(b) $r^2 = 4r\cos\theta$

The Cartesian equation:

$$r^2 = 4r\cos\theta$$

$$x^2 + y^2 = 4x \qquad \text{Substitution}$$

$$x^2 - 4x + y^2 = 0$$

$$x^2 - 4x + 4 + y^2 = 4 \qquad \text{Completing the square}$$

$$(x-2)^2 + y^2 = 4 \qquad \text{Factoring}$$

The graph: Circle, radius 2, center $(h, k) = (2, 0)$

(c) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

The Cartesian equation:

$$r(2\cos\theta - \sin\theta) = 4$$

$$2r\cos\theta - r\sin\theta = 4 \qquad \text{Multiplying by } r$$

$$2x - y = 4 \qquad \text{Substitution}$$

$$y = 2x - 4 \qquad \text{Solve for } y.$$

The graph: Line, slope $m = 2$, y-intercept $b = -4$ ■

Exercises 11.3

Polar Coordinates

1. Which polar coordinate pairs label the same point?
   a. $(3, 0)$ b. $(-3, 0)$ c. $(2, 2\pi/3)$
   d. $(2, 7\pi/3)$ e. $(-3, \pi)$ f. $(2, \pi/3)$
   g. $(-3, 2\pi)$ h. $(-2, -\pi/3)$
2. Which polar coordinate pairs label the same point?
   a. $(-2, \pi/3)$ b. $(2, -\pi/3)$ c. (r, θ)
   d. $(r, \theta + \pi)$ e. $(-r, \theta)$ f. $(2, -2\pi/3)$
   g. $(-r, \theta + \pi)$ h. $(-2, 2\pi/3)$
3. Plot the following points (given in polar coordinates). Then find all the polar coordinates of each point.
   a. $(2, \pi/2)$ b. $(2, 0)$
   c. $(-2, \pi/2)$ d. $(-2, 0)$
4. Plot the following points (given in polar coordinates). Then find all the polar coordinates of each point.
   a. $(3, \pi/4)$ b. $(-3, \pi/4)$
   c. $(3, -\pi/4)$ d. $(-3, -\pi/4)$

Polar to Cartesian Coordinates

5. Find the Cartesian coordinates of the points in Exercise 1.
6. Find the Cartesian coordinates of the following points (given in polar coordinates).
   a. $(\sqrt{2}, \pi/4)$ b. $(1, 0)$
   c. $(0, \pi/2)$ d. $(-\sqrt{2}, \pi/4)$
   e. $(-3, 5\pi/6)$ f. $(5, \tan^{-1}(4/3))$
   g. $(-1, 7\pi)$ h. $(2\sqrt{3}, 2\pi/3)$

Cartesian to Polar Coordinates

7. Find the polar coordinates, $0 \le \theta < 2\pi$ and $r \ge 0$, of the following points given in Cartesian coordinates.
   a. $(1, 1)$ b. $(-3, 0)$
   c. $(\sqrt{3}, -1)$ d. $(-3, 4)$
8. Find the polar coordinates, $-\pi \le \theta < \pi$ and $r \ge 0$, of the following points given in Cartesian coordinates.
   a. $(-2, -2)$ b. $(0, 3)$
   c. $(-\sqrt{3}, 1)$ d. $(5, -12)$
9. Find the polar coordinates, $0 \le \theta < 2\pi$ and $r \le 0$, of the following points given in Cartesian coordinates.
   a. $(3, 3)$ b. $(-1, 0)$
   c. $(-1, \sqrt{3})$ d. $(4, -3)$
10. Find the polar coordinates, $-\pi \le \theta < 2\pi$ and $r \le 0$, of the following points given in Cartesian coordinates.
    a. $(-2, 0)$ b. $(1, 0)$
    c. $(0, -3)$ d. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Graphing Sets of Polar Coordinate Points

Graph the sets of points whose polar coordinates satisfy the equations and inequalities in Exercises 11–26.

11. $r = 2$
12. $0 \le r \le 2$
13. $r \ge 1$
14. $1 \le r \le 2$

15. $0 \le \theta \le \pi/6, \quad r \ge 0$

16. $\theta = 2\pi/3, \quad r \le -2$

17. $\theta = \pi/3, \quad -1 \le r \le 3$

18. $\theta = 11\pi/4, \quad r \ge -1$

19. $\theta = \pi/2, \quad r \ge 0$

20. $\theta = \pi/2, \quad r \le 0$

21. $0 \le \theta \le \pi, \quad r = 1$

22. $0 \le \theta \le \pi, \quad r = -1$

23. $\pi/4 \le \theta \le 3\pi/4, \quad 0 \le r \le 1$

24. $-\pi/4 \le \theta \le \pi/4, \quad -1 \le r \le 1$

25. $-\pi/2 \le \theta \le \pi/2, \quad 1 \le r \le 2$

26. $0 \le \theta \le \pi/2, \quad 1 \le |r| \le 2$

Polar to Cartesian Equations

Replace the polar equations in Exercises 27–52 with equivalent Cartesian equations. Then describe or identify the graph.

27. $r \cos\theta = 2$

28. $r \sin\theta = -1$

29. $r \sin\theta = 0$

30. $r \cos\theta = 0$

31. $r = 4 \csc\theta$

32. $r = -3 \sec\theta$

33. $r\cos\theta + r\sin\theta = 1$

34. $r\sin\theta = r\cos\theta$

35. $r^2 = 1$

36. $r^2 = 4r\sin\theta$

37. $r = \dfrac{5}{\sin\theta - 2\cos\theta}$

38. $r^2 \sin 2\theta = 2$

39. $r = \cot\theta \csc\theta$

40. $r = 4\tan\theta \sec\theta$

41. $r = \csc\theta\, e^{r\cos\theta}$

42. $r\sin\theta = \ln r + \ln\cos\theta$

43. $r^2 + 2r^2\cos\theta\sin\theta = 1$

44. $\cos^2\theta = \sin^2\theta$

45. $r^2 = -4r\cos\theta$

46. $r^2 = -6r\sin\theta$

47. $r = 8\sin\theta$

48. $r = 3\cos\theta$

49. $r = 2\cos\theta + 2\sin\theta$

50. $r = 2\cos\theta - \sin\theta$

51. $r\sin\left(\theta + \dfrac{\pi}{6}\right) = 2$

52. $r\sin\left(\dfrac{2\pi}{3} - \theta\right) = 5$

Cartesian to Polar Equations

Replace the Cartesian equations in Exercises 53–66 with equivalent polar equations.

53. $x = 7$

54. $y = 1$

55. $x = y$

56. $x - y = 3$

57. $x^2 + y^2 = 4$

58. $x^2 - y^2 = 1$

59. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

60. $xy = 2$

61. $y^2 = 4x$

62. $x^2 + xy + y^2 = 1$

63. $x^2 + (y - 2)^2 = 4$

64. $(x - 5)^2 + y^2 = 25$

65. $(x - 3)^2 + (y + 1)^2 = 4$

66. $(x + 2)^2 + (y - 5)^2 = 16$

67. Find all polar coordinates of the origin.

68. **Vertical and horizontal lines**

    a. Show that every vertical line in the xy-plane has a polar equation of the form $r = a\sec\theta$.

    b. Find the analogous polar equation for horizontal lines in the xy-plane.

11.4 Graphing Polar Coordinate Equations

It is often helpful to graph an equation expressed in polar coordinates in the Cartesian xy-plane. This section describes some techniques for graphing these equations using symmetries and tangents to the graph.

Symmetry

Figure 11.27 illustrates the standard polar coordinate tests for symmetry. The following summary says how the symmetric points are related.

Symmetry Tests for Polar Graphs in the Cartesian xy-Plane

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph (Figure 11.27a).
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph (Figure 11.27b).
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph (Figure 11.27c).