MATH 17A - DISCUSSION #2

Problem 1: Conservation of whooping cranes Whooping Cranes are large, heron-like birds that live in the west/mid-west of America (Alberta and Wisconsin) in the summer, and migrate south (Texas and Florida, respectively) in the winter. A combination of hunting and habitat loss nearly drove them to extinction in the 1930s. In 1967, the Whooping Crane was declared endangered, and it has been the subject of intense conservation efforts ever since. The conservation effort has had limited success due, in part, to the longevity of the birds (they can live to be over 25 years), and their long, difficult migration.

Below is a table showing the number of wild Whooping Cranes each decade from 1940 to 2010. It comes from a paper by Butler, Harris and Strobel, published in the journal Biological Conservation in 2013.

Table 1: Data from Butler et al. 2013

Year	time since start (years)	Number of Cranes
1940	0	26
1950	10	32
1960	20	37
1970	30	58
1980	40	79
1990	50	146
2000	60	180
2010	70	283

(a) Plot the raw data with the following commands:

```
t=seq(from = 0, to = 70, by = 10)
N=c(26,32,37,58,79,146,180,283)
plot(t,N)
```

(b) We begin by investigating whether the population exhibits exponential growth. We will plot the data on a semilog graph and find the best fit line to $\log_{10}(N)$ vs. t. Assuming you have already entered t and N into R (if not use the code above), the commands for making the semilog plot with the data and best fit line are as follows:

```
logN = log10(N)
F = lm( logN~t )
plot(t,logN)
lines(t,fitted(F))
```

(c) The equation of the best fit line defines a model; i.e. an equation which gives an approximation to the number of cranes as a function of time. To get the equation for the best-fit line, type in "`F`" (what we named the output from the fitting command) at the prompt in R. If the linear fit is of the form $y = mx + b$, then the coefficient called "intercept" is b and the other coefficient is the slope. Let $N(t)$ represent the model equation for the number of whooping cranes as a function of time (where time is measured in **years** since 1940).

- Use the equation of the best fit line to find the equation for $N(t)$.
- Plot the original data and a graph of $N(t)$ on the same plot. The commands for plotting the data are given in part (a). To add the model, evaluate your expression for $N(t)$ at the same time points, and use the `lines` command to add the curve to the plot.

(d) Discuss and answer the following (in complete sentences):

- Could you predict the number of cranes 10 years in the future? 100 years? Explain. Would you have confidence in these predictions?
- Even though we don't have data for the years between our 10 year measurements, the model predicts the number of cranes at any year. In fact, the model predicts the number of cranes down to the minute. Do you think the model is reliable in its predictions of the number of cranes over very short intervals?

Problem 2: Exploring Recursions In class we examined a model of population growth of the form

$$N_{k+1} = RN_k$$

where N_k is the population size at generation k. We know that this model results in exponential growth or decay depending on the value of R. We know this because we can write down the solution to the recursion equation, i.e. we can express N as an explicit function of k. For many recursion relations we cannot find a formula for the solution. We must generate solutions by performing the recursion. This sounds tedious, but it is fast and straightforward using a computer.

(a) Consider a population growing by 3% per year with an initial size of one million. This is described by the recursion relation and initial condition

$$\begin{aligned} N_{k+1} &= 1.03N_k \\ N_0 &= 1, \end{aligned}$$

where N_k is the population size in year k in millions. Compute the solution for

100 years using the recursion and plot the result using the code below.

```
N=1
kmax=100
for(i in 1:kmax){N[i+1]=1.03*N[i]}
plot(0:kmax,N,xlab="year",ylab="population in millions")
```

The solution values are stored in an array named `N`. Note that the first entry of the array, accessed by `N[1]`, is N_0, and the second element of the array is N_1 and so on. This code consists of a loop in which the line of code is executed repeatedly with the value of i changing each time. This performs `kmax` iterations of the recursion and fills in the values in the array.

You can look at the values by simply entering
`N`
on the R command line.

(b) Now consider a model of a population described by the recursion

$$N_{k+1} = N_k + 0.10 N_k \left(1 - \frac{N_k}{10}\right) \tag{1}$$

- Find all fixed points of this recursion defined by equation (1).
- We can modify the code from part (a) to perform this recursion by chaning the line with the for loop to

  ```
  for(i in 1:kmax){N[i+1]=N[i] + 0.10*N[i]*(1-N[i]/10)}
  ```

  Find and plot the solution to equation (1) for 100 years for the following initial conditions: $N_0 = 1$, $N_0 = 0$, $N_0 = 10$, $N_0 = 12$.
- You should make four plots. For each plot, write a sentence commenting on the solution.

(c) Repeat part (b) for the model

$$N_{k+1} = N_k + 2.10 N_k \left(1 - \frac{N_k}{10}\right) \tag{2}$$

Specifically:

- Find all fixed points of the recursion defined by equation (2).
- Using R, find and plot the solution to equation (2) for 100 years for the following initial conditions: $N_0 = 1$, $N_0 = 0$, $N_0 = 10$, $N_0 = 12$.
- You should make four plots. For each plot, write a sentence commenting on the solution.