and calculate

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_0^1 2\pi(1 - y)\sqrt{2}\, dy$$

$$= 2\pi\sqrt{2}\left[y - \frac{y^2}{2}\right]_0^1 = 2\pi\sqrt{2}\left(1 - \frac{1}{2}\right)$$

$$= \pi\sqrt{2}.$$

The results agree, as they should. ■

Exercises 6.4

Finding Integrals for Surface Area

In Exercises 1–8:

a. Set up an integral for the area of the surface generated by revolving the given curve about the indicated axis.

T **b.** Graph the curve to see what it looks like. If you can, graph the surface too.

T **c.** Use your utility's integral evaluator to find the surface's area numerically.

1. $y = \tan x, \quad 0 \le x \le \pi/4; \quad x\text{-axis}$

2. $y = x^2, \quad 0 \le x \le 2; \quad x\text{-axis}$

3. $xy = 1, \quad 1 \le y \le 2; \quad y\text{-axis}$

4. $x = \sin y, \quad 0 \le y \le \pi; \quad y\text{-axis}$

5. $x^{1/2} + y^{1/2} = 3$ from (4, 1) to (1, 4); x-axis

6. $y + 2\sqrt{y} = x, \quad 1 \le y \le 2; \quad y\text{-axis}$

7. $x = \displaystyle\int_0^y \tan t\, dt, \quad 0 \le y \le \pi/3; \quad y\text{-axis}$

8. $y = \displaystyle\int_1^x \sqrt{t^2 - 1}\, dt, \quad 1 \le x \le \sqrt{5}; \quad x\text{-axis}$

Finding Surface Area

9. Find the lateral (side) surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$, about the x-axis. Check your answer with the geometry formula

$$\text{Lateral surface area} = \frac{1}{2} \times \text{base circumference} \times \text{slant height}.$$

10. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$, about the y-axis. Check your answer with the geometry formula

$$\text{Lateral surface area} = \frac{1}{2} \times \text{base circumference} \times \text{slant height}.$$

11. Find the surface area of the cone frustum generated by revolving the line segment $y = (x/2) + (1/2), 1 \le x \le 3$, about the x-axis. Check your result with the geometry formula

$$\text{Frustum surface area} = \pi(r_1 + r_2) \times \text{slant height}.$$

12. Find the surface area of the cone frustum generated by revolving the line segment $y = (x/2) + (1/2), 1 \le x \le 3$, about the y-axis. Check your result with the geometry formula

$$\text{Frustum surface area} = \pi(r_1 + r_2) \times \text{slant height}.$$

Find the areas of the surfaces generated by revolving the curves in Exercises 13–23 about the indicated axes. If you have a grapher, you may want to graph these curves to see what they look like.

13. $y = x^3/9, \quad 0 \le x \le 2; \quad x\text{-axis}$

14. $y = \sqrt{x}, \quad 3/4 \le x \le 15/4; \quad x\text{-axis}$

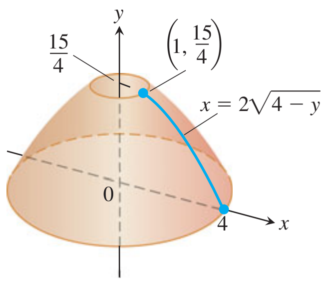
15. $y = \sqrt{2x - x^2}, \quad 0.5 \le x \le 1.5; \quad x\text{-axis}$

16. $y = \sqrt{x + 1}, \quad 1 \le x \le 5; \quad x\text{-axis}$

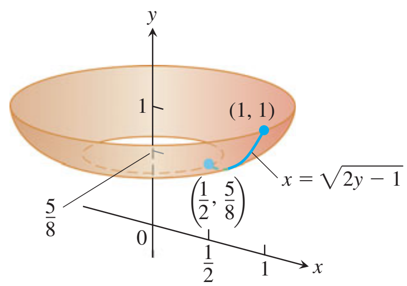
17. $x = y^3/3, \quad 0 \le y \le 1; \quad y\text{-axis}$

18. $x = (1/3)y^{3/2} - y^{1/2}, \quad 1 \le y \le 3; \quad y\text{-axis}$

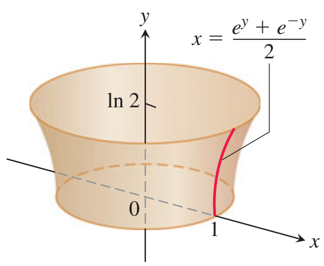
19. $x = 2\sqrt{4 - y}, \quad 0 \le y \le 15/4; \quad y\text{-axis}$



20. $x = \sqrt{2y - 1}, \quad 5/8 \le y \le 1; \quad y\text{-axis}$



21. $x = (e^y + e^{-y})/2, \quad 0 \le y \le \ln 2; \quad y\text{-axis}$



22. $y = (1/3)(x^2 + 2)^{3/2}$, $\quad 0 \le x \le \sqrt{2}$; $\quad y$-axis (*Hint:* Express $ds = \sqrt{dx^2 + dy^2}$ in terms of dx, and evaluate the integral $S = \int 2\pi x\, ds$ with appropriate limits.)

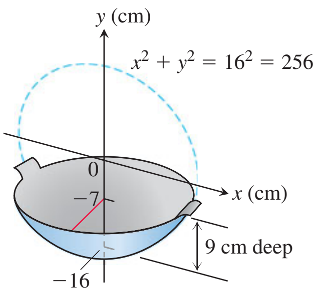
23. $x = (y^4/4) + 1/(8y^2)$, $\quad 1 \le y \le 2$; $\quad x$-axis (*Hint:* Express $ds = \sqrt{dx^2 + dy^2}$ in terms of dy, and evaluate the integral $S = \int 2\pi y\, ds$ with appropriate limits.)

24. Write an integral for the area of the surface generated by revolving the curve $y = \cos x, -\pi/2 \le x \le \pi/2$, about the x-axis. In Section 8.4 we will see how to evaluate such integrals.

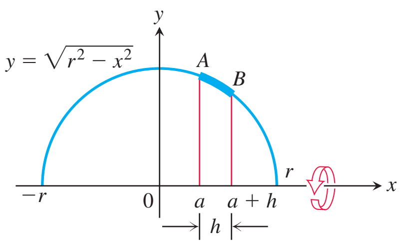
25. Testing the new definition Show that the surface area of a sphere of radius a is still $4\pi a^2$ by using Equation (3) to find the area of the surface generated by revolving the curve $y = \sqrt{a^2 - x^2}$, $\; -a \le x \le a$, about the x-axis.

26. Testing the new definition The lateral (side) surface area of a cone of height h and base radius r should be $\pi r\sqrt{r^2 + h^2}$, the semiperimeter of the base times the slant height. Show that this is still the case by finding the area of the surface generated by revolving the line segment $y = (r/h)x, 0 \le x \le h$, about the x-axis.

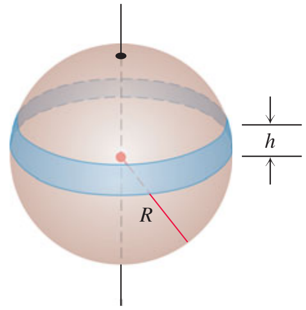
T **27. Enameling woks** Your company decided to put out a deluxe version of a wok you designed. The plan is to coat it inside with white enamel and outside with blue enamel. Each enamel will be sprayed on 0.5 mm thick before baking. (See accompanying figure.) Your manufacturing department wants to know how much enamel to have on hand for a production run of 5000 woks. What do you tell them? (Neglect waste and unused material and give your answer in liters. Remember that $1\text{ cm}^3 = 1\text{ mL}$, so $1\text{ L} = 1000\text{ cm}^3$.)



28. Slicing bread Did you know that if you cut a spherical loaf of bread into slices of equal width, each slice will have the same amount of crust? To see why, suppose the semicircle $y = \sqrt{r^2 - x^2}$ shown here is revolved about the x-axis to generate a sphere. Let AB be an arc of the semicircle that lies above an interval of length h on the x-axis. Show that the area swept out by AB does not depend on the location of the interval. (It does depend on the length of the interval.)



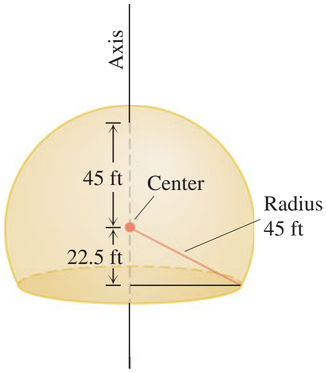
29. The shaded band shown here is cut from a sphere of radius R by parallel planes h units apart. Show that the surface area of the band is $2\pi Rh$.



30. Here is a schematic drawing of the 90-ft dome used by the U.S. National Weather Service to house radar in Bozeman, Montana.

a. How much outside surface is there to paint (not counting the bottom)?

T **b.** Express the answer to the nearest square foot.

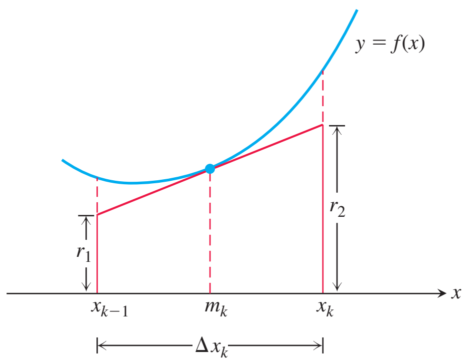


31. An alternative derivation of the surface area formula Assume f is smooth on $[a, b]$ and partition $[a, b]$ in the usual way. In the kth subinterval $[x_{k-1}, x_k]$, construct the tangent line to the curve at the midpoint $m_k = (x_{k-1} + x_k)/2$, as in the accompanying figure.

a. Show that

$$r_1 = f(m_k) - f'(m_k)\frac{\Delta x_k}{2} \quad \text{and} \quad r_2 = f(m_k) + f'(m_k)\frac{\Delta x_k}{2}.$$

b. Show that the length L_k of the tangent line segment in the kth subinterval is $L_k = \sqrt{(\Delta x_k)^2 + (f'(m_k)\,\Delta x_k)^2}$.



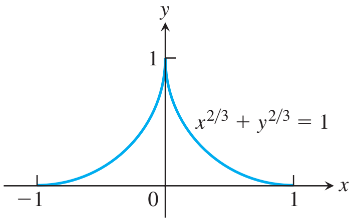
c. Show that the lateral surface area of the frustum of the cone swept out by the tangent line segment as it revolves about the x-axis is $2\pi f(m_k)\sqrt{1 + (f'(m_k))^2}\,\Delta x_k$.

d. Show that the area of the surface generated by revolving $y = f(x)$ about the x-axis over $[a, b]$ is

$$\lim_{n\to\infty} \sum_{k=1}^{n} \begin{pmatrix}\text{lateral surface area}\\ \text{of } k\text{th frustum}\end{pmatrix} = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx.$$

32. The surface of an astroid Find the area of the surface generated by revolving about the x-axis the portion of the astroid $x^{2/3} + y^{2/3} = 1$ shown in the accompanying figure.

(*Hint:* Revolve the first-quadrant portion $y = (1 - x^{2/3})^{3/2}$, $0 \le x \le 1$, about the x-axis and double your result.)



6.5 Work and Fluid Forces

In everyday life, *work* means an activity that requires muscular or mental effort. In science, the term refers specifically to a force acting on an object and the object's subsequent displacement. This section shows how to calculate work. The applications run from compressing railroad car springs and emptying subterranean tanks to forcing subatomic particles to collide and lifting satellites into orbit.

Work Done by a Constant Force

When an object moves a distance d along a straight line as a result of being acted on by a force of constant magnitude F in the direction of motion, we define the **work** W done by the force on the object with the formula

$$W = Fd \qquad \text{(Constant-force formula for work).} \tag{1}$$

From Equation (1) we see that the unit of work in any system is the unit of force multiplied by the unit of distance. In SI units (SI stands for *Système International*, or International System), the unit of force is a newton, the unit of distance is a meter, and the unit of work is a newton-meter ($\text{N}\cdot\text{m}$). This combination appears so often it has a special name, the **joule**. In the British system, the unit of work is the foot-pound, a unit sometimes used in applications.

Joules
The joule, abbreviated J, is named after the English physicist James Prescott Joule (1818–1889). The defining equation is

$$1 \text{ joule} = (1 \text{ newton})(1 \text{ meter}).$$

In symbols, $1\text{ J} = 1\text{ N}\cdot\text{m}$.

EXAMPLE 1 Suppose you jack up the side of a 2000-lb car 1.25 ft to change a tire. The jack applies a constant vertical force of about 1000 lb in lifting the side of the car (but because of the mechanical advantage of the jack, the force you apply to the jack itself is only about 30 lb). The total work performed by the jack on the car is $1000 \times 1.25 = 1250$ ft-lb. In SI units, the jack has applied a force of 4448 N through a distance of 0.381 m to do $4448 \times 0.381 \approx 1695$ J of work. ■

Work Done by a Variable Force Along a Line

If the force you apply varies along the way, as it will if you are stretching or compressing a spring, the formula $W = Fd$ has to be replaced by an integral formula that takes the variation in F into account.

Suppose that the force performing the work acts on an object moving along a straight line, which we take to be the x-axis. We assume that the magnitude of the force is a continuous function F of the object's position x. We want to find the work done over the interval from $x = a$ to $x = b$. We partition $[a, b]$ in the usual way and choose an arbitrary point c_k in each subinterval $[x_{k-1}, x_k]$. If the subinterval is short enough, the continuous function F