A DANGER OF BEING SMALL: GETTING WET

“A man coming out of a bath carries with him a film of water about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed.”


How do you think Haldane came up with these conclusions? Did he go out and weigh humans, mice, and flies before and after dipping them in water? Probably not. In fact, these statements are probably not that precise. The main point of Haldane’s statement is that as you get smaller the more dangerous getting wet becomes. Let’s build an idealized mathematical model to see why this is.

(a) Let $R$ denote the ratio between the mass of the water film clinging to an animal’s body after getting wet ($M_W$) and the dry animal’s body mass ($M_B$). Below we will express $R$ as a function of $M_B$. Explain why $R$ as a function of $M_B$ something we’d like to know?

(b) We will make the following assumptions:

- assume that all animals are approximately the same shape, and model the body of an animal as a simple geometrical shape; e.g. a sphere or a cube.
- assume for simplicity that the volume of water ($V_w$) on the surface is given by the product of surface area of the animal ($A$) and the thickness of the water film ($\ell$); i.e. $V_w = A\ell$. We will take Haldane’s estimate of the thickness of the water film ($\ell = 0.05$ cm).
- assume that the density of an animal ($\rho$) is the same as the density of water ($\rho = 1$ g/cm$^3 = 10^{-3}$ kg/cm$^3$).

Under the above assumptions, find $R$ as a function of $M_B$.

(c) Sketch a graph of $R$ as a function of $M_B$ from the previous part.

(d) Use R/R-studio to compute $R$ for a human (60 kg), a cat (5 kg), a rat (0.25 kg), a mouse (0.02 kg), a shrew (0.004 kg), a bee (0.0001 kg), a housefly (0.000002 kg),
and a mosquito (0.0000025 kg). Plot the data points corresponding to these animals on your graph of $R$ vs $M_B$ from the previous part. Try making plots with log scales for none, one, or both of the variables. Comment on your choice of scaling for the axes.

You can use the line of code below to enter the data:

```r
Mb=c(60, 5, 0.25, 0.02, 4e-3, 1e-4, 2e-5, 2.5e-6)
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(e) What can you conclude from your mathematical analysis? Do your calculations agree with Haldane’s assertions? Do you see any issues with your/Haldane’s conclusions that getting wet poses serious dangers to small animals?

(f) Consider the assumptions that you made in coming up with your mathematical model. Are they reasonable assumptions? How could you make your model more precise? Do you think your basic conclusions would change significantly if you constructed a more precise (but probably more complicated) model?