Conservation of whooping cranes

Whooping Cranes are large, heron-like birds that live in the west/mid-west of America (Alberta and Wisconsin) in the summer, and migrate south (Texas and Florida, respectively) in the winter. A combination of hunting and habitat loss nearly drove them to extinction in the 1930s. In 1967, the Whooping Crane was declared endangered, and it has been the subject of intense conservation efforts ever since. The conservation effort has had limited success due, in part, to the longevity of the birds (they can live to be over 25 years), and their long, difficult migration.

Below is a table showing the number of wild Whooping Cranes each decade from 1940 to 2010. It comes from a paper by Butler, Harris and Strobel, published in the journal Biological Conservation in 2013.

<table>
<thead>
<tr>
<th>Year</th>
<th>time since start (years)</th>
<th>Number of Cranes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1940</td>
<td>0</td>
<td>26</td>
</tr>
<tr>
<td>1950</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>1960</td>
<td>20</td>
<td>37</td>
</tr>
<tr>
<td>1970</td>
<td>30</td>
<td>58</td>
</tr>
<tr>
<td>1980</td>
<td>40</td>
<td>79</td>
</tr>
<tr>
<td>1990</td>
<td>50</td>
<td>146</td>
</tr>
<tr>
<td>2000</td>
<td>60</td>
<td>180</td>
</tr>
<tr>
<td>2010</td>
<td>70</td>
<td>283</td>
</tr>
</tbody>
</table>

a) Plot the raw data with the following commands:

```r
t=seq(from = 0, to = 70, by = 10)
N=c(26,32,37,58,79,146,180,283)
plot(t,N)
```

Make a guess: is the data linear, power law or exponential? To confirm your intuition we will look at both possibilities by making a log-log plot (part b) as well as a log-linear plot (part c) and see which has a better fit to a line.
b) Type in the following lines of code to plot the data on a log-log scale then fit a line to that data

```r
t=seq(from = 10, to = 70, by = 10)
logt = log(t, base=10)
N=c(32,37,58,79,146,180,283)
G<-lm(log(N, base = 10)~logt)
plot(logt, log(N, base = 10))
lines(logt, fitted(G))
```

Does that look like a good fit? We will next try a log-linear and then decide which was a better fit.

c) Type in the following lines of code to plot the data as a straight line and then fit a line to that data

```r
t=seq(from = 0, to = 70, by = 10)
N=c(26,32,37,58,79,146,180,283)
F<-lm(log(N, base = 10)~t)
plot(t, log(N, base = 10))
lines(t, fitted(F))
```

d) Go back to your guess and look at your log-log and log-linear fits to lines. What is the functional relationship between \( N \) and \( t \) (i.e. power law, exponential, etc.)?

To get the equation for the best-fit line, type in “F” or “G” at the prompt in R-studio. If the linear fit is of the form \( y = mx + b \), then the coefficient called “intercept” is \( b \) and the other coefficient is the slope.

e) Based on your previous answer, find an equation for \( N(t) \), which is a model for the number of whooping cranes as a function of time (where time is measured in \textbf{years} since 1940).

f) In part (e), your answer should be of the form \( N(t) = ac^t \), or \( N(t) = at^c \) where \( a \) and \( c \) are constants. You only have to find the model for one (power law OR exponential depending on your answer to (d)). What do the parameters \( a \) and \( c \) represent?

g) Your equation for \( N(t) \) (from part e) is a \textit{model}. It makes some assumptions, and we could test these assumptions. A critical assumption of the model is that the growth rate, \( c \), does not vary with time. Therefore, even though we don’t have data for the years between our 10 year measurements, the model predicts them (i.e. the
model predicts there were 247 cranes in 2009). In fact, the model even predicts the number of cranes down to the minute. Discuss and answer the following (in complete sentences): Do you think the model is reliable in its predictions of the number of cranes over very short intervals? Are there reasons to suspect that growth rate might not be a constant over some time intervals? Hint: whooping cranes have a very specific breeding season, and migrate from Canada down to Texas.

h) The number of cranes at 10 year intervals forms a sequence, \( N_0 = 26, N_1 = 32, N_2 = 37, \) etc. Use your model from part e (i.e. your equation for \( N(t) \)) to generate a sequence of the predicted number of cranes, \( P_0, P_1, \) etc., up to \( P_7. \)

i) Discuss and answer the following (in complete sentences): Are the predicted values close? Do you think you could do better? Where is the prediction the best? Where is it the worst? Could you predict the number of cranes 10 years in the future? 100 years? Would you have confidence in these predictions?

j) Find a recursion, of the form \( P_{i+1} = rP_i \) (where \( r \) is a constant that you must determine) that describes the predicted number of cranes. Don’t forget that your prediction in (e) assumes that time is measured in years while the data \( P_0, P_1, \) etc., up to \( P_7 \) is given in 10 year increments. How does that effect your value for \( r? \) What does the constant \( r \) represent? How does it relate to the parameter \( c \) in your answers to parts (e) and (f)?

k) Check your answer to part (e) with a “for” loop in R-studio. Here’s an example (with made up numbers) for how it works. Pretend my recursion from part j) is \( P_{i+1} = 2P_i, \) and suppose that \( P_0 = 10. \) Then, the following code will generate the sequence \( P_i, \) stored in the variable \( P. \) Note that the first entry of \( P \) will be \( P_0, \) the second \( P_1, \) etc.

```
P=10
for(i in 1:7){P[i+1]=2*P[i]}
P
```

Bonus questions: In what year will there be 1,000 cranes? How about 10,000 – the estimated number of cranes that existed before humans arrived in America?