Math 236A, Fall 2012.

**Homework Assignment 3**

**Due:** Oct. 26, 2012

1. Let \( \tau_a \) be the first time a standard Brownian motion in one dimension hits \( a > 0 \).
   (a) Compute the density of \( \tau_a \).
   (b) Either from (a) or (much more easily) by using an appropriate martingale, compute \( E(\exp(-\lambda \tau_a)) \), for any \( \lambda > 0 \).
   (c) Let \( b > 0 \) and let \( \tau_{-b} \) be the first time the Brownian motion hits \( -b \). Show that
   \[
   E(\exp(-\lambda \tau_{-b}) 1_{\{\tau_a < \tau_{-b}\}}) = E(\exp(-\lambda \tau_a) 1_{\{\tau_a < \tau_{-b}\}}) \cdot E(\exp(-\lambda \tau_{a+b})).
   \]
   (d) Let \( \tau = \tau_a \wedge \tau_{-a} \). Compute \( E(\exp(-\lambda \tau)) \).

2. Let \( B \) be the Brownian motion in two dimensions, started at \((0,a), a > 0\). Let now \( \tau \) be the first time \( B \) hits the line \( ax \). Also, let \( X \) be the x-coordinate of the point \( B(\tau) \).
   (a) Determine the density of \( X \) when \( \alpha = 0 \). \textit{(Hint.} Condition on the value of the stopping time \( \tau_a \) from problem 1(a).\textit{)}
   (b) Now determine the density of \( X \) when \( \alpha \neq 0 \). \textit{(Hint. Use orthogonal invariance.)}