Discussion Problems 7 (Tue., Nov. 15)

1. Assume that a function $f$ is defined, continuous and differentiable for all $x$. Give a precise argument for your answer on the following two questions.
   (a) If $f(1) = 1$ and $f(2) = 3$, is it possible that $f'(x) > 3$ for all $x$?
   (b) If $f(0) = 5$ and $f'(0) = -1$, is it possible that $f(x) \leq 5$ for all $x$?

2. Consider the function $f(x) = x^4 - 2x^2$ on the domain $D = [0, 2]$.
   (a) Find the range of $y = f(x)$ on $D$.
   (b) Find the range of $y = f(x)^2$ on $D$.
   (c) Find the range of $y = \cos \left( \frac{\pi}{4} f(x) \right)$ on $D$.
   (d) Find the range of $y = \sin \left( \frac{\pi}{4} f(x) \right)$ on $D$. 