
Discussion Problems 8 (Thu., Mar. 8)

1. The population of a certain monkey was 25,000 on Feb. 10, 2017, and 26,000 on Feb. 10, 2018. Assuming exponential growth, when will it be 27,000?

2. A side wall of a swimming pool has a lid of one of the two shapes: (a) isosceles triangle; and (b) circle, as in the picture. In each case, determine the hydrostatic force acting on the lid.

3. A reservoir is obtained by rotating the curve $y = x^4$, $0 \leq x \leq 1$, around the $y$-axis (both $x$ and $y$ have units m). It is completely filled with liquid of density $\rho$. (a) Determine the work needed to pump the liquid out the top of the reservoir. (b) Determine the work needed to lift the full reservoir by 10m (so that its bottom is 10m higher than initially; assume that only the liquid inside the reservoir has mass). (c) Now assume that the liquid is leaking at the constant rate as the reservoir is lifted, so that the reservoir is empty exactly at the time it is lifted by 10m. Determine the work spent.

4. Determine the center of mass of the region bounded by $y = 2 \sin(2x)$, $0 \leq x \leq \pi/2$ and $y = 0$.

5. (a) Determine the center of mass of the region $R$ bounded by $y = x^3$ and $y = \sqrt{x}$. (b) Rotate $R$ around the line $y = (3/2)x + 1$. Compute the volume of the resulting solid.

6. A regular hexagon (with side length 1) is rotated around an axis as in the picture. Determine the volume of the resulting solid. (Minimize your work!)