangle$ (b) 10
5. $\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$ [assuming counterclockwise]
7. $\left\langle \frac{8}{\sqrt{17}}, -\frac{2}{\sqrt{17}} \right\rangle$
9. Length = 2, direction is $\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$.
11. $\mathbf{v}(\pi/2) = 2(-\mathbf{i})$
13. Length = 7, direction is $\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$.
15. $\frac{8}{\sqrt{33}}\mathbf{i} - \frac{2}{\sqrt{33}}\mathbf{j} + \frac{8}{\sqrt{33}}\mathbf{k}$
17. $|\mathbf{v}| = \sqrt{2}$, $|\mathbf{u}| = 3$, $\mathbf{v} \cdot \mathbf{u} = \mathbf{u} \cdot \mathbf{v} = 3$, $\mathbf{v} \times \mathbf{u} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$,
 $\mathbf{u} \times \mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $|\mathbf{v} \times \mathbf{u}| = 3$, $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$,
 $|\mathbf{u}| \cos \theta = \frac{3}{\sqrt{2}}$, $\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{3}{2}(\mathbf{i} + \mathbf{j})$
19. $\frac{4}{3}(2\mathbf{i} + \mathbf{j} - \mathbf{k})$

23. $2\sqrt{7}$ 25. (a) $\sqrt{14}$ (b) 1 29. $\sqrt{78}/3$
31. $x = 1 - 3t$, $y = 2$, $z = 3 + 7t$ 33. $\sqrt{2}$
35. $2x + y + z = 5$ 37. $-9x + y + 7z = 4$
39. $\left(0, -\frac{1}{2}, -\frac{3}{2}\right)$, $(-1, 0, -3)$, $(1, -1, 0)$ 41. $\pi/3$
43. $x = -5 + 5t$, $y = 3 - t$, $z = -3t$
45. (b) $x = -12t$, $y = 19/12 + 15t$, $z = 1/6 + 6t$
47. Yes; $\mathbf{v}$ is parallel to the plane.
49. 3 51. $-3\mathbf{j} + 3\mathbf{k}$
53. $\frac{2}{\sqrt{35}}(5\mathbf{i} - \mathbf{j} - 3\mathbf{k})$ 55. $\left(\frac{11}{9}, \frac{26}{9}, -\frac{7}{9}\right)$
57. $(1, -2, -1)$; $x = 1 - 5t$, $y = -2 + 3t$, $z = -1 + 4t$
59. $2x + 7y + 2z + 10 = 0$
61. (a) No (b) No (c) No (d) No (e) Yes
63. $11/\sqrt{107}$
65. $x^2 + y^2 + z^2 = 4$



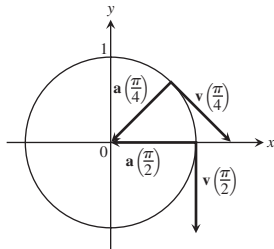
ADDITIONAL AND ADVANCED EXERCISES, pp. 759–762

1. $(26, 23, -1/3)$ 3. $|\mathbf{F}| = 20 \text{ lb}$
 5. (a) $|\mathbf{F}_1| = 80 \text{ lb}$, $|\mathbf{F}_2| = 60 \text{ lb}$, $\mathbf{F}_1 = \langle -48, 64 \rangle$,
 $\mathbf{F}_2 = \langle 48, 36 \rangle$, $\alpha = \tan^{-1} \frac{4}{3}$, $\beta = \tan^{-1} \frac{3}{4}$
 (b) $|\mathbf{F}_1| = \frac{2400}{13} \approx 184.615 \text{ lb}$, $|\mathbf{F}_2| = \frac{1000}{13} \approx 76.923 \text{ lb}$,
 $\mathbf{F}_1 = \left\langle \frac{-12,000}{169}, \frac{28,800}{169} \right\rangle \approx \langle -71.006, 170.414 \rangle$,
 $\mathbf{F}_2 = \left\langle \frac{12,000}{169}, \frac{5,000}{169} \right\rangle \approx \langle 71.006, 29.586 \rangle$,
 $\alpha = \tan^{-1} \frac{12}{5}$, $\beta = \tan^{-1} \frac{5}{12}$
 9. (a) $\theta = \tan^{-1} \sqrt{2} \approx 54.74^\circ$ (b) $\theta = \tan^{-1} 2\sqrt{2} \approx 70.53^\circ$
 13. (a) $\frac{6}{\sqrt{14}}$ (b) $2x - y + 2z = 8$
 (c) $x - 2y + z = 3 + 5\sqrt{6}$ and $x - 2y + z = 3 - 5\sqrt{6}$
 15. $\frac{32}{41}\mathbf{i} + \frac{23}{41}\mathbf{j} - \frac{13}{41}\mathbf{k}$
 17. (a) $0, 0$ (b) $-10\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $-9\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$
 (c) $-4\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$, $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$
 (d) $-10\mathbf{i} - 10\mathbf{k}$, $-12\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$
 19. The formula is always true.

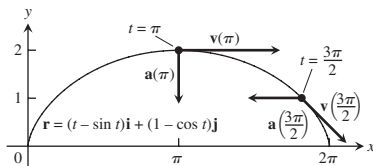
Chapter 13

SECTION 13.1, pp. 770–772

1. $\mathbf{i} - \frac{1}{2}\mathbf{j} + \mathbf{k}$ 3. $2\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{\pi}{4}\mathbf{k}$
 5. $y = x^2 - 2x$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{a} = 2\mathbf{j}$
 7. $y = \frac{2}{9}x^2$, $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{a} = 3\mathbf{i} + 8\mathbf{j}$
 9. $t = \frac{\pi}{4}$: $\mathbf{v} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$, $\mathbf{a} = \frac{-\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$;
 $t = \pi/2$: $\mathbf{v} = -\mathbf{j}$, $\mathbf{a} = -\mathbf{i}$



11. $t = \pi$: $\mathbf{v} = 2\mathbf{i}$, $\mathbf{a} = -\mathbf{j}$; $t = \frac{3\pi}{2}$: $\mathbf{v} = \mathbf{i} - \mathbf{j}$, $\mathbf{a} = -\mathbf{i}$



13. $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$; $\mathbf{a} = 2\mathbf{j}$; speed: 3; direction: $\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$;
 $\mathbf{v}(1) = 3\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$
 15. $\mathbf{v} = (-2 \sin t)\mathbf{i} + (3 \cos t)\mathbf{j} + 4\mathbf{k}$;
 $\mathbf{a} = (-2 \cos t)\mathbf{i} - (3 \sin t)\mathbf{j}$; speed: $2\sqrt{5}$;
 direction: $(-1/\sqrt{5})\mathbf{i} + (2/\sqrt{5})\mathbf{j}$;
 $\mathbf{v}(\pi/2) = 2\sqrt{5}\left[(-1/\sqrt{5})\mathbf{i} + (2/\sqrt{5})\mathbf{j}\right]$
 17. $\mathbf{v} = \left(\frac{2}{t+1}\right)\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$; $\mathbf{a} = \left(\frac{-2}{(t+1)^2}\right)\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;
 speed: $\sqrt{6}$; direction: $\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}$;
 $\mathbf{v}(1) = \sqrt{6}\left(\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}\right)$
 19. $\pi/2$ 21. $\pi/2$ 23. $x = t$, $y = -1$, $z = 1 + t$
 25. $x = t$, $y = \frac{1}{3}t$, $z = t$ 27. $4, -2$ 29. $2, -2$
 31. E 33. D 35. C
 37. (i): It has constant speed 1. (ii): Yes
 (iii): Counterclockwise (iv): Yes
 (b) (i): It has constant speed 2. (ii): Yes
 (iii): Counterclockwise (iv): Yes
 (c) (i): It has constant speed 1. (ii): Yes
 (iii): Counterclockwise (iv): Yes
 (iv): It starts at $(0, -1)$ instead of $(1, 0)$.
 (d) (i): It has constant speed 1. (ii): Yes
 (iii): Clockwise (iv): Yes
 (i): It has variable speed. (ii): No
 (iii): Counterclockwise (iv): Yes
 39. $\mathbf{v} = 2\sqrt{5}\mathbf{i} + \sqrt{5}\mathbf{j}$

SECTION 13.2, pp. 777–781

1. $(1/4)\mathbf{i} + 7\mathbf{j} + (3/2)\mathbf{k}$ 3. $\left(\frac{\pi + 2\sqrt{2}}{2}\right)\mathbf{j} + 2\mathbf{k}$
 5. $(\ln 4)\mathbf{i} + (\ln 4)\mathbf{j} + (\ln 2)\mathbf{k}$
 7. $\frac{e-1}{2}\mathbf{i} + \frac{e-1}{e}\mathbf{j} + \mathbf{k}$ 9. $\mathbf{i} - \mathbf{j} + \frac{\pi}{4}\mathbf{k}$
 11. $\mathbf{r}(t) = \left(\frac{-t^2}{2} + 1\right)\mathbf{i} + \left(\frac{-t^2}{2} + 2\right)\mathbf{j} + \left(\frac{-t^2}{2} + 3\right)\mathbf{k}$
 13. $\mathbf{r}(t) = ((t+1)^{3/2} - 1)\mathbf{i} + (-e^{-t} + 1)\mathbf{j} + (\ln(t+1) + 1)\mathbf{k}$
 15. $\mathbf{r}(t) = (3 + \ln|\sec t|)\mathbf{i} + (-2 + 2 \sin(t/2))\mathbf{j}$
 $+ (1 - (1/2) \ln|\sec 2t + \tan 2t|)\mathbf{k}$
 17. $\mathbf{r}(t) = 8t\mathbf{i} + 8t\mathbf{j} + (-16t^2 + 100)\mathbf{k}$
 19. $\mathbf{r}(t) = (e^t - 2t + 2)\mathbf{i} + (-e^{-t} + 3t + 2)\mathbf{j} + (e^{2t} - 2t + 1)\mathbf{k}$
 21. $\mathbf{r}(t) = \left(\frac{3}{2}t^2 + \frac{6}{\sqrt{11}}t + 1\right)\mathbf{i} - \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t - 2\right)\mathbf{j}$
 $+ \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t + 3\right)\mathbf{k} = \left(\frac{1}{2}t^2 + \frac{2t}{\sqrt{11}}\right)(3\mathbf{i} - \mathbf{j} + \mathbf{k})$
 $+ (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$
 23. 50 sec
 25. (a) 72.2 sec; 25,510 m (b) 4020 m (c) 6378 m
 27. (a) $v_0 \approx 9.9 \text{ m/sec}$ (b) $\alpha \approx 18.4^\circ$ or 71.6°
 29. 39.3° or 50.7° 35. (b) $\mathbf{v}_0$ would bisect $\angle AOR$.

37. (a) (Assuming that “x” is zero at the point of impact)
 $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where $x(t) = (35 \cos 27^\circ)t$ and $y(t) = 4 + (35 \sin 27^\circ)t - 16t^2$.
 (b) At $t \approx 0.497$ sec, it reaches its maximum height of about 7.945 ft.
 (c) Range ≈ 37.45 ft; flight time ≈ 1.201 sec
 (d) At $t \approx 0.254$ and $t \approx 0.740$ sec, when it is ≈ 29.532 and ≈ 14.376 ft from where it will land
 (e) Yes. It changes things because the ball won’t clear the net.

39. 4.00 ft, 7.80 ft/sec

47. (a) $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$; where

$$x(t) = \left(\frac{1}{0.08}\right)(1 - e^{-0.08t})(152 \cos 20^\circ - 17.6) \text{ and}$$

$$y(t) = 3 + \left(\frac{152}{0.08}\right)(1 - e^{-0.08t})(\sin 20^\circ)$$

$$+ \left(\frac{32}{0.08^2}\right)(1 - 0.08t - e^{-0.08t})$$

- (b) At $t \approx 1.527$ sec it reaches a maximum height of about 41.893 feet.
 (c) Range ≈ 351.734 ft; flight time ≈ 3.181 sec
 (d) At $t \approx 0.877$ and 2.190 sec, when it is about 106.028 and 251.530 ft from home plate
 (e) No

SECTION 13.3, pp. 784–785

1. $\mathbf{T} = \left(-\frac{2}{3} \sin t\right)\mathbf{i} + \left(\frac{2}{3} \cos t\right)\mathbf{j} + \frac{\sqrt{5}}{3}\mathbf{k}$, 3π

3. $\mathbf{T} = \frac{1}{\sqrt{1+t}}\mathbf{i} + \frac{\sqrt{t}}{\sqrt{1+t}}\mathbf{k}$, $\frac{52}{3}$

5. $\mathbf{T} = -\cos t \mathbf{j} + \sin t \mathbf{k}$, $\frac{3}{2}$

7. $\mathbf{T} = \left(\frac{\cos t - t \sin t}{t+1}\right)\mathbf{i} + \left(\frac{\sin t + t \cos t}{t+1}\right)\mathbf{j}$
 $+ \left(\frac{\sqrt{2}t^{1/2}}{t+1}\right)\mathbf{k}$, $\frac{\pi^2}{2} + \pi$

9. $(0, 5, 24\pi)$

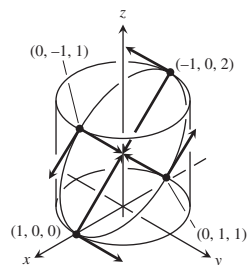
11. $s(t) = 5t$, $L = \frac{5\pi}{2}$

13. $s(t) = \sqrt{3}e^t - \sqrt{3}$, $L = \frac{3\sqrt{3}}{4}$

15. $\sqrt{2} + \ln(1 + \sqrt{2})$

17. (a) Cylinder is $x^2 + y^2 = 1$; plane is $x + z = 1$.

(b) and (c)



(d) $L = \int_0^{2\pi} \sqrt{1 + \sin^2 t} dt$ (e) $L \approx 7.64$

SECTION 13.4, pp. 790–791

1. $\mathbf{T} = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}$, $\mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}$, $\kappa = \cos t$

3. (a) $\mathbf{T} = \frac{1}{\sqrt{1+t^2}}\mathbf{i} - \frac{t}{\sqrt{1+t^2}}\mathbf{j}$, $\mathbf{N} = \frac{-t}{\sqrt{1+t^2}}\mathbf{i} -$

$$\frac{1}{\sqrt{1+t^2}}\mathbf{j}$$
, $\kappa = \frac{1}{2(\sqrt{1+t^2})^3}$

5. (b) $\cos x$

7. (b) $\mathbf{N} = \frac{-2e^{2t}}{\sqrt{1+4e^{4t}}}\mathbf{i} + \frac{1}{\sqrt{1+4e^{4t}}}\mathbf{j}$

(c) $\mathbf{N} = -\frac{1}{2}(\sqrt{4-t^2}\mathbf{i} + t\mathbf{j})$

9. $\mathbf{T} = \frac{3 \cos t}{5}\mathbf{i} - \frac{3 \sin t}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$,

$$\mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}$$
, $\kappa = \frac{3}{25}$

11. $\mathbf{T} = \left(\frac{\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{\cos t + \sin t}{\sqrt{2}}\right)\mathbf{j}$,

$$\mathbf{N} = \left(\frac{-\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{-\sin t + \cos t}{\sqrt{2}}\right)\mathbf{j}$$
, $\kappa = \frac{1}{e^t \sqrt{2}}$

13. $\mathbf{T} = \frac{t}{\sqrt{t^2+1}}\mathbf{i} + \frac{1}{\sqrt{t^2+1}}\mathbf{j}$,

$$\mathbf{N} = \frac{\mathbf{i}}{\sqrt{t^2+1}} - \frac{t\mathbf{j}}{\sqrt{t^2+1}}$$
, $\kappa = \frac{1}{t(t^2+1)^{3/2}}$

15. $\mathbf{T} = \left(\operatorname{sech} \frac{t}{a}\right)\mathbf{i} + \left(\tanh \frac{t}{a}\right)\mathbf{j}$,

$$\mathbf{N} = \left(-\tanh \frac{t}{a}\right)\mathbf{i} + \left(\operatorname{sech} \frac{t}{a}\right)\mathbf{j}$$
,

$$\kappa = \frac{1}{a} \operatorname{sech}^2 \frac{t}{a}$$

19. $1/(2b)$

21. $\left(x - \frac{\pi}{2}\right)^2 + y^2 = 1$

23. $\kappa(x) = 2/(1 + 4x^2)^{3/2}$

25. $\kappa(x) = |\sin x|/(1 + \cos^2 x)^{3/2}$

27. maximum curvature $2/(3\sqrt{3})$ at $x = 1/\sqrt{2}$

SECTION 13.5, p. 797

1. $\mathbf{a} = |\mathbf{a}|\mathbf{N}$ 3. $\mathbf{a}(1) = \frac{4}{3}\mathbf{T} + \frac{2\sqrt{5}}{3}\mathbf{N}$ 5. $\mathbf{a}(0) = 2\mathbf{N}$

7. $\mathbf{r}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k}$, $\mathbf{T}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$,

$$\mathbf{N}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$$
, $\mathbf{B}\left(\frac{\pi}{4}\right) = \mathbf{k}$; osculating plane:

$$z = -1$$
; normal plane: $-x + y = 0$; rectifying plane:

$$x + y = \sqrt{2}$$

9. $\mathbf{B} = \left(\frac{4}{5} \cos t\right)\mathbf{i} - \left(\frac{4}{5} \sin t\right)\mathbf{j} - \frac{3}{5}\mathbf{k}$, $\tau = -\frac{4}{25}$

11. $\mathbf{B} = \mathbf{k}$, $\tau = 0$ 13. $\mathbf{B} = -\mathbf{k}$, $\tau = 0$ 15. $\mathbf{B} = \mathbf{k}$, $\tau = 0$

17. Yes. If the car is moving on a curved path ($\kappa \neq 0$), then

$$a_N = \kappa|\mathbf{v}|^2 \neq 0 \text{ and } \mathbf{a} \neq \mathbf{0}.$$

23. $\kappa = \frac{1}{t}$, $\rho = t$

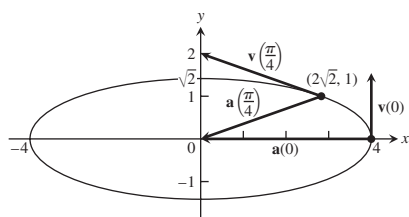
27. Components of $\mathbf{v}$: $-1.8701, 0.7089, 1.0000$
 Components of $\mathbf{a}$: $-1.6960, -2.0307, 0$
 Speed: 2.2361 ; Components of $\mathbf{T}$: $-0.8364, 0.3170, 0.4472$
 Components of $\mathbf{N}$: $-0.4143, -0.8998, -0.1369$
 Components of $\mathbf{B}$: $0.3590, -0.2998, 0.8839$; Curvature: 0.5060
 Torsion: 0.2813 ; Tangential component of acceleration: 0.7746
 Normal component of acceleration: 2.5298
29. Components of $\mathbf{v}$: $2.0000, 0, -0.1629$
 Components of $\mathbf{a}$: $0, -1.0000, -0.0086$; Speed: 2.0066
 Components of $\mathbf{T}$: $0.9967, 0, -0.0812$
 Components of $\mathbf{N}$: $-0.0007, -1.0000, -0.0086$
 Components of $\mathbf{B}$: $-0.0812, 0.0086, 0.9967$;
 Curvature: 0.2484
 Torsion: 0.0411 ; Tangential component of acceleration: 0.0007
 Normal component of acceleration: 1.0000

SECTION 13.6, p. 801

1. $\mathbf{v} = 2\mathbf{u}_r + 2\theta\mathbf{u}_\theta$
 $\mathbf{a} = -4\theta\mathbf{u}_r + 8\mathbf{u}_\theta$
3. $\mathbf{v} = (3a \sin \theta)\mathbf{u}_r + 3a(1 - \cos \theta)\mathbf{u}_\theta$
 $\mathbf{a} = 9a(2 \cos \theta - 1)\mathbf{u}_r + (18a \sin \theta)\mathbf{u}_\theta$
5. $\mathbf{v} = 2ae^{a\theta}\mathbf{u}_r + 2e^{a\theta}\mathbf{u}_\theta$
 $\mathbf{a} = 4e^{a\theta}(a^2 - 1)\mathbf{u}_r + 8ae^{a\theta}\mathbf{u}_\theta$
7. $\mathbf{v} = (-8 \sin 4t)\mathbf{u}_r + (4 \cos 4t)\mathbf{u}_\theta$
 $\mathbf{a} = (-40 \cos 4t)\mathbf{u}_r - (32 \sin 4t)\mathbf{u}_\theta$
13. $\approx 29.93 \times 10^{10} \text{ m}$ 15. $\approx 2.25 \times 10^9 \text{ km}^2/\text{sec}$
17. $\approx 1.876 \times 10^{27} \text{ kg}$

PRACTICE EXERCISES, pp. 802–803

1. $\frac{x^2}{16} + \frac{y^2}{2} = 1$



At $t = 0$: $a_T = 0$, $a_N = 4$, $\kappa = 2$;

At $t = \frac{\pi}{4}$: $a_T = \frac{7}{3}$, $a_N = \frac{4\sqrt{2}}{3}$, $\kappa = \frac{4\sqrt{2}}{27}$

3. $|\mathbf{v}|_{\max} = 1$ 5. $\kappa = 1/5$ 7. $dy/dt = -x$; clockwise
11. Shot put is on the ground, about 66 ft 3 in. from the stopboard.
15. Length $= \frac{\pi}{4}\sqrt{1 + \frac{\pi^2}{16}} + \ln\left(\frac{\pi}{4} + \sqrt{1 + \frac{\pi^2}{16}}\right)$
17. $\mathbf{T}(0) = \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N}(0) = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$;
 $\mathbf{B}(0) = -\frac{1}{3\sqrt{2}}\mathbf{i} + \frac{1}{3\sqrt{2}}\mathbf{j} + \frac{4}{3\sqrt{2}}\mathbf{k}$; $\kappa = \frac{\sqrt{2}}{3}$; $\tau = \frac{1}{6}$
19. $\mathbf{T}(\ln 2) = \frac{1}{\sqrt{17}}\mathbf{i} + \frac{4}{\sqrt{17}}\mathbf{j}$; $\mathbf{N}(\ln 2) = -\frac{4}{\sqrt{17}}\mathbf{i} + \frac{1}{\sqrt{17}}\mathbf{j}$;
 $\mathbf{B}(\ln 2) = \mathbf{k}$; $\kappa = \frac{8}{17\sqrt{17}}$; $\tau = 0$
21. $\mathbf{a}(0) = 10\mathbf{T} + 6\mathbf{N}$
23. $\mathbf{T} = \left(\frac{1}{\sqrt{2}} \cos t\right)\mathbf{i} - (\sin t)\mathbf{j} + \left(\frac{1}{\sqrt{2}} \cos t\right)\mathbf{k}$;

$$\mathbf{N} = \left(-\frac{1}{\sqrt{2}} \sin t\right)\mathbf{i} - (\cos t)\mathbf{j} - \left(\frac{1}{\sqrt{2}} \sin t\right)\mathbf{k};$$

$$\mathbf{B} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{k}; \kappa = \frac{1}{\sqrt{2}}; \tau = 0$$

25. $\frac{\pi}{3}$ 27. $x = 1 + t$, $y = t$, $z = -t$ 31. $\kappa = \frac{1}{a}$

ADDITIONAL AND ADVANCED EXERCISES, pp. 804–805

1. (a) $\frac{d\theta}{dt}\bigg|_{\theta=2\pi} = 2\sqrt{\frac{\pi gb}{a^2 + b^2}}$
 (b) $\theta = \frac{gbt^2}{2(a^2 + b^2)}$, $z = \frac{gb^2t^2}{2(a^2 + b^2)}$
 (c) $\mathbf{v}(t) = \frac{gbt}{\sqrt{a^2 + b^2}}\mathbf{T}$;
 $\frac{d^2\mathbf{r}}{dt^2} = \frac{bg}{\sqrt{a^2 + b^2}}\mathbf{T} + a\left(\frac{bgt}{a^2 + b^2}\right)^2\mathbf{N}$

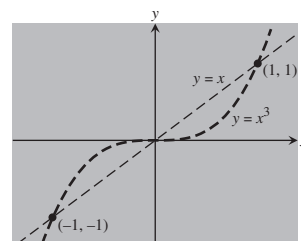
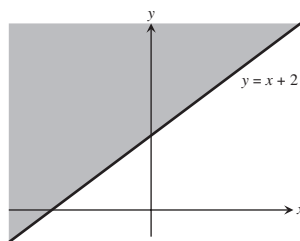
There is no component in the direction of $\mathbf{B}$.

5. (a) $\frac{dx}{dt} = \dot{r} \cos \theta - r\dot{\theta} \sin \theta$, $\frac{dy}{dt} = \dot{r} \sin \theta + r\dot{\theta} \cos \theta$
 (b) $\frac{dr}{dt} = \dot{x} \cos \theta + \dot{y} \sin \theta$, $r\frac{d\theta}{dt} = -\dot{x} \sin \theta + \dot{y} \cos \theta$
7. (a) $\mathbf{a}(1) = -9\mathbf{u}_r - 6\mathbf{u}_\theta$, $\mathbf{v}(1) = -\mathbf{u}_r + 3\mathbf{u}_\theta$
 (b) 6.5 in.
9. (c) $\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta + \dot{z}\mathbf{k}$, $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{k}$

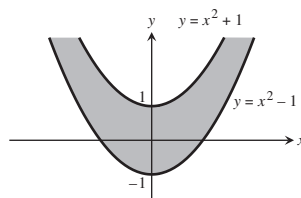
Chapter 14

SECTION 14.1, pp. 812–814

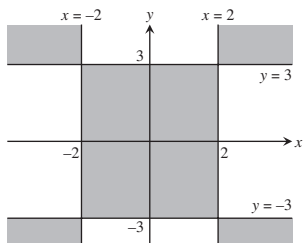
1. (a) 0 (b) 0 (c) 58 (d) 33
3. (a) $4/5$ (b) $8/5$ (c) 3 (d) 0
5. Domain: all points (x, y) on or above line $y = x + 2$
7. Domain: all points (x, y) not lying on the graph of $y = x$ or $y = x^3$



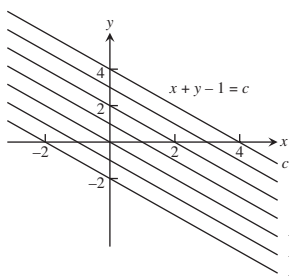
9. Domain: all points (x, y) satisfying $x^2 - 1 \leq y \leq x^2 + 1$



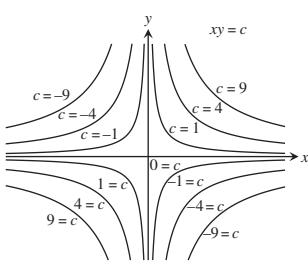
11. Domain: all points (x, y) for which $(x - 2)(x + 2)(y - 3)(y + 3) \geq 0$



13.

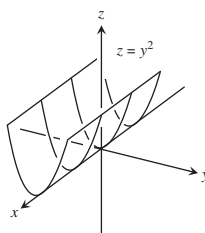


15.

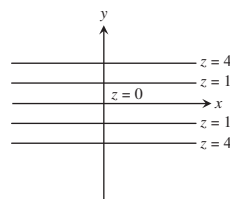


17. (a) All points in the xy -plane (b) All reals
(c) The lines $y - x = c$ (d) No boundary points
(e) Both open and closed (f) Unbounded
19. (a) All points in the xy -plane (b) $z \geq 0$
(c) For $f(x, y) = 0$, the origin; for $f(x, y) \neq 0$, ellipses with the center $(0, 0)$, and major and minor axes along the x - and y -axes, respectively
(d) No boundary points (e) Both open and closed
(f) Unbounded
21. (a) All points in the xy -plane (b) All reals
(c) For $f(x, y) = 0$, the x - and y -axes; for $f(x, y) \neq 0$, hyperbolas with the x - and y -axes as asymptotes
(d) No boundary points (e) Both open and closed
(f) Unbounded
23. (a) All (x, y) satisfying $x^2 + y^2 < 16$ (b) $z \geq 1/4$
(c) Circles centered at the origin with radii $r < 4$
(d) Boundary is the circle $x^2 + y^2 = 16$
(e) Open (f) Bounded
25. (a) $(x, y) \neq (0, 0)$ (b) All reals
(c) The circles with center $(0, 0)$ and radii $r > 0$
(d) Boundary is the single point $(0, 0)$
(e) Open (f) Unbounded
27. (a) All (x, y) satisfying $-1 \leq y - x \leq 1$
(b) $-\pi/2 \leq z \leq \pi/2$
(c) Straight lines of the form $y - x = c$ where $-1 \leq c \leq 1$
(d) Boundary is two straight lines $y = 1 + x$ and $y = -1 + x$
(e) Closed (f) Unbounded
29. (a) Domain: all points (x, y) outside the circle $x^2 + y^2 = 1$
(b) Range: all reals
(c) Circles centered at the origin with radii $r > 1$
(d) Boundary: $x^2 + y^2 = 1$
(e) Open (f) Unbounded
31. (f), (h) 33. (a), (i) 35. (d), (j)

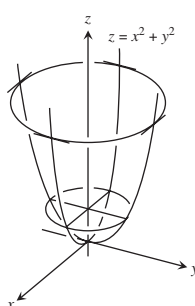
37. (a)



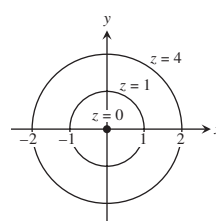
(b)



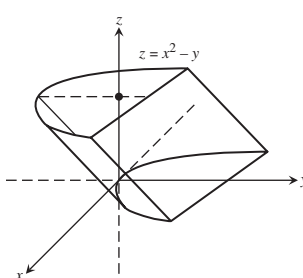
39. (a)



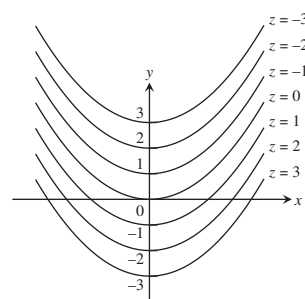
(b)



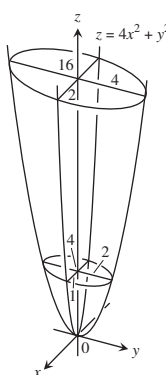
41. (a)



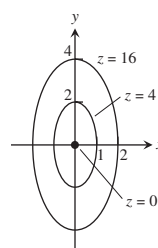
(b)



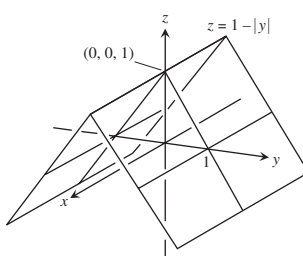
43. (a)



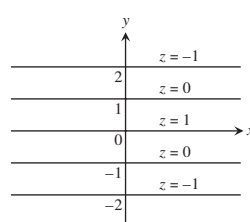
(b)



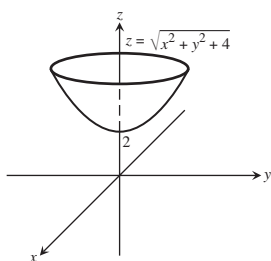
45. (a)



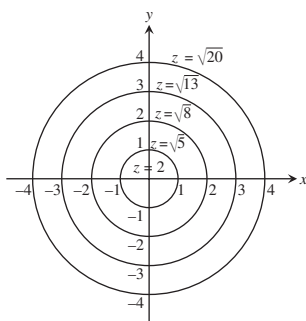
(b)



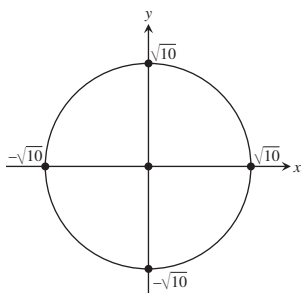
47. (a)



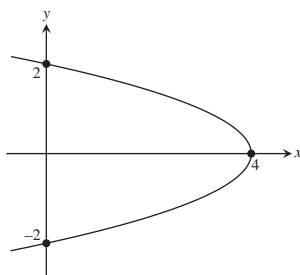
(b)



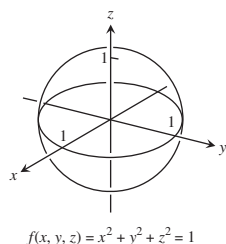
49. $x^2 + y^2 = 10$



51. $x + y^2 = 4$

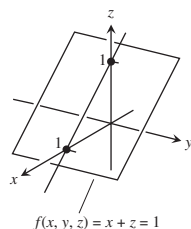


53.



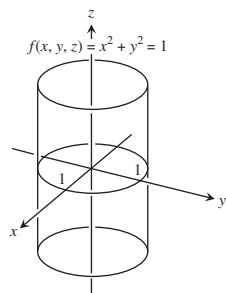
$f(x, y, z) = x^2 + y^2 + z^2 = 1$

55.



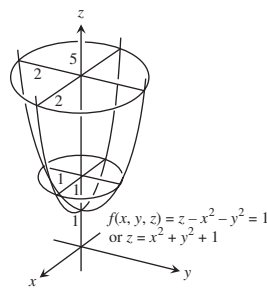
$f(x, y, z) = x + z = 1$

57.



$f(x, y, z) = x^2 + y^2 = 1$

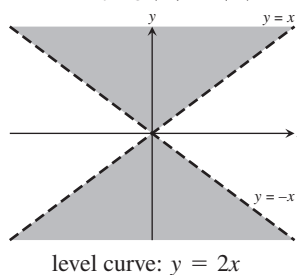
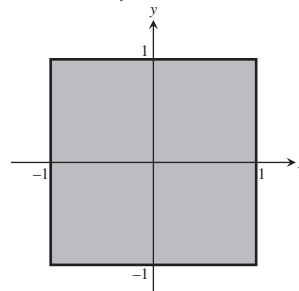
59.



$f(x, y, z) = z - x^2 - y^2 = 1$
or $z = x^2 + y^2 + 1$

61. $\sqrt{x - y} - \ln z = 2$

63. $x^2 + y^2 + z^2 = 4$

65. Domain: all points (x, y) satisfying $|x| < |y|$ level curve: $y = 2x$ 67. Domain: all points (x, y) satisfying $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ level curve:
 $\sin^{-1} y - \sin^{-1} x = \frac{\pi}{2}$

SECTION 14.2, pp. 820–823

1. $5/2$ 3. $2\sqrt{6}$ 5. 1 7. $1/2$ 9. 1
 11. $1/4$ 13. 0 15. -1 17. 2 19. $1/4$
 21. 1 23. 3 25. $19/12$ 27. 2 29. 3
 31. (a) All (x, y) (b) All (x, y) except $(0, 0)$
 33. (a) All (x, y) except where $x = 0$ or $y = 0$ (b) All (x, y)
 35. (a) All (x, y, z)
 (b) All (x, y, z) except the interior of the cylinder $x^2 + y^2 = 1$
 37. (a) All (x, y, z) with $z \neq 0$ (b) All (x, y, z) with $x^2 + z^2 \neq 1$
 39. (a) All points (x, y, z) satisfying $z > x^2 + y^2 + 1$
 (b) All points (x, y, z) satisfying $z \neq \sqrt{x^2 + y^2}$
 41. Consider paths along $y = x, x > 0$, and along $y = x, x < 0$.
 43. Consider the paths $y = kx^2, k$ a constant.
 45. Consider the paths $y = mx, m$ a constant, $m \neq -1$.
 47. Consider the paths $y = kx^2, k$ a constant, $k \neq 0$.
 49. Consider the paths $x = 1$ and $y = x$.
 51. Along $y = 1$ the limit is 0; along $y = e^x$ the limit is $1/2$.
 53. Along $y = 0$ the limit is 1; along $y = -\sin x$ the limit is 0.
 55. (a) 1 (b) 0 (c) Does not exist
 59. The limit is 1. 61. The limit is 0.
 63. (a) $f(x, y)|_{y=mx} = \sin 2\theta$ where $\tan \theta = m$ 65. 0
 67. Does not exist 69. $\pi/2$ 71. $f(0, 0) = \ln 3$
 73. $\delta = 0.1$ 75. $\delta = 0.005$ 77. $\delta = 0.04$
 79. $\delta = \sqrt{0.015}$ 81. $\delta = 0.005$

SECTION 14.3, pp. 833–835

1. $\frac{\partial f}{\partial x} = 4x, \frac{\partial f}{\partial y} = -3$ 3. $\frac{\partial f}{\partial x} = 2x(y + 2), \frac{\partial f}{\partial y} = x^2 - 1$
 5. $\frac{\partial f}{\partial x} = 2y(xy - 1), \frac{\partial f}{\partial y} = 2x(xy - 1)$

7. $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}}, \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}$
9. $\frac{\partial f}{\partial x} = \frac{-1}{(x + y)^2}, \frac{\partial f}{\partial y} = \frac{-1}{(x + y)^2}$
11. $\frac{\partial f}{\partial x} = \frac{-y^2 - 1}{(xy - 1)^2}, \frac{\partial f}{\partial y} = \frac{-x^2 - 1}{(xy - 1)^2}$
13. $\frac{\partial f}{\partial x} = e^{x+y+1}, \frac{\partial f}{\partial y} = e^{x+y+1}$ 15. $\frac{\partial f}{\partial x} = \frac{1}{x + y}, \frac{\partial f}{\partial y} = \frac{1}{x + y}$
17. $\frac{\partial f}{\partial x} = 2 \sin(x - 3y) \cos(x - 3y),$
 $\frac{\partial f}{\partial y} = -6 \sin(x - 3y) \cos(x - 3y)$
19. $\frac{\partial f}{\partial x} = yx^{y-1}, \frac{\partial f}{\partial y} = x^y \ln x$ 21. $\frac{\partial f}{\partial x} = -g(x), \frac{\partial f}{\partial y} = g(y)$
23. $f_x = y^2, f_y = 2xy, f_z = -4z$
25. $f_x = 1, f_y = -y(y^2 + z^2)^{-1/2}, f_z = -z(y^2 + z^2)^{-1/2}$
27. $f_x = \frac{yz}{\sqrt{1 - x^2y^2z^2}}, f_y = \frac{xz}{\sqrt{1 - x^2y^2z^2}}, f_z = \frac{xy}{\sqrt{1 - x^2y^2z^2}}$
29. $f_x = \frac{1}{x + 2y + 3z}, f_y = \frac{2}{x + 2y + 3z}, f_z = \frac{3}{x + 2y + 3z}$
31. $f_x = -2xe^{-(x^2+y^2+z^2)}, f_y = -2ye^{-(x^2+y^2+z^2)}, f_z = -2ze^{-(x^2+y^2+z^2)}$
33. $f_x = \operatorname{sech}^2(x + 2y + 3z), f_y = 2 \operatorname{sech}^2(x + 2y + 3z),$
 $f_z = 3 \operatorname{sech}^2(x + 2y + 3z)$
35. $\frac{\partial f}{\partial t} = -2\pi \sin(2\pi t - \alpha), \frac{\partial f}{\partial \alpha} = \sin(2\pi t - \alpha)$
37. $\frac{\partial h}{\partial \rho} = \sin \phi \cos \theta, \frac{\partial h}{\partial \phi} = \rho \cos \phi \cos \theta, \frac{\partial h}{\partial \theta} = -\rho \sin \phi \sin \theta$
39. $W_P(P, V, \delta, v, g) = V, W_V(P, V, \delta, v, g) = P + \frac{\delta v^2}{2g},$
 $W_\delta(P, V, \delta, v, g) = \frac{Vv^2}{2g}, W_v(P, V, \delta, v, g) = \frac{V\delta v}{g},$
 $W_g(P, V, \delta, v, g) = -\frac{V\delta v^2}{2g^2}$
41. $\frac{\partial f}{\partial x} = 1 + y, \frac{\partial f}{\partial y} = 1 + x, \frac{\partial^2 f}{\partial x^2} = 0, \frac{\partial^2 f}{\partial y^2} = 0, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 1$
43. $\frac{\partial g}{\partial x} = 2xy + y \cos x, \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x,$
 $\frac{\partial^2 g}{\partial x^2} = 2y - y \sin x, \frac{\partial^2 g}{\partial y^2} = -\cos y,$
 $\frac{\partial^2 g}{\partial y \partial x} = \frac{\partial^2 g}{\partial x \partial y} = 2x + \cos x$
45. $\frac{\partial r}{\partial x} = \frac{1}{x + y}, \frac{\partial r}{\partial y} = \frac{1}{x + y}, \frac{\partial^2 r}{\partial x^2} = \frac{-1}{(x + y)^2}, \frac{\partial^2 r}{\partial y^2} = \frac{-1}{(x + y)^2},$
 $\frac{\partial^2 r}{\partial y \partial x} = \frac{\partial^2 r}{\partial x \partial y} = \frac{-1}{(x + y)^2}$
47. $\frac{\partial w}{\partial x} = x^2y \sec^2(xy) + 2x \tan(xy), \frac{\partial w}{\partial y} = x^3 \sec^2(xy),$
 $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = 2x^3y \sec^2(xy) \tan(xy) + 3x^2 \sec^2(xy)$
 $\frac{\partial^2 w}{\partial x^2} = 4xy \sec^2(xy) + 2x^2y^2 \sec^2(xy) \tan(xy) + 2 \tan(xy)$
 $\frac{\partial^2 w}{\partial y^2} = 2x^4 \sec^2(xy) \tan(xy)$
49. $\frac{\partial w}{\partial x} = \sin(x^2y) + 2x^2y \cos(x^2y), \frac{\partial w}{\partial y} = x^3 \cos(x^2y),$
 $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = 3x^2 \cos(x^2y) - 2x^4y \sin(x^2y)$
 $\frac{\partial^2 w}{\partial x^2} = 6xy \cos(x^2y) - 4x^3y^2 \sin(x^2y)$
 $\frac{\partial^2 w}{\partial y^2} = -x^5 \sin(x^2y)$
51. $\frac{\partial f}{\partial x} = 2xy^3 - 4x^3, \frac{\partial f}{\partial y} = 3x^2y^2 + 5y^4,$
 $\frac{\partial^2 f}{\partial x^2} = 2y^3 - 12x^2, \frac{\partial^2 f}{\partial y^2} = 6x^2y + 20y^3,$
 $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 6xy^2$
53. $\frac{\partial z}{\partial x} = 2x \cos(2x - y^2) + \sin(2x - y^2),$
 $\frac{\partial z}{\partial y} = -2xy \cos(2x - y^2),$
 $\frac{\partial^2 z}{\partial x^2} = 4 \cos(2x - y^2) - 4x \sin(2x - y^2),$
 $\frac{\partial^2 z}{\partial y^2} = -4xy^2 \sin(2x - y^2) - 2x \cos(2x - y^2),$
 $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 4xy \sin(2x - y^2) - 2y \cos(2x - y^2)$
55. $\frac{\partial w}{\partial x} = \frac{2}{2x + 3y}, \frac{\partial w}{\partial y} = \frac{3}{2x + 3y}, \frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = \frac{-6}{(2x + 3y)^2}$
57. $\frac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4, \frac{\partial w}{\partial y} = 2xy + 3x^2y^2 + 4x^3y^3,$
 $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = 2y + 6xy^2 + 12x^2y^3$
59. $\frac{\partial \omega}{\partial x} = \frac{2x}{y^3}, \frac{\partial \omega}{\partial y} = \frac{-3x^2}{y^4}$
 $\frac{\partial^2 \omega}{\partial y \partial x} = \frac{-6x}{y^4}, \frac{\partial^2 \omega}{\partial x \partial y} = \frac{-6x}{y^4}$
61. (a) x first (b) y first (c) x first
 (d) x first (e) y first (f) y first
63. $f_x(1, 2) = -13, f_y(1, 2) = -2$
65. $f_x(-2, 3) = 1/2, f_y(-2, 3) = 3/4$
67. (a) 3 (b) 2 69. 12

$$71. \frac{\partial f}{\partial x} = 3x^2y^2 - 2x \Rightarrow$$

$$f(x, y) = x^3y^2 - x^2 + g(y) \Rightarrow$$

$$\frac{\partial f}{\partial y} = 2x^3y + g'(y) = 2x^3y + 64 \Rightarrow$$

$$g'(y) = 64 \Rightarrow g(y) = 64y \text{ works} \Rightarrow$$

$$f(x, y) = x^3y^2 - x^2 + 64y \text{ works}$$

$$73. \frac{\partial^2 f}{\partial y \partial x} = \frac{2x - 2y}{(x + y)^3} \neq \frac{\partial^2 f}{\partial x \partial y} = \frac{2y - 2x}{(x + y)^3} \text{ so impossible} \quad 75. -2$$

$$77. \frac{\partial A}{\partial a} = \frac{a}{bc \sin A}, \frac{\partial A}{\partial b} = \frac{c \cos A - b}{bc \sin A}$$

$$79. v_x = \frac{\ln v}{(\ln u)(\ln v) - 1}$$

$$81. f_x(x, y) = 0 \text{ for all points } (x, y),$$

$$f_y(x, y) = \begin{cases} 3y^2, & y \geq 0 \\ -2y, & y < 0 \end{cases}$$

$$f_{xy}(x, y) = f_{yx}(x, y) = 0 \text{ for all points } (x, y)$$

99. Yes

SECTION 14.4, pp. 842-844

$$1. (a) \frac{dw}{dt} = 0, \quad (b) \frac{dw}{dt}(\pi) = 0$$

$$3. (a) \frac{dw}{dt} = 1, \quad (b) \frac{dw}{dt}(3) = 1$$

$$5. (a) \frac{dw}{dt} = 4t \tan^{-1} t + 1, \quad (b) \frac{dw}{dt}(1) = \pi + 1$$

$$7. (a) \frac{\partial z}{\partial u} = 4 \cos v \ln(u \sin v) + 4 \cos v,$$

$$\frac{\partial z}{\partial v} = -4u \sin v \ln(u \sin v) + \frac{4u \cos^2 v}{\sin v}$$

$$(b) \frac{\partial z}{\partial u} = \sqrt{2}(\ln 2 + 2), \frac{\partial z}{\partial v} = -2\sqrt{2}(\ln 2 - 2)$$

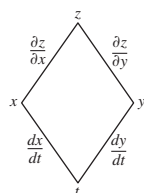
$$9. (a) \frac{\partial w}{\partial u} = 2u + 4uv, \frac{\partial w}{\partial v} = -2v + 2u^2$$

$$(b) \frac{\partial w}{\partial u} = 3, \frac{\partial w}{\partial v} = -\frac{3}{2}$$

$$11. (a) \frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = \frac{z}{(z - y)^2}, \frac{\partial u}{\partial z} = \frac{-y}{(z - y)^2}$$

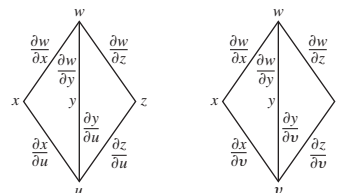
$$(b) \frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = 1, \frac{\partial u}{\partial z} = -2$$

$$13. \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

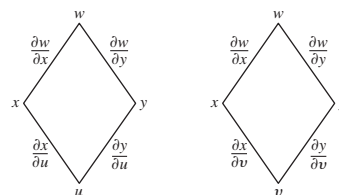


$$15. \frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u},$$

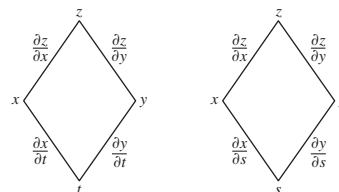
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial v} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial v}$$



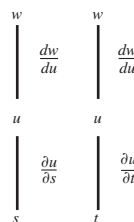
$$17. \frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u}, \frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial v}.$$



$$19. \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}, \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}$$

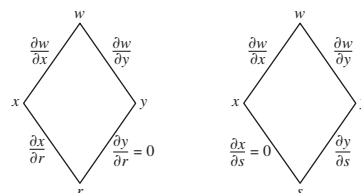


$$21. \frac{\partial w}{\partial s} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial s}, \frac{\partial w}{\partial t} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial t}$$



$$23. \frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} \text{ since } \frac{\partial y}{\partial r} = 0,$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} = \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} \text{ since } \frac{\partial x}{\partial s} = 0$$



25. $4/3$ 27. $-4/5$ 29. 20 31. $\frac{\partial z}{\partial x} = \frac{1}{4}, \frac{\partial z}{\partial y} = -\frac{3}{4}$

33. $\frac{\partial z}{\partial x} = -1, \frac{\partial z}{\partial y} = -1$ 35. 12 37. -7

39. $\frac{\partial z}{\partial u} = 2, \frac{\partial z}{\partial v} = 1$ 41. $\frac{\partial w}{\partial t} = 2t e^{s^3+t^2}, \frac{\partial w}{\partial s} = 3s^2 e^{s^3+t^2}$

43. 23 45. -16, 2 47. -0.00005 amp/sec

53. $(\cos 1, \sin 1, 1)$ and $(\cos(-2), \sin(-2), -2)$

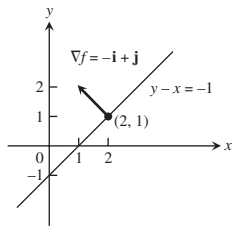
55. (a) Maximum at $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$; minimum at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

(b) Max = 6, min = 2

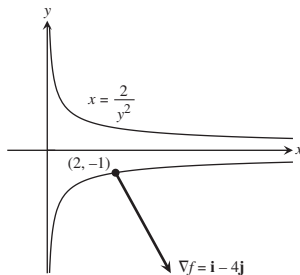
57. 5°C/sec 59. $2x\sqrt{x^8+x^3} + \int_0^{x^2} \frac{3x^2}{2\sqrt{t^4+x^3}} dt$

SECTION 14.5, pp. 852-853

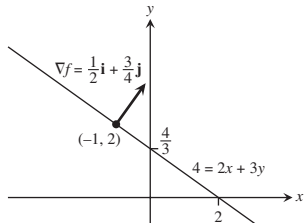
1.



3.



5.



7. $\nabla f = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ 9. $\nabla f = -\frac{26}{27}\mathbf{i} + \frac{23}{54}\mathbf{j} - \frac{23}{54}\mathbf{k}$

11. -4 13. $21/13$ 15. 3 17. 2

19. $\mathbf{u} = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$, $(D_{\mathbf{u}}f)_{P_0} = \sqrt{2}$; $-\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$, $(D_{-\mathbf{u}}f)_{P_0} = -\sqrt{2}$

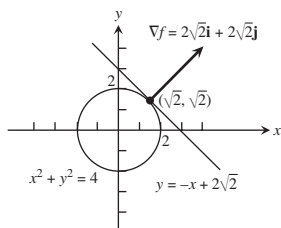
21. $\mathbf{u} = \frac{1}{3\sqrt{3}}\mathbf{i} - \frac{5}{3\sqrt{3}}\mathbf{j} - \frac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{\mathbf{u}}f)_{P_0} = 3\sqrt{3}$;

$-\mathbf{u} = -\frac{1}{3\sqrt{3}}\mathbf{i} + \frac{5}{3\sqrt{3}}\mathbf{j} + \frac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{-\mathbf{u}}f)_{P_0} = -3\sqrt{3}$

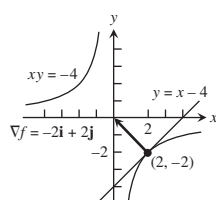
23. $\mathbf{u} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$, $(D_{\mathbf{u}}f)_{P_0} = 2\sqrt{3}$;

$-\mathbf{u} = -\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$, $(D_{-\mathbf{u}}f)_{P_0} = -2\sqrt{3}$

25.



27.



29. (a) $\mathbf{u} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$, $D_{\mathbf{u}}f(1, -1) = 5$

(b) $\mathbf{u} = -\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$, $D_{\mathbf{u}}f(1, -1) = -5$

(c) $\mathbf{u} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$, $\mathbf{u} = -\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$

(d) $\mathbf{u} = -\mathbf{j}$, $\mathbf{u} = \frac{24}{25}\mathbf{i} - \frac{7}{25}\mathbf{j}$

(e) $\mathbf{u} = -\mathbf{i}$, $\mathbf{u} = \frac{7}{25}\mathbf{i} + \frac{24}{25}\mathbf{j}$

31. $\mathbf{u} = \frac{7}{\sqrt{53}}\mathbf{i} - \frac{2}{\sqrt{53}}\mathbf{j}$, $-\mathbf{u} = -\frac{7}{\sqrt{53}}\mathbf{i} + \frac{2}{\sqrt{53}}\mathbf{j}$

33. No, the maximum rate of change is $\sqrt{185} < 14$.

35. $-7/\sqrt{5}$ 41. $r(t) = (-3 - 6t)\mathbf{i} + (4 + 8t)\mathbf{j}$, $-\infty < t < \infty$

43. $r(t) = (3 + 6t)\mathbf{i} + (-2 - 4t)\mathbf{j} + (1 + 2t)\mathbf{k}$, $-\infty < t < \infty$

SECTION 14.6, pp. 860-863

1. (a) $x + y + z = 3$

(b) $x = 1 + 2t$, $y = 1 + 2t$, $z = 1 + 2t$

3. (a) $2x - z - 2 = 0$

(b) $x = 2 - 4t$, $y = 0$, $z = 2 + 2t$

5. (a) $2x + 2y + z - 4 = 0$

(b) $x = 2t$, $y = 1 + 2t$, $z = 2 + t$

7. (a) $x + y + z - 1 = 0$

(b) $x = t$, $y = 1 + t$, $z = t$

9. (a) $-x + 3y + z/e = 2$

(b) $x = 2 - t$, $y = 1 + 3t$, $z = e + (1/e)t$

11. $2x - z - 2 = 0$

13. $x - y + 2z - 1 = 0$

15. $x = 1$, $y = 1 + 2t$, $z = 1 - 2t$

17. $x = 1 - 2t$, $y = 1$, $z = \frac{1}{2} + 2t$

19. $x = 1 + 90t$, $y = 1 - 90t$, $z = 3$

21. $df = \frac{9}{11,830} \approx 0.0008$ 23. $dg = 0$

25. (a) $\frac{\sqrt{3}}{2} \sin \sqrt{3} - \frac{1}{2} \cos \sqrt{3} \approx 0.935^\circ\text{C/ft}$

(b) $\sqrt{3} \sin \sqrt{3} - \cos \sqrt{3} \approx 1.87^\circ\text{C/sec}$

27. (a) $L(x, y) = 1$ (b) $L(x, y) = 2x + 2y - 1$

29. (a) $L(x, y) = 3x - 4y + 5$ (b) $L(x, y) = 3x - 4y + 5$

31. (a) $L(x, y) = 1 + x$ (b) $L(x, y) = -y + \frac{\pi}{2}$

33. (a) $W(20, 25) = 11^\circ\text{F}$, $W(30, -10) = -39^\circ\text{F}$, $W(15, 15) = 0^\circ\text{F}$

(b) $W(10, -40) \approx -65.5^\circ\text{F}$, $W(50, -40) \approx -88^\circ\text{F}$,

$W(60, 30) \approx 10.2^\circ\text{F}$

(c) $L(v, T) \approx -0.36(v - 25) + 1.337(T - 5) - 17.4088$

(d) i) $L(24, 6) \approx -15.7^\circ\text{F}$

ii) $L(27, 2) \approx -22.1^\circ\text{F}$

iii) $L(5, -10) \approx -30.2^\circ\text{F}$

35. $L(x, y) = 7 + x - 6y$; 0.06 37. $L(x, y) = x + y + 1$; 0.08

39. $L(x, y) = 1 + x$; 0.0222

41. (a) $L(x, y, z) = 2x + 2y + 2z - 3$ (b) $L(x, y, z) = y + z$

(c) $L(x, y, z) = 0$

43. (a) $L(x, y, z) = x$

(b) $L(x, y, z) = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$

(c) $L(x, y, z) = \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z$

45. (a) $L(x, y, z) = 2 + x$
 (b) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$
 (c) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$
 47. $L(x, y, z) = 2x - 6y - 2z + 6, 0.0024$
 49. $L(x, y, z) = x + y - z - 1, 0.00135$
 51. Maximum error (estimate) ≤ 0.31 in magnitude
 53. Pay more attention to the smaller of the two dimensions. It will generate the larger partial derivative.
 55. f is most sensitive to a change in d .
 61. (a) 1.75% (b) 1.75%

SECTION 14.7, pp. 870–872

1. $f(-3, 3) = -5$, local minimum 3. $f(-2, 1)$, saddle point
 5. $f\left(3, \frac{3}{2}\right) = \frac{17}{2}$, local maximum
 7. $f(2, -1) = -6$, local minimum 9. $f(1, 2)$, saddle point
 11. $f\left(\frac{16}{7}, 0\right) = -\frac{16}{7}$, local maximum
 13. $f(0, 0)$, saddle point; $f\left(-\frac{2}{3}, \frac{2}{3}\right) = \frac{170}{27}$, local maximum
 15. $f(0, 0) = 0$, local minimum; $f(1, -1)$, saddle point
 17. $f(0, \pm\sqrt{5})$, saddle points; $f(-2, -1) = 30$, local maximum; $f(2, 1) = -30$, local minimum
 19. $f(0, 0)$, saddle point; $f(1, 1) = 2$, $f(-1, -1) = 2$, local maxima
 21. $f(0, 0) = -1$, local maximum
 23. $f(n\pi, 0)$, saddle points, for every integer n
 25. $f(2, 0) = e^{-4}$, local minimum
 27. $f(0, 0) = 0$, local minimum; $f(0, 2)$, saddle point
 29. $f\left(\frac{1}{2}, 1\right) = \ln\left(\frac{1}{4}\right) - 3$, local maximum
 31. Absolute maximum: 1 at $(0, 0)$; absolute minimum: -5 at $(1, 2)$
 33. Absolute maximum: 4 at $(0, 2)$; absolute minimum: 0 at $(0, 0)$
 35. Absolute maximum: 11 at $(0, -3)$; absolute minimum: -10 at $(4, -2)$
 37. Absolute maximum: 4 at $(2, 0)$; absolute minimum: $\frac{3\sqrt{2}}{2}$ at $\left(3, -\frac{\pi}{4}\right)$, $\left(3, \frac{\pi}{4}\right)$, $\left(1, -\frac{\pi}{4}\right)$, and $\left(1, \frac{\pi}{4}\right)$
 39. $a = -3$, $b = 2$
 41. Hottest is $2\frac{1}{4}^\circ$ at $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$; coldest is $-\frac{1}{4}^\circ$ at $\left(\frac{1}{2}, 0\right)$.
 43. (a) $f(0, 0)$, saddle point (b) $f(1, 2)$, local minimum
 (c) $f(1, -2)$, local minimum; $f(-1, -2)$, saddle point
 49. $\left(\frac{1}{6}, \frac{1}{3}, \frac{355}{36}\right)$ 51. $\left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$ 53. 3, 3, 3 55. 12
 57. $\frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}}$ 59. 2 ft $\times$ 2 ft $\times$ 1 ft
 61. Points $(0, 2, 0)$ and $(0, -2, 0)$ have distance 2 from the origin.
 63. (a) On the semicircle, max $f = 2\sqrt{2}$ at $t = \pi/4$, min $f = -2$ at $t = \pi$. On the quarter circle, max $f = 2\sqrt{2}$ at $t = \pi/4$, min $f = 2$ at $t = 0, \pi/2$.

- (b) On the semicircle, max $g = 2$ at $t = \pi/4$, min $g = -2$ at $t = 3\pi/4$. On the quarter circle, max $g = 2$ at $t = \pi/4$, min $g = 0$ at $t = 0, \pi/2$.
 (c) On the semicircle, max $h = 8$ at $t = 0, \pi$; min $h = 4$ at $t = \pi/2$. On the quarter circle, max $h = 8$ at $t = 0$, min $h = 4$ at $t = \pi/2$.
 65. i) min $f = -1/2$ at $t = -1/2$; no max
 ii) max $f = 0$ at $t = -1, 0$; min $f = -1/2$ at $t = -1/2$
 iii) max $f = 4$ at $t = 1$; min $f = 0$ at $t = 0$
 69. $y = -\frac{20}{13}x + \frac{9}{13}$, $y|_{x=4} = -\frac{71}{13}$

SECTION 14.8, pp. 879–882

1. $\left(\pm\frac{1}{\sqrt{2}}, \frac{1}{2}\right), \left(\pm\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ 3. 39 5. $(3, \pm 3\sqrt{2})$
 7. (a) 8 (b) 64
 9. $r = 2$ cm, $h = 4$ cm
 11. Length $= 4\sqrt{2}$, width $= 3\sqrt{2}$
 13. $f(0, 0) = 0$ is minimum; $f(2, 4) = 20$ is maximum.
 15. Lowest $= 0^\circ$, highest $= 125^\circ$
 17. $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ 19. 1 21. $(0, 0, 2), (0, 0, -2)$
 23. $f(1, -2, 5) = 30$ is maximum; $f(-1, 2, -5) = -30$ is minimum.
 25. 3, 3, 3 27. $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ units
 29. $(\pm 4/3, -4/3, -4/3)$ 31. $\approx 24,322$ units
 33. $U(8, 14) = \$128$ 37. $f(2/3, 4/3, -4/3) = \frac{4}{3}$
 39. $(2, 4, 4)$ 41. Maximum is $1 + 6\sqrt{3}$ at $(\pm\sqrt{6}, \sqrt{3}, 1)$; minimum is $1 - 6\sqrt{3}$ at $(\pm\sqrt{6}, -\sqrt{3}, 1)$.
 43. Maximum is 4 at $(0, 0, \pm 2)$; minimum is 2 at $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$.

SECTION 14.9, p. 886

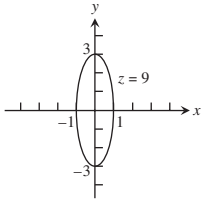
1. Quadratic: $x + xy$; cubic: $x + xy + \frac{1}{2}xy^2$
 3. Quadratic: xy ; cubic: xy
 5. Quadratic: $y + \frac{1}{2}(2xy - y^2)$;
 cubic: $y + \frac{1}{2}(2xy - y^2) + \frac{1}{6}(3x^2y - 3xy^2 + 2y^3)$
 7. Quadratic: $\frac{1}{2}(2x^2 + 2y^2) = x^2 + y^2$; cubic: $x^2 + y^2$
 9. Quadratic: $1 + (x + y) + (x + y)^2$;
 cubic: $1 + (x + y) + (x + y)^2 + (x + y)^3$
 11. Quadratic: $1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$; $E(x, y) \leq 0.00134$

SECTION 14.10, p. 890

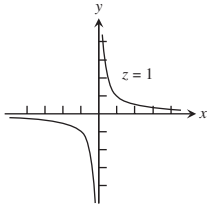
1. (a) 0 (b) $1 + 2z$ (c) $1 + 2z$
 3. (a) $\frac{\partial U}{\partial P} + \frac{\partial U}{\partial T}\left(\frac{V}{nR}\right)$ (b) $\frac{\partial U}{\partial P}\left(\frac{nR}{V}\right) + \frac{\partial U}{\partial T}$
 5. (a) 5 (b) 5 7. $\left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta$ $\left(\frac{\partial r}{\partial x}\right)_y = \frac{x}{\sqrt{x^2 + y^2}}$

PRACTICE EXERCISES, pp. 891–894

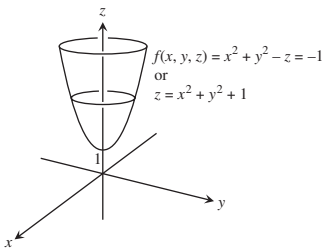
1. Domain: all points in the xy -plane; range: $z \geq 0$. Level curves are ellipses with major axis along the y -axis and minor axis along the x -axis.



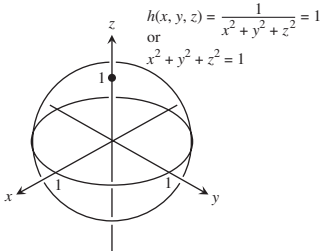
3. Domain: all (x, y) such that $x \neq 0$ and $y \neq 0$; range: $z \neq 0$. Level curves are hyperbolas with the x - and y -axes as asymptotes.



5. Domain: all points in xyz -space; range: all real numbers. Level surfaces are paraboloids of revolution with the z -axis as axis.



7. Domain: all (x, y, z) such that $(x, y, z) \neq (0, 0, 0)$; range: positive real numbers. Level surfaces are spheres with center $(0, 0, 0)$ and radius $r > 0$.



9. -2 11. $1/2$ 13. 1

15. Let $y = kx^2$, $k \neq 1$

17. No; $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist.

19. $\frac{\partial g}{\partial r} = \cos \theta + \sin \theta$, $\frac{\partial g}{\partial \theta} = -r \sin \theta + r \cos \theta$

21. $\frac{\partial f}{\partial R_1} = -\frac{1}{R_1^2}$, $\frac{\partial f}{\partial R_2} = -\frac{1}{R_2^2}$, $\frac{\partial f}{\partial R_3} = -\frac{1}{R_3^2}$

23. $\frac{\partial P}{\partial n} = \frac{RT}{V}$, $\frac{\partial P}{\partial R} = \frac{nT}{V}$, $\frac{\partial P}{\partial T} = \frac{nR}{V}$, $\frac{\partial P}{\partial V} = -\frac{nRT}{V^2}$

25. $\frac{\partial^2 g}{\partial x^2} = 0$, $\frac{\partial^2 g}{\partial y^2} = \frac{2x}{y^3}$, $\frac{\partial^2 g}{\partial y \partial x} = \frac{\partial^2 g}{\partial x \partial y} = -\frac{1}{y^2}$

27. $\frac{\partial^2 f}{\partial x^2} = -30x + \frac{2 - 2x^2}{(x^2 + 1)^2}$, $\frac{\partial^2 f}{\partial y^2} = 0$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 1$

29. $\left. \frac{dw}{dt} \right|_{t=0} = -1$

31. $\left. \frac{\partial w}{\partial r} \right|_{(r,s)=(\pi,0)} = 2$, $\left. \frac{\partial w}{\partial s} \right|_{(r,s)=(\pi,0)} = 2 - \pi$

33. $\left. \frac{df}{dt} \right|_{t=1} = -(\sin 1 + \cos 2)(\sin 1) + (\cos 1 + \cos 2)(\cos 1) - 2(\sin 1 + \cos 1)(\sin 2)$

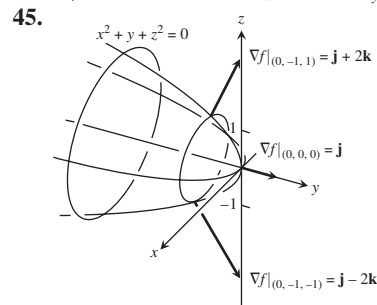
35. $\left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} = -1$

37. Increases most rapidly in the direction $\mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$;
decreases most rapidly in the direction $-\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$;

$D_{\mathbf{u}}f = \frac{\sqrt{2}}{2}$; $D_{-\mathbf{u}}f = -\frac{\sqrt{2}}{2}$; $D_{\mathbf{u}_1}f = -\frac{7}{10}$ where $\mathbf{u}_1 = \frac{\mathbf{v}}{|\mathbf{v}|}$

39. Increases most rapidly in the direction $\mathbf{u} = \frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$;
decreases most rapidly in the direction $-\mathbf{u} = -\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$;
 $D_{\mathbf{u}}f = 7$; $D_{-\mathbf{u}}f = -7$; $D_{\mathbf{u}_1}f = 7$ where $\mathbf{u}_1 = \frac{\mathbf{v}}{|\mathbf{v}|}$

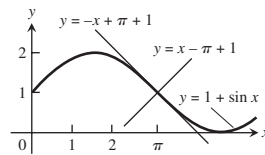
41. $\pi/\sqrt{2}$ 43. (a) $f_x(1, 2) = f_y(1, 2) = 2$ (b) $14/5$



47. Tangent: $4x - y - 5z = 4$; normal line: $x = 2 + 4t$, $y = -1 - t$, $z = 1 - 5t$

49. $2y - z - 2 = 0$

51. Tangent: $x + y = \pi + 1$; normal line: $y = x - \pi + 1$



53. $x = 1 - 2t$, $y = 1$, $z = 1/2 + 2t$

55. Answers will depend on the upper bound used for $|f_{xx}|$, $|f_{xy}|$, $|f_{yy}|$. With $M = \sqrt{2}/2$, $|E| \leq 0.0142$. With $M = 1$, $|E| \leq 0.02$.

57. $L(x, y, z) = y - 3z$, $L(x, y, z) = x + y - z - 1$

59. Be more careful with the diameter.

61. $dI = 0.038$, % change in $I = 15.83\%$, more sensitive to voltage change

63. (a) 5% 65. Local minimum of -8 at $(-2, -2)$

67. Saddle point at $(0, 0)$, $f(0, 0) = 0$; local maximum of $1/4$ at $(-1/2, -1/2)$

69. Saddle point at $(0, 0)$, $f(0, 0) = 0$; local minimum of -4 at $(0, 2)$; local maximum of 4 at $(-2, 0)$; saddle point at $(-2, 2)$, $f(-2, 2) = 0$

71. Absolute maximum: 28 at (0, 4); absolute minimum: $-9/4$ at $(3/2, 0)$
 73. Absolute maximum: 18 at (2, -2); absolute minimum: $-17/4$ at $(-2, 1/2)$
 75. Absolute maximum: 8 at (-2, 0); absolute minimum: -1 at (1, 0)
 77. Absolute maximum: 4 at (1, 0); absolute minimum: -4 at (0, -1)
 79. Absolute maximum: 1 at (0, ± 1) and (1, 0); absolute minimum: -1 at (-1, 0)
 81. Maximum: 5 at (0, 1); minimum: $-1/3$ at (0, $-1/3$)
 83. Maximum: $\sqrt{3}$ at $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; minimum: $-\sqrt{3}$ at $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$
 85. Width = $\left(\frac{c^2V}{ab}\right)^{1/3}$, depth = $\left(\frac{b^2V}{ac}\right)^{1/3}$, height = $\left(\frac{a^2V}{bc}\right)^{1/3}$
 87. Maximum: $\frac{3}{2}$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{2}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\sqrt{2}\right)$; minimum: $\frac{1}{2}$ at $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \sqrt{2}\right)$
 89. $\frac{\partial w}{\partial x} = \cos \theta \frac{\partial w}{\partial r} - \frac{\sin \theta}{r} \frac{\partial w}{\partial \theta}$, $\frac{\partial w}{\partial y} = \sin \theta \frac{\partial w}{\partial r} + \frac{\cos \theta}{r} \frac{\partial w}{\partial \theta}$
 95. $(t, -t \pm 4, t)$, t a real number
 101. (a) $(2y + x^2z)e^{yz}$ (b) $x^2e^{yz}\left(y - \frac{z}{2y}\right)$ (c) $(1 + x^2y)e^{yz}$

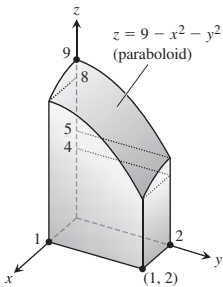
ADDITIONAL AND ADVANCED EXERCISES, pp. 894–896

1. $f_{xy}(0, 0) = -1$, $f_{yx}(0, 0) = 1$
 7. (c) $\frac{r^2}{2} = \frac{1}{2}(x^2 + y^2 + z^2)$ 13. $V = \frac{\sqrt{3}abc}{2}$
 17. $f(x, y) = \frac{y}{2} + 4$, $g(x, y) = \frac{x}{2} + \frac{9}{2}$
 19. $y = 2 \ln |\sin x| + \ln 2$
 21. (a) $\frac{1}{\sqrt{53}}(2\mathbf{i} + 7\mathbf{j})$ (b) $\frac{-1}{\sqrt{29,097}}(98\mathbf{i} - 127\mathbf{j} + 58\mathbf{k})$
 23. $w = e^{-c^2\pi^2t} \sin \pi x$

Chapter 15

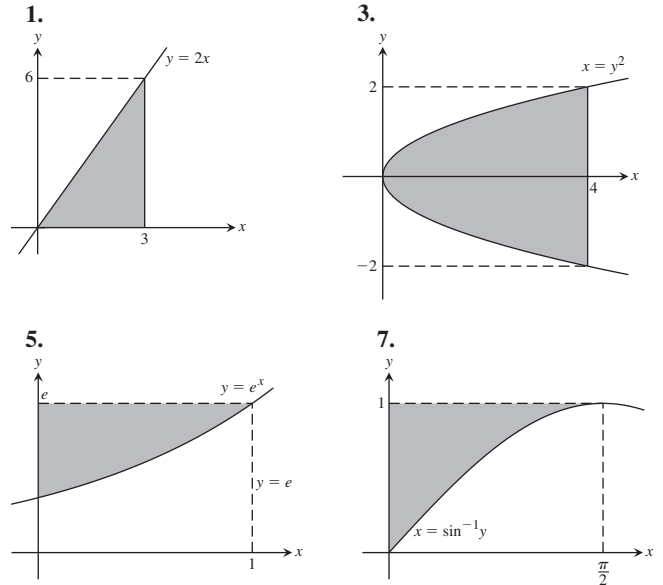
SECTION 15.1, pp. 901–902

1. 24 3.1 5. 16 7. $2 \ln 2 - 1$ 9. $(3/2)(5 - e)$
 11. $3/2$ 13. $\ln 2$ 15. $3/2, -2$ 17. 14 19. 0
 21. $1/2$ 23. $2 \ln 2$ 25. $(\ln 2)^2$
 27.



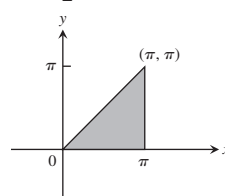
29. $8/3$ 31. 1 33. $\sqrt{2}$ 35. $2/27$
 37. $\frac{3}{2} \ln 3 - 1$ 39. (a) $1/3$ (b) $2/3$

SECTION 15.2, pp. 909–911

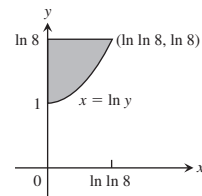


9. (a) $0 \leq x \leq 2, x^3 \leq y \leq 8$
 (b) $0 \leq y \leq 8, 0 \leq x \leq y^{1/3}$
 11. (a) $0 \leq x \leq 3, x^2 \leq y \leq 3x$
 (b) $0 \leq y \leq 9, \frac{y}{3} \leq x \leq \sqrt{y}$
 13. (a) $0 \leq x \leq 9, 0 \leq y \leq \sqrt{x}$
 (b) $0 \leq y \leq 3, y^2 \leq x \leq 9$
 15. (a) $0 \leq x \leq \ln 3, e^{-x} \leq y \leq 1$
 (b) $\frac{1}{3} \leq y \leq 1, -\ln y \leq x \leq \ln 3$
 17. (a) $0 \leq x \leq 1, x \leq y \leq 3 - 2x$
 (b) $0 \leq y \leq 1, 0 \leq x \leq y \cup 1 \leq y \leq 3, 0 \leq x \leq \frac{3-y}{2}$

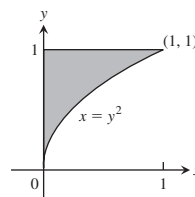
19. $\frac{\pi^2}{2} + 2$



21. $8 \ln 8 - 16 + e$

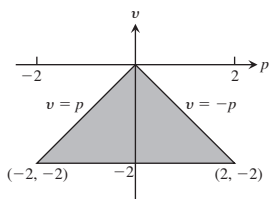


23. $e - 2$

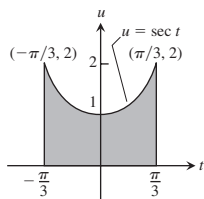


25. $\frac{3}{2} \ln 2$ 27. $-1/10$

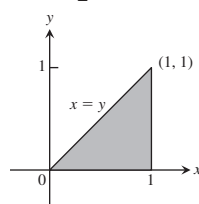
29. 8



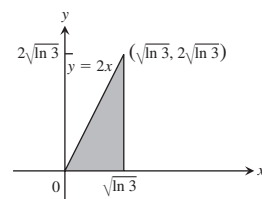
31. 2π



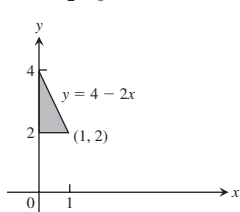
49. $\frac{e-2}{2}$



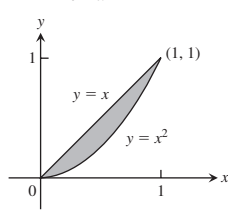
51. 2



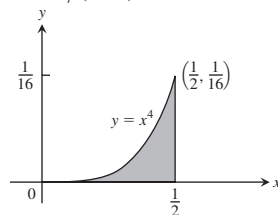
33. $\int_2^4 \int_0^{(4-y)/2} dx dy$



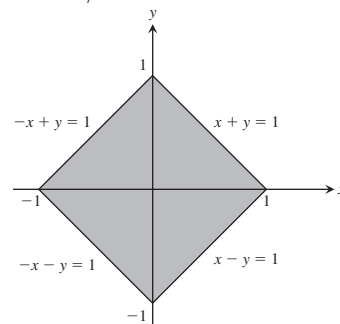
35. $\int_0^1 \int_{x^2}^{2x} dy dx$



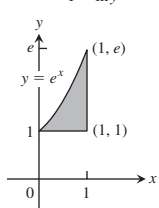
53. $1/(80\pi)$



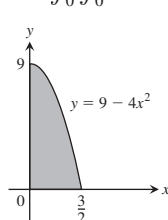
55. $-2/3$



37. $\int_1^e \int_{\ln y}^1 dx dy$



39. $\int_0^9 \int_0^{(\sqrt{9-y})/2} 16x dx dy$



57. $4/3$

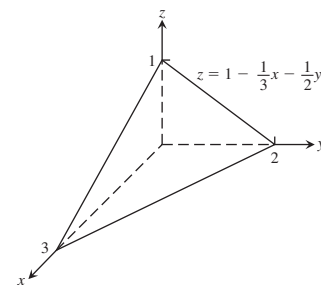
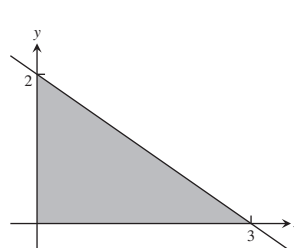
59. $625/12$

61. 16

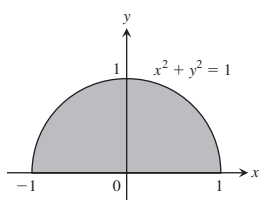
63. 20

65. $2(1 + \ln 2)$

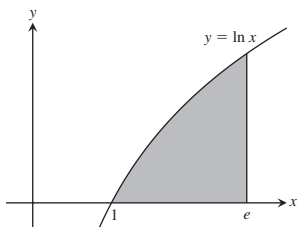
67.



41. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 3y dy dx$



43. $\int_0^1 \int_{e^y}^e xy dx dy$



69. 1

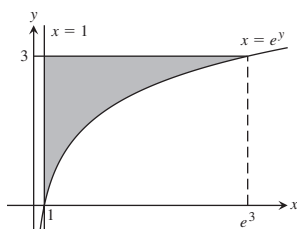
71. π^2

73. $-\frac{3}{32}$

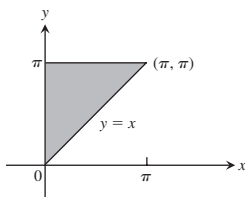
75. $\frac{20\sqrt{3}}{9}$

77. $\int_0^1 \int_x^{2-x} (x^2 + y^2) dy dx = \frac{4}{3}$

45. $\int_1^{e^3} \int_{\ln x}^3 (x + y) dy dx$



47. 2



79. R is the set of points (x, y) such that $x^2 + 2y^2 < 4$.

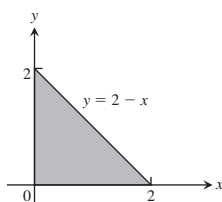
81. No, by Fubini's Theorem, the two orders of integration must give the same result.

85. 0.603 87. 0.233

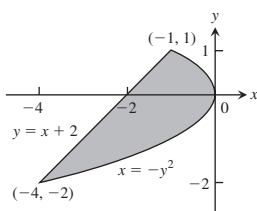
SECTION 15.3, p. 914

$$1. \int_0^2 \int_0^{2-x} dy \, dx = 2 \quad \text{or}$$

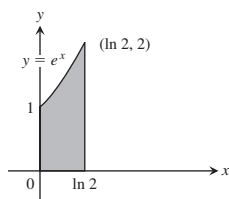
$$\int_0^2 \int_0^{2-y} dx \, dy = 2$$



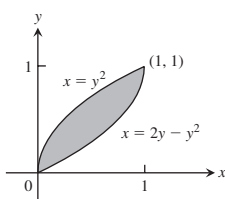
$$3. \int_{-2}^1 \int_{y-2}^{-y^2} dx \, dy = \frac{9}{2}$$



$$5. \int_0^{\ln 2} \int_0^{e^x} dy \, dx = 1$$

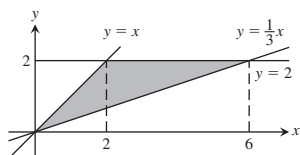


$$7. \int_0^1 \int_{y^2}^{2y-y^2} dx \, dy = \frac{1}{3}$$



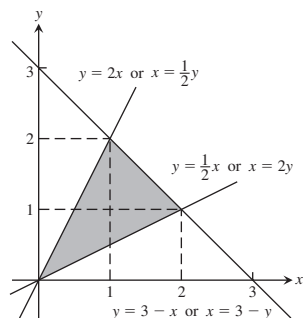
$$9. \int_0^2 \int_y^{3y} 1 \, dx \, dy = 4 \quad \text{or}$$

$$\int_0^2 \int_{x/3}^x 1 \, dy \, dx + \int_2^6 \int_{x/3}^2 1 \, dy \, dx = 4$$

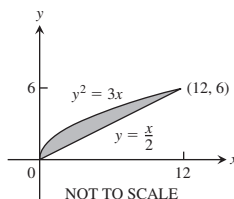
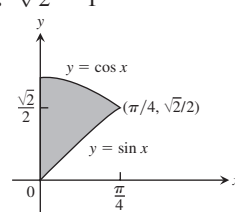
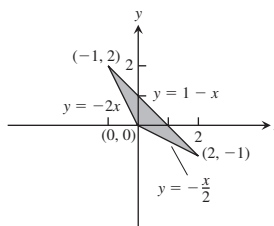


$$11. \int_0^1 \int_{x/2}^{2x} 1 \, dy \, dx + \int_1^2 \int_{x/2}^{3-x} 1 \, dy \, dx = \frac{3}{2} \quad \text{or}$$

$$\int_0^1 \int_{y/2}^{2y} 1 \, dx \, dy + \int_1^2 \int_{y/2}^{3-y} 1 \, dx \, dy = \frac{3}{2}$$



13. 12

15. $\sqrt{2} - 1$ 17. $\frac{3}{2}$ 19. (a) 0 (b) $4/\pi^2$ 21. $8/3$ 23. $\pi - 2$ 25. $40,000(1 - e^{-2}) \ln(7/2) \approx 43,329$

SECTION 15.4, pp. 919-921

$$1. \frac{\pi}{2} \leq \theta \leq 2\pi, 0 \leq r \leq 9 \quad 3. \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}, 0 \leq r \leq \csc \theta$$

$$5. 0 \leq \theta \leq \frac{\pi}{6}, 1 \leq r \leq 2\sqrt{3} \sec \theta;$$

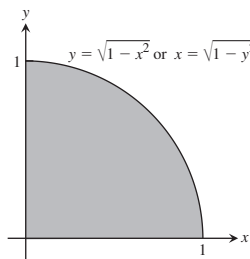
$$\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}, 1 \leq r \leq 2 \csc \theta$$

$$7. -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 2 \cos \theta \quad 9. \frac{\pi}{2}$$

$$11. 2\pi \quad 13. 36 \quad 15. 2 - \sqrt{3} \quad 17. (1 - \ln 2)\pi$$

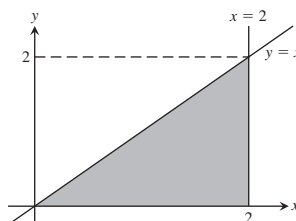
$$19. (2 \ln 2 - 1)(\pi/2) \quad 21. \frac{2(1 + \sqrt{2})}{3}$$

23.



$$\int_0^1 \int_0^{\sqrt{1-x^2}} xy \, dy \, dx \quad \text{or} \quad \int_0^1 \int_0^{\sqrt{1-y^2}} xy \, dx \, dy$$

25.



$$\int_0^2 \int_0^x y^2 (x^2 + y^2) \, dy \, dx \quad \text{or} \quad \int_0^2 \int_y^2 y^2 (x^2 + y^2) \, dx \, dy$$

27. $2(\pi - 2)$ 29. 12π 31. $(3\pi/8) + 1$ 33. $\frac{2a}{3}$
35. $\frac{2a}{3}$ 37. $2\pi(2 - \sqrt{e})$ 39. $\frac{4}{3} + \frac{5\pi}{8}$
41. (a) $\frac{\sqrt{\pi}}{2}$ (b) 1 43. $\pi \ln 4$, no 45. $\frac{1}{2}(a^2 + 2h^2)$
47. $\frac{8}{9}(3\pi - 4)$

SECTION 15.5, pp. 929–931

1. $1/6$
3. $\int_0^1 \int_0^{2-2x} \int_0^{3-3x-3y/2} dz \, dy \, dx, \int_0^2 \int_0^{1-y/2} \int_0^{3-3x-3y/2} dz \, dx \, dy,$
 $\int_0^1 \int_0^{3-3x} \int_0^{2-2x-2z/3} dy \, dz \, dx, \int_0^3 \int_0^{1-z/3} \int_0^{2-2x-2z/3} dy \, dx \, dz,$
 $\int_0^2 \int_0^{3-3y/2} \int_0^{1-y/2-z/3} dx \, dz \, dy, \int_0^3 \int_0^{2-2z/3} \int_0^{1-y/2-z/3} dx \, dy \, dz.$

The value of all six integrals is 1.

5. $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} 1 \, dz \, dx \, dy, \int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{x^2+y^2}^{8-x^2-y^2} 1 \, dz \, dx \, dy,$
 $\int_{-2}^2 \int_4^{8-y^2} \int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1 \, dx \, dz \, dy + \int_{-2}^2 \int_{y^2}^4 \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1 \, dx \, dz \, dy,$
 $\int_4^8 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1 \, dx \, dy \, dz + \int_0^4 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1 \, dx \, dy \, dz,$
 $\int_{-2}^2 \int_4^{8-x^2} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1 \, dy \, dz \, dx + \int_{-2}^2 \int_{x^2}^4 \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1 \, dy \, dz \, dx,$
 $\int_4^8 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1 \, dy \, dx \, dz + \int_0^4 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1 \, dy \, dx \, dz.$

The value of all six integrals is 16π .

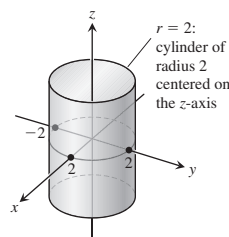
7. 1 9. 6 11. $\frac{5(2 - \sqrt{3})}{4}$ 13. 18
15. $7/6$ 17. 0 19. $\frac{1}{2} - \frac{\pi}{8}$
21. (a) $\int_{-1}^1 \int_0^{1-x^2} \int_{x^2}^{1-z} dy \, dz \, dx$ (b) $\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{x^2}^{1-z} dy \, dx \, dz$
(c) $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} dx \, dy \, dz$ (d) $\int_0^1 \int_0^{1-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx \, dz \, dy$
(e) $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{1-y} dz \, dx \, dy$
23. $2/3$ 25. $20/3$ 27. 1 29. $16/3$ 31. $8\pi - \frac{32}{3}$
33. 2 35. 4π 37. $31/3$ 39. 1 41. $2 \sin 4$
43. 4 45. $a = 3$ or $a = 13/3$
47. The domain is the set of all points (x, y, z) such that $4x^2 + 4y^2 + z^2 \leq 4$.

SECTION 15.6, pp. 938–941

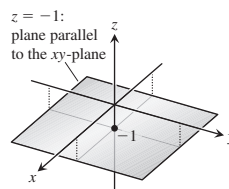
1. $\bar{x} = 5/14, \bar{y} = 38/35$ 3. $\bar{x} = 64/35, \bar{y} = 5/7$
5. $\bar{x} = \bar{y} = 4a/(3\pi)$
7. $I_x = I_y = 4\pi \text{ gm/cm}^2, I_0 = 8\pi \text{ gm/cm}^2$
9. $\bar{x} = -1, \bar{y} = 1/4$ 11. $I_x = 64/105$
13. $\bar{x} = 3/8, \bar{y} = 17/16$ 15. $\bar{x} = 11/3, \bar{y} = 14/27, I_y = 432$
17. $\bar{x} = 0, \bar{y} = 13/31, I_y = 7/5$
19. $\bar{x} = 0, \bar{y} = 7/10; I_x = 9/10 \text{ kg/m}^2, I_y = 3/10 \text{ kg/m}^2,$
 $I_0 = 6/5 \text{ kg/m}^2$
21. $I_x = \frac{M}{3}(b^2 + c^2), I_y = \frac{M}{3}(a^2 + c^2), I_z = \frac{M}{3}(a^2 + b^2)$
23. $\bar{x} = \bar{y} = 0, \bar{z} = 12/5, I_x = 7904/105 \approx 75.28,$
 $I_y = 4832/63 \approx 76.70, I_z = 256/45 \approx 5.69$
25. (a) $\bar{x} = \bar{y} = 0, \bar{z} = 8/3$ (b) $c = 2\sqrt{2}$
27. $I_L = 1386$
29. (a) $4/3 \text{ gm}$ (b) $\bar{x} = 4/5 \text{ cm}, \bar{y} = \bar{z} = 2/5 \text{ cm}$
31. (a) $5/2$ (b) $\bar{x} = \bar{y} = \bar{z} = 8/15$ (c) $I_x = I_y = I_z = 11/6$
33. 3 kg
37. (a) $I_{\text{c.m.}} = \frac{abc(a^2 + b^2)}{12}, R_{\text{c.m.}} = \sqrt{\frac{a^2 + b^2}{12}}$
(b) $I_L = \frac{abc(a^2 + 7b^2)}{3}, R_L = \sqrt{\frac{a^2 + 7b^2}{3}}$
39. $\mu_x = \mu_y = 7/12$
41. $\mu_x = 3/4, \mu_y = 2/3$
43. $f(x, y) = 1/6, P(X < Y) = 2/3$

SECTION 15.7, pp. 949–953

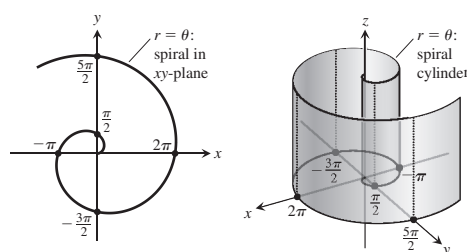
1.



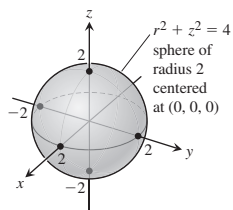
3.



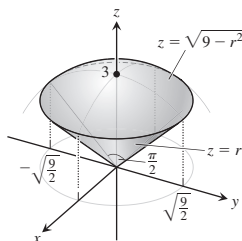
5.



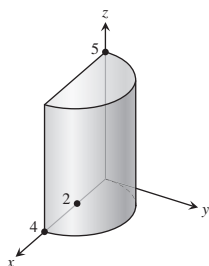
7.



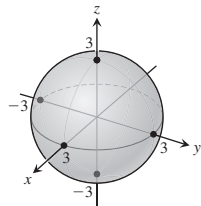
9. $r \leq z \leq \sqrt{9 - r^2}$: cone with vertex angle $\frac{\pi}{2}$ below a sphere of radius 3 centered at $(0, 0, 0)$, and its interior



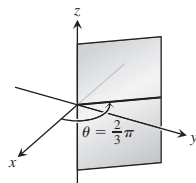
11. $0 \leq r \leq 4 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq z \leq 5$: half-cylinder of height 5, radius 2, and tangent to the z -axis, and its interior



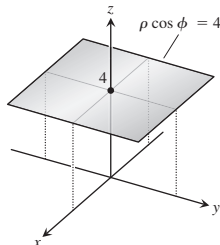
13. $\rho = 3$: sphere of radius 3 centered at $(0, 0, 0)$



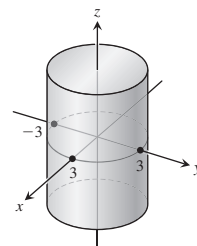
15. $\theta = \frac{2}{3}\pi$: closed half-plane along the z -axis



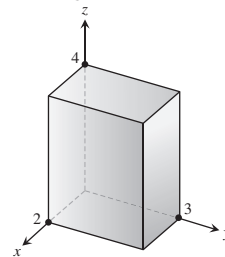
17. $\rho \cos \phi = 4$: plane with z -intercept 4 and parallel to the xy -plane



19. $0 \leq \rho \leq 3 \csc \phi \Rightarrow 0 \leq \rho \sin \phi \leq 3$: cylinder of radius 3 centered on the z -axis, and its interior



21. $0 \leq \rho \cos \theta \sin \phi \leq 2$, $0 \leq \rho \sin \theta \sin \phi \leq 3$, $0 \leq \rho \cos \phi \leq 4$: rectangular box $2 \times 3 \times 4$, and its interior



23. $\frac{4\pi(\sqrt{2} - 1)}{3}$ 25. $\frac{17\pi}{5}$ 27. $\pi(6\sqrt{2} - 8)$ 29. $\frac{3\pi}{10}$

31. $\pi/3$

33. (a) $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

(b) $\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^1 r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{\sqrt{3}}^2 \int_0^{\sqrt{4-z^2}} r \, dr \, dz \, d\theta$

(c) $\int_0^1 \int_0^{\sqrt{4-r^2}} \int_0^{2\pi} r \, d\theta \, dz \, dr$

35. $\int_{-\pi/2}^{\pi/2} \int_0^{\cos \theta} \int_0^{3r^2} f(r, \theta, z) \, dz \, r \, dr \, d\theta$

37. $\int_0^{\pi} \int_0^{2 \sin \theta} \int_0^{4-r \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$

39. $\int_{-\pi/2}^{\pi/2} \int_1^{1+\cos \theta} \int_0^4 f(r, \theta, z) \, dz \, r \, dr \, d\theta$

41. $\int_0^{\pi/4} \int_0^{\sec \theta} \int_0^{2-r \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$ 43. π^2

45. $\pi/3$ 47. 5π 49. 2π 51. $\left(\frac{8-5\sqrt{2}}{2}\right)\pi$

53. (a) $\int_0^{2\pi} \int_0^{\pi/6} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{\csc \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$

(b) $\int_0^{2\pi} \int_1^2 \int_{\pi/6}^{\sin^{-1}(1/\rho)} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta +$

$\int_0^{2\pi} \int_0^2 \int_0^{\pi/6} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta +$

$\int_0^{2\pi} \int_0^1 \int_{\pi/6}^{\pi/2} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta$

55.
$$\int_0^{2\pi} \int_0^{\pi/2} \int_{\cos \phi}^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{31\pi}{6}$$

57.
$$\int_0^{2\pi} \int_0^{\pi} \int_0^{1-\cos \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{8\pi}{3}$$

59.
$$\int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^{2\cos \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{\pi}{3}$$

61. (a)
$$8 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

(b)
$$8 \int_0^{\pi/2} \int_0^2 \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$$

(c)
$$8 \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} dz \, dy \, dx$$

63. (a)
$$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec \phi}^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

(b)
$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$$

(c)
$$\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} dz \, dy \, dx \quad \text{(d) } 5\pi/3$$

65. $8\pi/3$ 67. $9/4$ 69. $\frac{3\pi-4}{18}$ 71. $\frac{2\pi a^3}{3}$

73. $5\pi/3$ 75. $\pi/2$ 77. $\frac{4(2\sqrt{2}-1)\pi}{3}$ 79. 16π

81. $5\pi/2$ 83. $\frac{4\pi(8-3\sqrt{3})}{3}$ 85. $2/3$ 87. $3/4$

89. $\bar{x} = \bar{y} = 0, \bar{z} = 3/8$ 91. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3/8)$

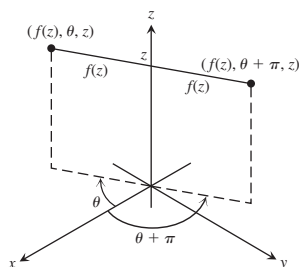
93. $\bar{x} = \bar{y} = 0, \bar{z} = 5/6$ 95. $I_x = \pi/4$ 97. $\frac{a^4 h \pi}{10}$

99. (a) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{4}{5}\right), I_z = \frac{\pi}{12}$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{6}\right), I_z = \frac{\pi}{14}$

101. $\frac{3M}{\pi R^3}$

103. The surface's equation $r = f(z)$ tells us that the point $(r, \theta, z) = (f(z), \theta, z)$ will lie on the surface for all θ . In particular, $(f(z), \theta + \pi, z)$ lies on the surface whenever $(f(z), \theta, z)$ lies on the surface, so the surface is symmetric with respect to the z -axis.



SECTION 15.8, pp. 961–963

1. (a) $x = \frac{u+v}{3}, y = \frac{v-2u}{3}, \frac{1}{3}$

(b) Triangular region with boundaries $u = 0, v = 0$, and $u + v = 3$

3. (a) $x = \frac{1}{5}(2u - v), y = \frac{1}{10}(3v - u); \frac{1}{10}$

(b) Triangular region with boundaries $3v = u, v = 2u$, and $3u + v = 10$

7. $64/5$ 9. $\int_1^2 \int_1^3 (u+v) \frac{2u}{v} \, du \, dv = 8 + \frac{52}{3} \ln 2$

11. $\frac{\pi ab(a^2 + b^2)}{4}$ 13. $\frac{1}{3} \left(1 + \frac{3}{e^2}\right) \approx 0.4687$

15. $\frac{225}{16}$ 17. 12 19. $\frac{a^2 b^2 c^2}{6}$

21. (a) $\begin{vmatrix} \cos v & -u \sin v \\ \sin v & u \cos v \end{vmatrix} = u \cos^2 v + u \sin^2 v = u$

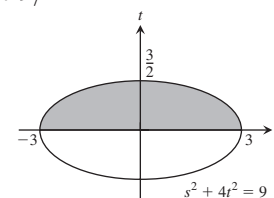
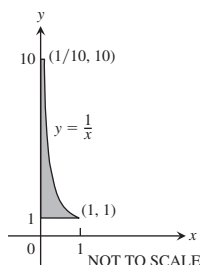
(b) $\begin{vmatrix} \sin v & u \cos v \\ \cos v & -u \sin v \end{vmatrix} = -u \sin^2 v - u \cos^2 v = -u$

27. $\frac{3}{2} \ln 2$

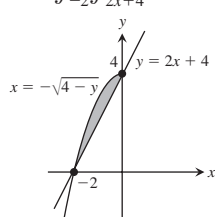
PRACTICE EXERCISES, pp. 963–965

1. $9e - 9$

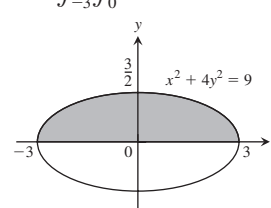
3. $9/2$



5. $\int_{-2}^0 \int_{2x+4}^{4-x^2} dy \, dx = \frac{4}{3}$



7. $\int_{-3}^3 \int_0^{(1/2)\sqrt{9-x^2}} y \, dy \, dx = \frac{9}{2}$



9. $\sin 4$ 11. $\frac{\ln 17}{4}$

13. $4/3$ 15. $4/3$ 17. $1/4$

19. π 21. $\frac{\pi-2}{4}$

23. 0 25. $8/35$ 27. $\pi/2$

29. $\frac{2(31 - 3^{5/2})}{3}$

31. (a) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 3 \, dz \, dx \, dy$

(b) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 3 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$ (c) $2\pi(8 - 4\sqrt{2})$

$$33. \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sec \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{\pi}{3}$$

$$35. \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} z^2 xy \, dz \, dy \, dx \\ + \int_1^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} z^2 xy \, dz \, dy \, dx$$

$$37. \text{(a)} \frac{8\pi(4\sqrt{2}-5)}{3} \quad \text{(b)} \frac{8\pi(4\sqrt{2}-5)}{3}$$

$$39. I_z = \frac{8\pi\delta(b^5 - a^5)}{15}$$

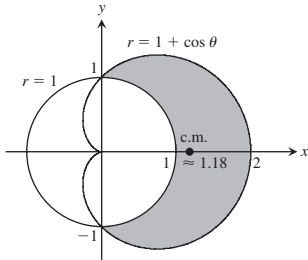
$$41. \bar{x} = \bar{y} = \frac{1}{2 - \ln 4} \quad 43. I_0 = 104 \quad 45. I_x = 2\delta$$

$$47. M = 4, M_x = 0, M_y = 0$$

$$49. \bar{x} = \frac{3\sqrt{3}}{\pi}, \bar{y} = 0$$

$$51. \text{(a)} \bar{x} = \frac{15\pi + 32}{6\pi + 48}, \bar{y} = 0$$

(b)



ADDITIONAL AND ADVANCED EXERCISES, pp. 966-967

$$1. \text{(a)} \int_{-3}^2 \int_x^{6-x^2} x^2 \, dy \, dx \quad \text{(b)} \int_{-3}^2 \int_x^{6-x^2} \int_0^{x^2} dz \, dy \, dx$$

$$\text{(c)} 125/4$$

$$3. 2\pi \quad 5. 3\pi/2$$

$$7. \text{(a)} \text{Hole radius} = 1, \text{sphere radius} = 2 \quad \text{(b)} 4\sqrt{3}\pi$$

$$9. \pi/4 \quad 11. \ln\left(\frac{b}{a}\right) \quad 15. 1/\sqrt[4]{3}$$

$$17. \text{Mass} = a^2 \cos^{-1}\left(\frac{b}{a}\right) - b\sqrt{a^2 - b^2},$$

$$I_0 = \frac{a^4}{2} \cos^{-1}\left(\frac{b}{a}\right) - \frac{b^3}{2} \sqrt{a^2 - b^2} - \frac{b^3}{6} (a^2 - b^2)^{3/2}$$

$$19. \frac{1}{ab}(e^{a^2b^2} - 1) \quad 21. \text{(b)} 1 \quad \text{(c)} 0$$

$$25. h = \sqrt{20} \text{ in.}, h = \sqrt{60} \text{ in.} \quad 27. 2\pi \left[\frac{1}{3} - \left(\frac{1}{3} \right) \frac{\sqrt{2}}{2} \right]$$

Chapter 16

SECTION 16.1, pp. 974-976

$$1. \text{Graph (c)} \quad 3. \text{Graph (g)} \quad 5. \text{Graph (d)} \quad 7. \text{Graph (f)}$$

$$9. \sqrt{2} \quad 11. \frac{13}{2} \quad 13. 3\sqrt{14} \quad 15. \frac{1}{6}(5\sqrt{5} + 9)$$

$$17. \sqrt{3} \ln\left(\frac{b}{a}\right) \quad 19. \text{(a)} 4\sqrt{5} \quad \text{(b)} \frac{1}{12}(17^{3/2} - 1)$$

$$21. \frac{15}{32}(e^{16} - e^{64}) \quad 23. \frac{1}{27}(40^{3/2} - 13^{3/2})$$

$$25. \frac{1}{6}(5^{3/2} + 7\sqrt{2} - 1) \quad 27. \frac{10\sqrt{5} - 2}{3} \quad 29. 8$$

$$31. \frac{1}{6}(17^{3/2} - 1) \quad 33. 2\sqrt{2} - 1$$

$$35. \text{(a)} 4\sqrt{2} - 2 \quad \text{(b)} \sqrt{2} + \ln(1 + \sqrt{2}) \quad 37. I_z = 2\pi\delta a^3$$

$$39. \text{(a)} I_z = 2\pi\sqrt{2}\delta \quad \text{(b)} I_z = 4\pi\sqrt{2}\delta \quad 41. I_x = 2\pi - 2$$

SECTION 16.2, pp. 986-989

$$1. \nabla f = -(xi + yj + zk)(x^2 + y^2 + z^2)^{-3/2}$$

$$3. \nabla g = -\left(\frac{2x}{x^2 + y^2}\right)\mathbf{i} - \left(\frac{2y}{x^2 + y^2}\right)\mathbf{j} + e^z\mathbf{k}$$

$$5. \mathbf{F} = -\frac{kx}{(x^2 + y^2)^{3/2}}\mathbf{i} - \frac{ky}{(x^2 + y^2)^{3/2}}\mathbf{j}, \text{ any } k > 0$$

$$7. \text{(a)} 9/2 \quad \text{(b)} 13/3 \quad \text{(c)} 9/2$$

$$9. \text{(a)} 1/3 \quad \text{(b)} -1/5 \quad \text{(c)} 0$$

$$11. \text{(a)} 2 \quad \text{(b)} 3/2 \quad \text{(c)} 1/2$$

$$13. -15/2 \quad 15. 36 \quad 17. \text{(a)} -5/6 \quad \text{(b)} 0 \quad \text{(c)} -7/12$$

$$19. 1/2 \quad 21. -\pi \quad 23. 69/4 \quad 25. -39/2 \quad 27. 25/6$$

$$29. \text{(a)} \text{Circ}_1 = 0, \text{circ}_2 = 2\pi, \text{flux}_1 = 2\pi, \text{flux}_2 = 0$$

$$\text{(b)} \text{Circ}_1 = 0, \text{circ}_2 = 8\pi, \text{flux}_1 = 8\pi, \text{flux}_2 = 0$$

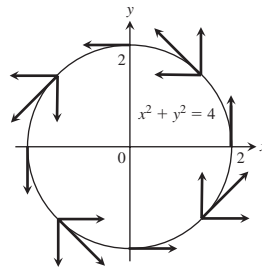
$$31. \text{Circ} = 0, \text{flux} = a^2\pi \quad 33. \text{Circ} = a^2\pi, \text{flux} = 0$$

$$35. \text{(a)} -\frac{\pi}{2} \quad \text{(b)} 0 \quad \text{(c)} 1 \quad 37. (.0001)\pi \text{ kg/s}$$

$$39. \text{(a)} 32 \quad \text{(b)} 32 \quad \text{(c)} 32 \quad 41. 115.2 \text{ J}$$

$$43. 5/3 - (3/2)\ln 2 \text{ m}^2/\text{s} \quad 45. 5/3 \text{ g/s}$$

47.



$$49. \text{(a)} \mathbf{G} = -y\mathbf{i} + x\mathbf{j} \quad \text{(b)} \mathbf{G} = \sqrt{x^2 + y^2} \mathbf{F}$$

$$51. \mathbf{F} = -\frac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2 + y^2}} \quad 55. 48 \quad 57. \pi \quad 59. 0 \quad 61. \frac{1}{2}$$

SECTION 16.3, pp. 998-1000

$$1. \text{Conservative} \quad 3. \text{Not conservative} \quad 5. \text{Not conservative}$$

$$7. f(x, y, z) = x^2 + \frac{3y^2}{2} + 2z^2 + C \quad 9. f(x, y, z) = xe^{y+2z} + C$$

$$11. f(x, y, z) = x \ln x - x + \tan(x + y) + \frac{1}{2} \ln(y^2 + z^2) + C$$

$$13. 49 \quad 15. -16 \quad 17. 1 \quad 19. 9 \ln 2 \quad 21. 0 \quad 23. -3$$

$$27. \mathbf{F} = \nabla\left(\frac{x^2 - 1}{y}\right) \quad 29. \text{(a)} 1 \quad \text{(b)} 1 \quad \text{(c)} 1$$

$$31. \text{(a)} 2 \quad \text{(b)} 2 \quad 33. \text{(a)} c = b = 2a \quad \text{(b)} c = b = 2$$

$$35. \text{It does not matter what path you use. The work will be the same on any path because the field is conservative.}$$

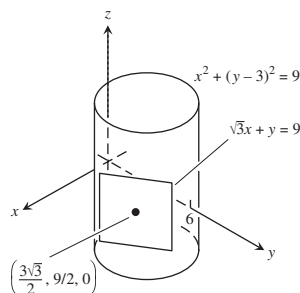
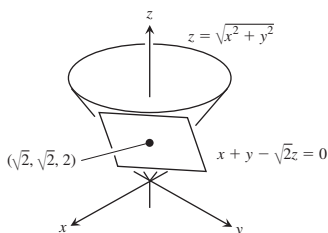
37. The force $\mathbf{F}$ is conservative because all partial derivatives of M , N , and P are zero. $f(x, y, z) = ax + by + cz + C$; $A = (xa, ya, za)$ and $B = (xb, yb, zb)$. Therefore, $\int \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A) = a(xb - xa) + b(yb - ya) + c(zb - za) = \mathbf{F} \cdot \overrightarrow{AB}$.

SECTION 16.4, pp. 1010–1012

1. $2y - 1$ 3. $ye^x - xe^y$ 5. $\sin y - \sin x$
 7. Flux = 0, circ = $2\pi a^2$ 9. Flux = $-\pi a^2$, circ = 0
 11. Flux = 2, circ = 0 13. Flux = -9 , circ = 9
 15. Flux = $-11/60$, circ = $-7/60$
 17. Flux = $64/9$, circ = 0 19. Flux = $1/2$, circ = $1/2$
 21. Flux = $1/5$, circ = $-1/12$ 23. 0 25. $2/33$ 27. 0
 29. -16π 31. πa^2 33. $3\pi/8$
 35. (a) 0 if C is traversed counterclockwise
 (b) $(h - k)(\text{area of the region})$ 45. (a) 0

SECTION 16.5, pp. 1020–1022

1. $\mathbf{r}(r, \theta) = (r \cos \theta)\mathbf{i} + (r \sin \theta)\mathbf{j} + r^2\mathbf{k}$, $0 \leq r \leq 2$, $0 \leq \theta \leq 2\pi$
 3. $\mathbf{r}(r, \theta) = (r \cos \theta)\mathbf{i} + (r \sin \theta)\mathbf{j} + (r/2)\mathbf{k}$, $0 \leq r \leq 6$, $0 \leq \theta \leq \pi/2$
 5. $\mathbf{r}(r, \theta) = (r \cos \theta)\mathbf{i} + (r \sin \theta)\mathbf{j} + \sqrt{9 - r^2}\mathbf{k}$, $0 \leq r \leq 3\sqrt{2}/2$, $0 \leq \theta \leq 2\pi$; Also:
 $\mathbf{r}(\phi, \theta) = (3 \sin \phi \cos \theta)\mathbf{i} + (3 \sin \phi \sin \theta)\mathbf{j} + (3 \cos \phi)\mathbf{k}$, $0 \leq \phi \leq \pi/4$, $0 \leq \theta \leq 2\pi$
 7. $\mathbf{r}(\phi, \theta) = (\sqrt{3} \sin \phi \cos \theta)\mathbf{i} + (\sqrt{3} \sin \phi \sin \theta)\mathbf{j} + (\sqrt{3} \cos \phi)\mathbf{k}$, $\pi/3 \leq \phi \leq 2\pi/3$, $0 \leq \theta \leq 2\pi$
 9. $\mathbf{r}(x, y) = x\mathbf{i} + y\mathbf{j} + (4 - y^2)\mathbf{k}$, $0 \leq x \leq 2$, $-2 \leq y \leq 2$
 11. $\mathbf{r}(u, v) = u\mathbf{i} + (3 \cos v)\mathbf{j} + (3 \sin v)\mathbf{k}$, $0 \leq u \leq 3$, $0 \leq v \leq 2\pi$
 13. (a) $\mathbf{r}(r, \theta) = (r \cos \theta)\mathbf{i} + (r \sin \theta)\mathbf{j} + (1 - r \cos \theta - r \sin \theta)\mathbf{k}$, $0 \leq r \leq 3$, $0 \leq \theta \leq 2\pi$
 (b) $\mathbf{r}(u, v) = (1 - u \cos v - u \sin v)\mathbf{i} + (u \cos v)\mathbf{j} + (u \sin v)\mathbf{k}$, $0 \leq u \leq 3$, $0 \leq v \leq 2\pi$
 15. $\mathbf{r}(u, v) = (4 \cos^2 v)\mathbf{i} + u\mathbf{j} + (4 \cos v \sin v)\mathbf{k}$, $0 \leq u \leq 3$, $-\pi/2 \leq v \leq \pi/2$; Another way: $\mathbf{r}(u, v) = (2 + 2 \cos v)\mathbf{i} + u\mathbf{j} + (2 \sin v)\mathbf{k}$, $0 \leq u \leq 3$, $0 \leq v \leq 2\pi$
 17. $\int_0^{2\pi} \int_0^1 \frac{\sqrt{5}}{2} r dr d\theta = \frac{\pi\sqrt{5}}{2}$
 19. $\int_0^{2\pi} \int_1^3 r\sqrt{5} dr d\theta = 8\pi\sqrt{5}$ 21. $\int_0^{2\pi} \int_1^4 1 du dv = 6\pi$
 23. $\int_0^{2\pi} \int_0^1 u\sqrt{4u^2 + 1} du dv = \frac{(5\sqrt{5} - 1)}{6}\pi$
 25. $\int_0^{2\pi} \int_{\pi/4}^{\pi} 2 \sin \phi d\phi d\theta = (4 + 2\sqrt{2})\pi$
 27. 29.



33. (b) $A = \int_0^{2\pi} \int_0^\pi [a^2 b^2 \sin^2 \phi \cos^2 \phi + b^2 c^2 \cos^4 \phi \cos^2 \theta + a^2 c^2 \cos^4 \phi \sin^2 \theta]^{1/2} d\phi d\theta$

35. $x_0 x + y_0 y = 25$ 37. $13\pi/3$ 39. 4
 41. $6\sqrt{6} - 2\sqrt{2}$ 43. $\pi\sqrt{c^2 + 1}$
 45. $\frac{\pi}{6}(17\sqrt{17} - 5\sqrt{5})$ 47. $3 + 2 \ln 2$
 49. $\frac{\pi}{6}(13\sqrt{13} - 1)$ 51. $5\pi\sqrt{2}$ 53. $\frac{2}{3}(5\sqrt{5} - 1)$

SECTION 16.6, pp. 1030–1032

1. $\iint_S x d\sigma = \int_0^3 \int_0^2 u\sqrt{4u^2 + 1} du dv = \frac{17\sqrt{17} - 1}{4}$
 3. $\iint_S x^2 d\sigma = \int_0^{2\pi} \int_0^\pi \sin^3 \phi \cos^2 \theta d\phi d\theta = \frac{4\pi}{3}$
 5. $\iint_S z d\sigma = \int_0^1 \int_0^1 (4 - u - v)\sqrt{3} dv du = 3\sqrt{3}$
 (for $x = u, y = v$)
 7. $\iint_S x^2 \sqrt{5 - 4z} d\sigma = \int_0^1 \int_0^{2\pi} u^2 \cos^2 v \cdot \sqrt{4u^2 + 1} \cdot u\sqrt{4u^2 + 1} dv du = \frac{11\pi}{12}$
 9. $9a^3$ 11. $\frac{abc}{4}(ab + ac + bc)$ 13. 2
 15. $\frac{1}{30}(\sqrt{2} + 6\sqrt{6})$ 17. $\sqrt{6}/30$ 19. -32 21. $\frac{\pi a^3}{6}$
 23. $13a^4/6$ 25. $2\pi/3$ 27. $-73\pi/6$ 29. 18
 31. $\frac{\pi a^3}{6}$ 33. $\frac{\pi a^2}{4}$ 35. $\frac{\pi a^3}{2}$ 37. -32 39. -4
 41. $3a^4$ 43. $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$
 45. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{14}{9}\right)$, $I_z = \frac{15\pi\sqrt{2}}{2}\delta$
 47. (a) $\frac{8\pi}{3}a^4\delta$ (b) $\frac{20\pi}{3}a^4\delta$ 49. $70/3 \text{ mg}$

SECTION 16.7, pp. 1043–1045

1. $-\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ 3. $(1 - y)\mathbf{i} + (1 - z)\mathbf{j} + (1 - x)\mathbf{k}$
 5. $x(z^2 - y^2)\mathbf{i} + y(x^2 - z^2)\mathbf{j} + z(y^2 - x^2)\mathbf{k}$ 7. 4π
 9. $-5/6$ 11. 0 13. -6π 15. $2\pi a^2$ 17. $-\pi$
 19. 12π 21. $-\pi/4$ 23. -15π 25. -8π
 33. $16I_y + 16I_x$

SECTION 16.8, pp. 1056–1057

1. 0 3. $(y^2 z + xz^2 + x^2 y)e^{xyz}$ 5. 0 7. 0
 9. -16 11. -8π 13. 3π 15. $-40/3$ 17. 12π
 19. $12\pi(4\sqrt{2} - 1)$ 23. No
 25. The integral's value never exceeds the surface area of S .
 27. $184/35$

PRACTICE EXERCISES, pp. 1058–1060

1. Path 1: $2\sqrt{3}$; path 2: $1 + 3\sqrt{2}$ 3. $4a^2$ 5. 0
 7. $8\pi \sin(1)$ 9. 0 11. $\pi\sqrt{3}$
 13. $2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$ 15. $\frac{abc}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$ 17. 50

19. $\mathbf{r}(\phi, \theta) = (6 \sin \phi \cos \theta)\mathbf{i} + (6 \sin \phi \sin \theta)\mathbf{j} + (6 \cos \phi)\mathbf{k}$,
 $\frac{\pi}{6} \leq \phi \leq \frac{2\pi}{3}, 0 \leq \theta \leq 2\pi$
21. $\mathbf{r}(r, \theta) = (r \cos \theta)\mathbf{i} + (r \sin \theta)\mathbf{j} + (1 + r)\mathbf{k}, 0 \leq r \leq 2,$
 $0 \leq \theta \leq 2\pi$
23. $\mathbf{r}(u, v) = (u \cos v)\mathbf{i} + 2u^2\mathbf{j} + (u \sin v)\mathbf{k}, 0 \leq u \leq 1,$
 $0 \leq v \leq \pi$
25. $\sqrt{6}$ 27. $\pi[\sqrt{2} + \ln(1 + \sqrt{2})]$ 29. Conservative
31. Not conservative 33. $f(x, y, z) = y^2 + yz + 2x + z$
35. Path 1: 2; path 2: $8/3$ 37. (a) $1 - e^{-2\pi}$ (b) $1 - e^{-2\pi}$
39. 0 41. (a) $4\sqrt{2} - 2$ (b) $\sqrt{2} + \ln(1 + \sqrt{2})$
43. $(\bar{x}, \bar{y}, \bar{z}) = \left(1, \frac{16}{15}, \frac{2}{3}\right); I_x = \frac{232}{45}, I_y = \frac{64}{15}, I_z = \frac{56}{9}$
45. $\bar{z} = \frac{3}{2}, I_z = \frac{7\sqrt{3}}{3}$ 47. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 49/12), I_z = 640\pi$
49. Flux: $3/2$; circ: $-1/2$ 53. 3
55. $\frac{2\pi}{3}(7 - 8\sqrt{2})$ 57. 0 59. π

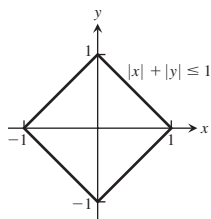
ADDITIONAL AND ADVANCED EXERCISES, pp. 1061-1062

1. 6π 3. $2/3$
5. (a) $\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ (b) $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{k}$
 (c) $\mathbf{F}(x, y, z) = z\mathbf{i}$
7. $\frac{16\pi R^3}{3}$ 9. $a = 2, b = 1$. The minimum flux is -4 .
11. (b) $\frac{16}{3}g$ (c) Work $= \left(\int_C gxy \, ds\right) \bar{y} = g \int_C xy^2 \, ds = \frac{16}{3}g$
13. (c) $\frac{4}{3}\pi w$ 19. False if $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$

Appendices

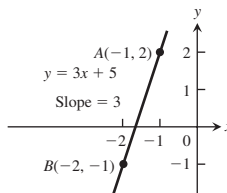
APPENDIX 1, p. AP-6

1. $0.\bar{1}, 0.\bar{2}, 0.\bar{3}, 0.\bar{8}, 0.\bar{9}$ or 1
3. $x < -2$
7. $3, -3$ 9. $7/6, 25/6$
11. $-2 \leq t \leq 4$
15. $(-\infty, -2] \cup [2, \infty)$
19. $(-3, -2) \cup (2, 3)$ 21. $(0, 1)$ 23. $(-\infty, 1]$
27. The graph of $|x| + |y| \leq 1$ is the interior and boundary of the "diamond-shaped" region.

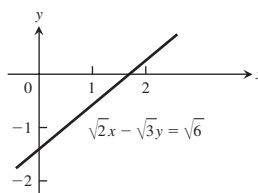


APPENDIX 3, pp. AP-17-AP-18

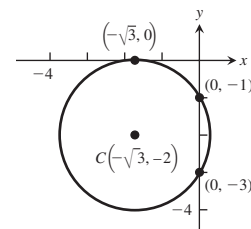
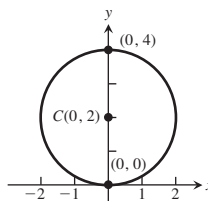
1. $2, -4; 2\sqrt{5}$ 3. Unit circle
5. $m_{\perp} = -\frac{1}{3}$



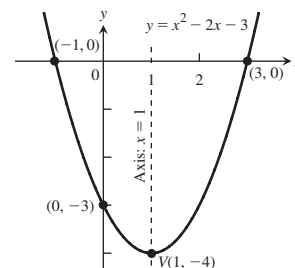
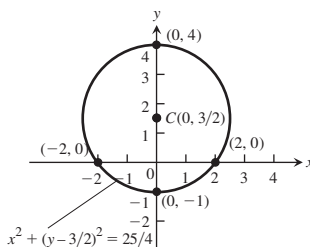
7. (a) $x = -1$ (b) $y = 4/3$ 9. $y = -x$
11. $y = -\frac{5}{4}x + 6$ 13. $y = 4x + 4$ 15. $y = -\frac{x}{2} + 12$
17. x-intercept $= \sqrt{3}$, y-intercept $= -\sqrt{2}$



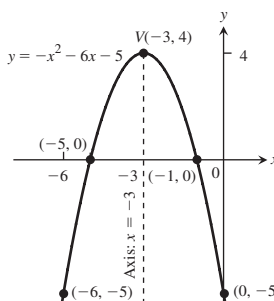
19. $(3, -3)$
21. $x^2 + (y - 2)^2 = 4$
23. $(x + \sqrt{3})^2 + (y + 2)^2 = 4$



25. $x^2 + (y - 3/2)^2 = 25/4$ 27.



29.



31. Exterior points of a circle of radius $\sqrt{7}$, centered at the origin

33. The washer between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ (points with distance from the origin between 1 and 2)

35. $(x + 2)^2 + (y - 1)^2 < 6$

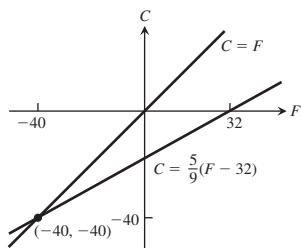
37. $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$

39. $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{3}\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$

41. (a) ≈ -2.5 degrees/inch (b) ≈ -16.1 degrees/inch
(c) ≈ -8.3 degrees/inch

43. 5.97 atm

45. Yes: $C = F = -40^\circ$



51. $k = -8, k = 1/2$

APPENDIX 7, pp. AP-33-AP-34

1. (a) $14 + 8i$ (b) $-7 - 4i$ (c) $-5i$

3. (a) By reflecting z across the real axis

(b) By reflecting z across the imaginary axis

(c) By reflecting z across the real axis and then multiplying the length of the vector by $1/|z|^2$

5. (a) Points on the circle $x^2 + y^2 = 4$

(b) Points inside the circle $x^2 + y^2 = 4$

(c) Points outside the circle $x^2 + y^2 = 4$

7. Points on a circle of radius 1, center $(-1, 0)$

9. Points on the line $y = -x$ 11. $4e^{2\pi i/3}$ 13. $1e^{2\pi i/3}$

15. $\cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$

17. $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ 19. $2i, -\sqrt{3} - i, \sqrt{3} - i$

21. $\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i, -\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i$ 23. $1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$

ANSWERS TO ODD-NUMBERED EXERCISES

Chapter 17

SECTION 17.1, pp. 17-6-17-7

1. $y = c_1 e^{-3x} + c_2 e^{4x}$
3. $y = c_1 e^{-4x} + c_2 e^x$
5. $y = c_1 e^{-2x} + c_2 e^{2x}$
7. $y = c_1 e^{-x} + c_2 e^{3x/2}$
9. $y = c_1 e^{-x/4} + c_2 e^{3x/2}$
11. $y = c_1 \cos 3x + c_2 \sin 3x$
13. $y = c_1 \cos 5x + c_2 \sin 5x$
15. $y = e^x(c_1 \cos 2x + c_2 \sin 2x)$
17. $y = e^{-x}(c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x)$
19. $y = e^{-2x}(c_1 \cos \sqrt{5}x + c_2 \sin \sqrt{5}x)$
21. $y = c_1 + c_2 x$
23. $y = c_1 e^{-2x} + c_2 x e^{-2x}$
25. $y = c_1 e^{-3x} + c_2 x e^{-3x}$
27. $y = c_1 e^{-x/2} + c_2 x e^{-x/2}$
29. $y = c_1 e^{-x/3} + c_2 x e^{-x/3}$
31. $y = -\frac{3}{4} e^{-5x} + \frac{3}{4} e^{-x}$
33. $y = \frac{1}{2\sqrt{3}} \sin 2\sqrt{3}x$
35. $y = -\cos 2\sqrt{2}x + \frac{1}{\sqrt{2}} \sin 2\sqrt{2}x$
37. $y = (1 - 2x)e^{2x}$
39. $y = 2(1 + 2x)e^{-3x/2}$
41. $y = c_1 e^{-x} + c_2 e^{3x}$
43. $y = c_1 e^{-x/2} + c_2 x e^{-x/2}$
45. $y = c_1 \cos \sqrt{5}x + c_2 \sin \sqrt{5}x$
47. $y = c_1 e^{-x/5} + c_2 x e^{-x/5}$
49. $y = e^{-x/2}(c_1 \cos x + c_2 \sin x)$
51. $y = c_1 e^{3x/4} + c_2 x e^{3x/4}$
53. $y = c_1 e^{-4x/3} + c_2 x e^{-4x/3}$
55. $y = c_1 e^{-x/2} + c_2 x e^{4x/3}$
57. $y = (1 + 2x)e^{-x}$
59. $y = \frac{15}{13} e^{-7x/3} + \frac{11}{13} e^{2x}$

SECTION 17.2, pp. 17-14-17-15

1. $y = c_1 e^{5x} + c_2 e^{-2x} + \frac{3}{10}$
3. $y = c_1 + c_2 e^x + \frac{1}{2} \cos x - \frac{1}{2} \sin x$
5. $y = c_1 \cos x + c_2 \sin x - \frac{1}{8} \cos 3x$
7. $y = c_1 e^{2x} + c_2 e^{-x} - 6 \cos x - 2 \sin x$
9. $y = c_1 e^x + c_2 e^{-x} - x^2 - 2 + \frac{1}{2} x e^x$
11. $y = c_1 e^{3x} + c_2 e^{-2x} - \frac{1}{4} e^{-x} + \frac{49}{50} \cos x + \frac{7}{50} \sin x$
13. $y = c_1 + c_2 e^{-5x} + x^3 + \frac{3}{5} x^2 - \frac{6}{25} x$
15. $y = c_1 + c_2 e^{3x} + 2x^2 + \frac{4}{3} x + \frac{1}{3} x e^{3x}$
17. $y = c_1 + c_2 e^{-x} + \frac{1}{2} x^2 - x$
19. $y = c_1 \cos x + c_2 \sin x - \frac{1}{2} x \cos x$
21. $y = (c_1 + c_2 x) e^{-x} + \frac{1}{2} x^2 e^{-x}$
23. $y = c_1 e^x + c_2 e^{-x} + \frac{1}{2} x e^x$
25. $y = e^{-2x}(c_1 \cos x + c_2 \sin x) + 2$
27. $y = A \cos x + B \sin x + x \sin x + \cos x \ln(\cos x)$
29. $y = c_1 + c_2 e^{5x} + \frac{1}{10} x^2 e^{5x} - \frac{1}{25} x e^{5x}$
31. $y = c_1 \cos x + c_2 \sin x - \frac{1}{2} x \cos x + x \sin x$

33. $y = c_1 + c_2 e^x + \frac{1}{2} e^{-x} + x e^x$
35. $y = c_1 e^{5x} + c_2 e^{-x} - \frac{1}{8} e^x - \frac{4}{5}$
37. $y = c_1 \cos x + c_2 \sin x - (\sin x)[\ln(\csc x + \cot x)]$
39. $y = c_1 + c_2 e^{8x} + \frac{1}{8} x e^{8x}$
41. $y = c_1 + c_2 e^x - x^4/4 - x^3 - 3x^2 - 6x$
43. $y = c_1 + c_2 e^{-2x} - \frac{1}{3} e^x + x^3/6 - x^2/4 + x/4$
45. $y = c_1 \cos x + c_2 \sin x + (x - \tan x) \cos x - \sin x \ln(\cos x)$
 $= c_1 \cos x + c_2' \sin x + x \cos x - (\sin x) \ln(\cos x)$
47. $y = c e^{3x} - \frac{1}{2} e^x$
49. $y = c e^{3x} + 5 x e^{3x}$
51. $y = 2 \cos x + \sin x - 1 + \sin x \ln(\sec x + \tan x)$
53. $y = -e^{-x} + 1 + \frac{1}{2} x^2 - x$
55. $y = 2(e^x - e^{-x}) \cos x - 3 e^{-x} \sin x$
57. $y = (1 - x + x^2) e^x$
59. $y_p = \frac{1}{4} x^2$

SECTION 17.3, pp. 17-20-17-21

1. $my'' + y' + y = 0, \quad y(0) = 2, \quad y'(0) = 2$
3. $\frac{25}{32} y'' + 40y = 0, \quad y(0) = \frac{7}{24}, \quad y'(0) = \frac{v_0}{12}$
5. $2q'' + 4q' + 10q = 20 \cos t, \quad q(0) = 2, \quad q'(0) = 3$
7. 0.0864 ft (above equilibrium)
9. $y(t) = 0.2917 \cos(7.1554t) + \frac{v_0}{85.8623} \sin(7.1554t)$ (in feet), or
 $y = 3.5 \cos(7.1554t) + \frac{v_0}{0.1398} \sin(7.1554t)$ (in inches).
11. 0.308 sec
13. 8.334 lb
15. 24.4949 ft/sec
17. -1.56 ft/sec² (acceleration upward)
19. $q(t) = -8e^{-3t} + 10e^{-2t}, \quad \lim_{t \rightarrow \infty} q(t) = 0$
21. $y(t) = 1 + 2e^{-t} - \frac{1}{3} e^{-2t} - \frac{2}{3} e^{-8t}$
23. $y(\pi) = -2$ m (above equilibrium)
25. $q(t) = \frac{1}{5} + \left(\frac{49\sqrt{199}}{995} \sin \frac{\sqrt{199}}{2} t + \frac{49}{5} \cos \frac{\sqrt{199}}{2} t \right) e^{-t/2}$

SECTION 17.4, p. 17-24

1. $y = \frac{c_1}{x^2} + c_2 x$
3. $y = \frac{c_1}{x^2} + c_2 x^3$
5. $y = c_1 x^2 + c_2 x^4$
7. $y = c_1 x^{-1/3} + c_2$
9. $y = x(c_1 + c_2 \ln x)$
11. $y = x[c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)]$
13. $y = \frac{1}{x}[c_1 \cos(3 \ln x) + c_2 \sin(3 \ln x)]$
15. $y = \frac{1}{\sqrt{x}}[c_1 \cos(\ln x) + c_2 \sin(\ln x)]$
17. $y = \frac{1}{x}(c_1 + c_2 \ln x)$
19. $y = c_1 + c_2 \ln x$

$$21. y = \frac{1}{\sqrt[3]{x}}(c_1 + c_2 \ln x) \quad 23. y = x^{-5/4}(c_1 + c_2 \ln x)$$

$$25. y = \frac{1}{2x^3} + \frac{x}{2} \quad 27. y = x$$

$$29. y = x [-\cos(\ln x) + 2 \sin(\ln x)]$$

SECTION 17.5, p. 17-29

$$1. y = c_0 + c_1 \left(x - x^2 + \frac{2}{3}x^3 - \cdots \right) = c_0 - \frac{c_1}{2}e^{-2x}$$

$$3. y = c_0(1 - 2x^2 + \cdots) + c_1 \left(x - \frac{2}{3}x^3 + \cdots \right)$$

$$= c_0 \cos 2x + c_1 \sin 2x$$

$$5. y = c_1 x + c_2 x^2$$

$$7. y = c_0 \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots \right) + c_1 \left(x + \frac{1}{6}x^3 + \cdots \right)$$

$$9. y = c_0 \left(1 - x^2 + \frac{5}{12}x^4 - \cdots \right) + c_1 x$$

$$11. y = c_0(1 - 3x^2 + \cdots) + c_1(x - x^3)$$

$$13. y = c_0 \left(1 + x^2 + \frac{2}{3}x^4 + \cdots \right) + c_1 \left(x + x^3 + \frac{3}{5}x^5 + \cdots \right)$$

$$15. y = c_0 \left(1 - \frac{3}{2}x^2 + \cdots \right) + c_1 \left(x - \frac{1}{2}x^3 + \cdots \right)$$

$$17. y = c_0 \left(1 - \frac{3}{2}x^2 + \frac{1}{8}x^4 + \cdots \right) + c_1 \left(x - \frac{1}{3}x^3 \right)$$