Math 235A, Fall 2025

**Homework 7**

1. Assume that you have $n$ pairs of socks, of $n$ different colors, in a drawer. You start pulling socks out one by one at random without replacement (each next sock is chosen from the remaining ones with equal probability). Let $T_n$ be the first time (measured in the number of pulled-out socks) you get a matching pair. Determine the distribution of $T_n$ exactly and find a simple (deterministic) sequence $a_n$ so that $T_n/a_n$ converges in distribution, as $n \to \infty$, to a non-trivial limit. Determine the asymptotics of $E(T_n)$.

2. Let $X_1, X_2, \dots$ be i.i.d. with $E(X_1) = 0$ and $E(X_1^4) < \infty$. Let $S_n = X_1 + \cdots + X_n$. Show that $|S_n/\sqrt{n}|^\alpha$ are uniformly integrable for any $0 \le \alpha < 4$.