A good strategy to run away? You are the captain of a US Coast Guard speedboat, patrolling the Pacific coast on a foggy day. All of a sudden, the fog lifts and you observe what is obviously a smuggling boat a distance \( d \) away. Unfortunately, the fog instantly descends and the visibility again becomes zero. Here are the assumptions you may make. First, the speed of the smuggling boat is known to be 1 (in appropriate units) and the speed of your boat is \( v > 1 \). Second, the smuggling boat will move in a straight line in an unknown direction from where it was spotted. Can you move your boat on a curve guaranteed to catch the smugglers? (You may suppose that the boats are points and that for the smugglers to be caught the two boats have to be at the same point at the same time.)
**Solution.** Set the coordinate system so that initially the smugglers are at the origin and the police is at \((d, 0)\). Move the police boat toward the origin until it is at the point where it would meet the smugglers if they proceeded to move on the positive \(x\) axis. This is the point \(\left(\frac{d}{v + 1}, 0\right)\). From now on, i.e. \(t \geq t_0 = \frac{d}{v + 1}\), move the police on the curve
\[
(x(t), y(t)) = (t \cdot \cos \theta, t \cdot \sin \theta),
\]
where \(\theta = \theta(t)\) is the angle to be determined, but \(\theta(t_0) = 0\). Note that this means that the police is at distance \(t\) from the origin at time \(t\), which is true for the smugglers as well! Therefore, if it is possible for the police to make \(\theta\) sweep all angles in \([0, 2\pi)\), we are done.

Now this becomes a calculus problem. First we compute \(x'(t), y'(t)\) and then, after a short calculation, obtain
\[
x'(t)^2 + y'(t)^2 = 1 + t^2(\theta')^2.
\]
As the derivative of the arc length equals, on the one hand, \(\sqrt{x'(t)^2 + y'(t)^2}\), and, on the other hand, \(v\), we get
\[
v^2 = 1 + t^2(\theta')^2.
\]
Thus
\[
\theta' = \sqrt{v^2 - 1}/t
\]
and
\[
\theta = \sqrt{v^2 - 1} \log(t/t_0).
\]
The police moves on the spiral, given in polar coordinates by
\[
r = \frac{d}{v + 1} \cdot \exp \left(\frac{\theta}{\sqrt{v^2 - 1}}\right).
\]