

Homework 7

Durrett, Chapter 5, 5.3.6 (ignore (i) and (ii); instead, use Theorem 5.3.11 to show recurrence when $C < 1/4$ and transience when $C > 1/4$), 5.3.7.

1. Let $S_n^{(d)}$ be the d -dimensional simple symmetric simple random walk. Determine the asymptotics of $P_0(S_{2n}^{(d)} = 0)$.

2. Let again $S_n^{(d)}$ be the d -dimensional simple symmetric random walk, Let

$$R_n = |\{S_0^{(d)}, S_1^{(d)}, \dots, S_n^{(d)}\}|$$

be the number of points visited through time n . Show that, as $n \rightarrow \infty$,

$$E_0(R_n)/n \rightarrow 1 - \rho_{00} = P_0(\text{no return to } 0).$$

(*Hint.* Show that the the following two expressions both approach the same limit as $E_0(R_n)/n$:

$$\frac{1}{n} \sum_{k=0}^n P_0(\text{visit new site at time } k), \frac{1}{n} \sum_{k=0}^n P_0(\text{no return to } 0 \text{ up to time } k).$$