Discussion Problems 1 (Tue., Oct. 9)

1. Compute the following limits:
   
   \[(a) \lim_{x \to 2} \frac{x^2 - 4}{x - 2}, \quad (b) \lim_{x \to 1} \frac{x - 1}{2 - \sqrt{x + 3}}, \quad (c) \lim_{x \to 8} \frac{\sqrt{2x} - 4}{\sqrt{x + 1} - 3} \]
   
   \[(d) \lim_{x \to 0} x \cdot \cos(x^3 + 17x^{-3}), \quad (e) \lim_{x \to 1} x \cdot \cos(x^3 + 17x^{-3}) \]

2. Let

   \[f(x) = \begin{cases} 
   x^2 + a & x \leq 1 \\
   1 - x & x > 1 
   \end{cases}\]

   (a) Determine \(a\) so that \(\lim_{x \to 1} f(x)\) exists.

   (b) Then graph the function \(y = f(x)\) and determine its range.

3. Let \(f(x) = |x| + |x + 1| + |x + 2|\). Determine domain and range of this function.

4. Let

   \[f(x) = \begin{cases} 
   \frac{1}{x^2} + a & x > 0 \\
   x & x \leq 0 
   \end{cases}\]

   Is it possible to choose a real number \(a\) so that the function \(y = f(x)\) has a limit as \(x \to 0\)?

5. Let

   \[f(x) = \frac{x - 1}{\sqrt{x} - 1}.\]

   Determine \(L = \lim_{x \to 1} f(x)\). Then find a \(\delta > 0\) so that \(|x - 1| < \delta\) implies \(|f(x) - L| < 0.00001\).

6. Let

   \[f(x) = \frac{x^3 - 8}{x - 2}.\]

   Demonstrate, by the precise definition of the limit, that \(\lim_{x \to 2} f(x) = L\) for some finite number \(L\). (First determine \(L\).)

7. Let

   \[f(x) = \frac{(x^2 + 8)(x - 2)}{x - 3}.\]

   Demonstrate, by the precise definition of the limit, that \(\lim_{x \to 2} f(x) = L\) for some finite number \(L\). (First determine \(L\).)