Homework 5
Math 128A
Due Friday, December 6th

1. Write programs to perform numerical integration of a given function, f, over a given interval $[a,b]$ using

   (i) composite trapezoidal rule
   (ii) composite Simpson's rule
   (iii) composite 3-point Gaussian quadrature (3 points per subinterval).

   Your routines should take as inputs: the integrand f, the endpoints a and b, and the number the number of subintervals n.

   (a) Note that each quadrature rule requires a different number of points for a given number of subintervals. For each of the three composite quadrature rules how many function evaluations are required?

   (b) Apply each of these composite quadratures to approximate

   $$\int_0^1 \frac{4}{1+x^2}\,dx.$$

   Make a table of the results for $n = 2, 4, 8, 16, 32$, and a table of the errors. How is the order of accuracy demonstrated in the table of errors? Make a log-log plot of the error vs. the number of function evaluations for each of the three composite quadratures on the same axes. Comment on your results.

   (c) Repeat the previous problem for

   $$\int_0^1 \sqrt{x}\,dx.$$

   Discuss your results. In particular, why are the results for the observed order of accuracy different from the previous problem?

2. The three point open Newton-Cotes formula is

   $$\int_a^b f(x)\,dx \approx \frac{4h}{3}\Big(2f(x_1) - f(x_2) + 2f(x_3)\Big),$$

   where $h = (b-a)/4$ and $x_j = jh + a$.

   (a) Derive this formula by integrating the appropriate interpolating polynomial.

   (b) Apply the formula to the monomials x^k for $k = 0, 1, \ldots$ for $a = 0$ and $b = 1$ to determine the degree of precision.

   (c) Derive the integration formula based on the unequally spaced points $x_1 = a + h$, $x_2 = a + 2h$, $x_3 = a + 7h/2$, and determine its degree of precision.

   (d) Using a mathematical argument, explain the origin of the difference in precision between these two integration formulas.