Math 236A, Fall 2022.

Homework Assignment 3

Due: Oct. 19, 2022

1. Let τ_x be the first time a standard Brownian motion in one dimension hits x. Below, assume $a, \lambda > 0$.

(a) Compute the density of τ_a .

(b) Either from (a) or (much more easily) by using an appropriate martingale, compute $E(\exp(-\lambda\tau_a))$.

(c) Let $\tau = \tau_a \wedge \tau_{-a}$. Show that τ and $B(\tau)$ are independent. Use this to compute $E(\exp(-\lambda\tau))$.

2. Let B be the Brownian motion in two dimensions, started at (0, a), a > 0. Let now τ be the first time B hits the line αx . Also, let X be the x-coordinate of the point $B(\tau)$.

(a) Determine the density of X when $\alpha = 0$. (*Hint*. Condition on the value of the stopping time τ_a from problem 1(a).)

(b) Now determine the density of X when $\alpha \neq 0$. (*Hint*. Use orthogonal invariance.)