Discussion Problems 2 (Tue., Oct. 16)

1. Compute the following limits:
   (a) \( \lim_{x \to \infty} \frac{2 + x - x^2}{3 + 5x + 7x^2} \)
   (b) \( \lim_{x \to \infty} \frac{(3x + 1)^3(x - 2)}{(x + 3)^3(x^2 - 8)} \)
   (c) \( \lim_{x \to \infty} \frac{x^2 \sin(x^3)}{x^3 + 1} \)
   (d) \( \lim_{x \to \infty} \frac{x + \sqrt{x^2 + 1} + \sqrt{x^2 + 2}}{\sqrt{x^2 + x + \sqrt{x^2 - x}}} \)

2. Does this limit exist: \( \lim_{x \to \infty} \frac{x \sin x}{x + 1} \)?

3. Compute the following limits:
   (a) \( \lim_{x \to 1^-} \frac{2 + \sqrt{x + 3}}{2 - \sqrt{x + 3}} \)
   (b) \( \lim_{x \to 1^+} \frac{(x - 1)^{3/2}}{2 - \sqrt{x + 3}} \)
   (c) \( \lim_{x \to 1} \frac{x - 1}{2 - \sqrt{x + 3}} \)

4. Graph the function \( y = \frac{2x^2 - 3x - 2}{x^2 - 1} \) using intercepts and asymptotes. Find all intersections between the function and its asymptotes.

5. Let
   \[ f(x) = \begin{cases} 
   x^2 & x \leq 1 \\
   Ax + B & 1 < x < 2 \\
   -2x^2 & 2 \leq x 
   \end{cases} \]
   (a) Determine \( A \) and \( B \) so that \( y = f(x) \) is continuous everywhere.
   (b) Then graph the function \( y = f(x) \) and determine its range.

6. Let \( f(x) = \frac{x^2 + 1}{x} \) and \( g(x) = \frac{x + 2}{x^2} \). Do the graphs of \( y = f(x) \) and \( y = g(x) \) intersect for some \( x > 0 \)?

7. Compute the following limits:
   (a) \( \lim_{x \to 0} \frac{\sin 7x}{\sin 3x} \)
   (b) \( \lim_{x \to \pi} \frac{\sin 7x}{\sin 3x} \)
   (c) \( \lim_{x \to \infty} \frac{\sin 7x}{3x} \)

8. (An additional practice problem on the definition of limit.) Prove, by definition, that
   \( \lim_{x \to 1} \frac{x}{(x + 1)(x - 2)} = \frac{1}{2} \).