Math 236A, Fall 2022.

Homework Assignment 6

Due: Nov. 23, 2022.

1. Assume that B_t^1 and B_t^2 are independent Brownian motions. Consider $X_t = \int_0^t B_s^1 dB_s^1$ and $Y_t = \int_0^t B_s^2 dB_s^1$. Then the two bracket processes $\langle X \rangle_t$ and $\langle Y \rangle_t$ are equal in distribution, but X_t and Y_t are not equal in distribution for any fixed t > 0. Prove this.

2. Assume that h_1, \ldots, h_n are *deterministic* real functions which are orthonormal in $L^2[0, t]$. Find the distribution of the vector $(\int_0^t h_1 dB_s, \ldots, \int_0^t h_n dB_s)$.

3. Find the stochastic integral representation of the following random variables (for a fixed *T*): $B_{T/2}B_T$, $\left(\int_0^T s \, dB_s\right)^2$, $\int_0^T s B_s \, ds$. You may (but are not required to) use Malliavin calculus.