Math/Stat 235A, Fall 2025

Homework 1, a review of combinatorial probability

1. Roll a single die 10 times. Compute the following probabilities: (a) that you get at least one 6, (b) that you get at least one 6 *and* at least one 5, (c) that you get three 1's, two 2's and five 3's.

2. Consider the following game. Roll a fair die and (independently) shuffle a full deck of cards. If you roll 3 or less, you lose immediately. Otherwise deal yourself as many cards from the top of the deck as the number that came up on the die. You win if all four aces are among the dealt cards. Compute the winning probability for this game.

3. A chocolate egg either contains a toy or is empty. Assume that each egg contains a toy with probability p, independently of other eggs. You have 5 eggs; you open the first one, see if it has a toy inside, then do the same for the second one, etc. Let E_1 be the event that you get at least 4 toys, and E_2 the event that you get at least two toys in succession. Compute $P(E_1)$ and $P(E_2)$. Are E_1 and E_2 independent?

4. Again, a chocolate egg either contains a toy or is empty. Assume that each egg contains a toy with probability p, independently of other eggs. There are only three kinds of toys: red, blue, and green. Assume that every non–empty egg contains one of the three toys chosen at random, again independently of other eggs. You buy 5 eggs. Compute the probability that you get a complete set of toys, that is, at least one red toy, at least one blue toy, and at least one green toy.

5. A lottery ticket consists of two rows, each containing 3 numbers from $1, 2, \ldots, 50$. The drawing consists of choosing 5 different numbers from $1, 2, \ldots, 50$ at random. A ticket wins if its first row contains *at least two* of the numbers drawn, *and* the second row contains *at least two* of the numbers drawn. Thus there are four types tickets, as shown in the table below. For example, if the numbers

Ticket 1	Ticket 2	Ticket 3	Ticket 4
1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	1 2 3	2 3 4	3 4 5

1, 3, 5, 6, 17 are drawn, then Ticket 1, Ticket 2 and Ticket 4 all win, while Ticket 3 loses. Compute the winning probabilities for all four tickets.

6. Assume that the set U has n elements. Select r independent random subsets $A_1, \ldots, A_r \subset U$. All A_i are chosen so that all 2^n choices are equally likely. Compute (in a closed form) the probability that the A_i are pairwise disjoint.

7. The following problem was reputedly set before job-seekers at a Wall Street brokerage.

As a broker, your job is to accommodate your client's wishes without placing any of your personal capital at risk. Your client wishes to place a \$1,000 bet on the outcome of the world series, a baseball contest decided in favor of whichever of two teams first wins 4 games. The client deposits his \$1,000 with you in advance of the series. At the end of the series he must receive from you either \$2,000 if his team wins, or nothing if his team loses. No market exists for bets on the entire world series. However, you can place even-odds bets, in any amounts, on each game individually.

What is your strategy for placing bets on the individual games in order to achieve the cumulative result demanded by your client?