Math 235A, Fall 2025

Homework 6

Durrett: 2.4.1, 2.4.2, 3.2.13, 3.2.14, 3.2.15 (You can use the following two facts, without a proof. (1) If the components of a random vector X with values in $\mathbb{R}^n$ are i.i.d. standard normal, and $A \in \mathbb{R}^{n\times n}$ is orthogonal, then the components of AX are also i.i.d. standard normal. (2) Uniform probability measure on the sphere $S^{n+1} \subset \mathbb{R}^n$ is the unique probability measure on S^{n+1} invariant under orthogonal transformations (a.k.a. the Haar measure).)

1. Let $X_1, X_2, \ldots$ be i.i.d. uniform on $[0, 1]$, and $M_n = \max\{X_1, \ldots, X_n\}$. Show that, as $n \to \infty$, $M_n \to 1$ a.s. and find a simple (deterministic) sequence a_n so that $Y_n = a_n(1 - M_n)$ converges in distribution to a non-trivial limit. Does F_{Y_n} converge *uniformly* to its limit?