3.3.1. We will give a proof for sup $K$; the proof for inf $K$ is the same. As $K$ is bounded, sup $K$ is a real number. Further, by Problem 3 on Discussion 8, sup $K \in \overline{K}$, as it is either in $K$ or a limit point of $K$. As $K$ is also closed, $\overline{K} = K$ and thus sup $K \in K$.

3.3.2. As a subset of the bounded set $K$, $K \cap F$ is bounded. As the intersection of two closed sets, $K \cap F$ is closed. By Heine-Borel Theorem, $K \cap F$ is compact.

3.3.5. (a) Not compact as not closed (and not bounded either). Take any sequence of rational numbers converging to an irrational number, say $\sqrt{2}/2$.
(b) Not compact as not closed. See the sequence in (a).
(c) Not compact as not bounded. Take $x_n = n$, which does not have a bounded subsequence, and therefore cannot have a convergent one.
(d) Compact as finite.
(e) Not compact as not closed. The sequence $x_n = 1/n$ of elements in the set converges to 0 and so does any of its subsequences. But 0 is not contained in the set.
(f) Compact. The only limit point is 1, which is included in the set, so the set is closed and bounded.

3.3.7. (a) Yes, as it is closed and bounded.
(b) No. Take $A = (0, 1)$, $K = [-1, 1]$; then $A \cap K = A$, which is not closed thus not compact.
(c) No. Take $F_n = [n, \infty)$. These are closed nested intervals, but $\cap_{n=1}^{\infty} F_n = \emptyset$.
(d) Yes, it is a finite union of closed sets (singletons), thus closed, and clearly bounded.
(e) No. The set may not be bounded, e.g., $\mathbb{N}$.

3.3.9 (b). Let $x_n$ be an increasing sequence of irrational numbers, with $x_1 < 0$, that converges to $\sqrt{2}/2$, say $x_n = \sqrt{2}/2 - 1/n$. Then $\mathcal{C} = \{(x_1, x_2), (x_2, x_3), \ldots\} \cup \{\sqrt{2}/2, 2\}$ is a pairwise disjoint infinite open cover of $[0, 1] \cap \mathbb{Q}$ with no finite subcover — in fact, if we remove any set from $\mathcal{C}$, we no longer have a cover.

3.3.10. Such sets must be finite: as singletons are closed, and every set is covered by its singletons, a compact set must be a finite union of singletons, thus finite.