Math 235A, Fall 2025

Homework 8

This is the last homework.

Durrett: 3.3.2(i), 3.3.3 (prove the second equality without using Ex. 2.1.5), 3.4.4, 3.4.5.

1. (a) Assume X_n are Poisson(λ_n), and $\lambda_n \to \infty$ and $n \to \infty$. Show that $(X_n - \lambda_n)/\sqrt{\lambda_n} \xrightarrow{d} \mathrm{N}(0,1)$. (You only need to understand Example 3.4.8 in Durrett for this.) (b) Compute the limit

$$\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!}.$$

2. Assume a random variable X has $0 < E(X^2) < \infty$ and has the property that if $X_1 \stackrel{d}{=} X$ and $X_2 \stackrel{d}{=} X$ are independent, then $X \stackrel{d}{=} (X_1 + X_2)/\sqrt{2}$. Show that X is Normal. (The finite second moment assumption is not necessary, but without it the problem is much harder.)