

MAT 17A - DISCUSSION #3

The falling penny

The concept of how a function's output changes as its input changes is central to many branches of science. 17A instructor Sam Walcott made a video of a falling penny in front of a piece of lined paper. Watch the video (Penny_Drop.mov); it's very short. Let's calculate the penny's velocity based on its change in position. The velocity is the change in the penny's position (y) with respect to time (t), and it is written as follows

$$v = \lim_{\Delta t \rightarrow 0} \frac{y(t + \Delta t) - y(t)}{\Delta t}$$

We will use the data from the movie to approximate the velocity. The movie goes very fast, and so to help with the calculation you are given six still frames (Frame_1.png, Frame_2.png, Frame_3.png, Frame_4.png, Frame_5.png, and Frame_6.png) equally spaced $1/23$ seconds apart in time. The movie and images can be obtained from

<https://www.math.ucdavis.edu/~guy/teaching/17a/ws3images/>

- (a) Fill out the table below to record the penny's position as a function of time. The movie is recorded at 23 frames per second, and the spacing between lines on a piece of legal paper is 0.0087 m (8.7 mm). For each frame, find where the center of the penny is and estimate the number of lines from the top of the sheet that corresponds to that position. To get you started, the first two columns have been filled out for you. Look at Frames 1 and 2 to understand where these data come from, and then fill out the remaining columns.

Frame	1	2	3	4	5	6
time (seconds)	0	$1/23$				
Position (lines from top)	0	1.5				
Position (m)	0	0.01305				

Table 1: Fill out the last four columns of this table to complete problem 1a

- (b) We can find the approximate speed of the penny by calculating the change in its position, and dividing by the change in time, $v = \Delta y / \Delta t$. For example, the change in position from Frame 1 to Frame 2 is $\Delta y = y_2 - y_1 = 0.01305 - 0\text{m} = 0.01305\text{m}$. The change in time from Frame 1 to Frame 2 is $\Delta t = t_2 - t_1 = 1/23 - 0\text{sec} = 0.04348\text{sec}$. These give $v = 0.01305/0.04348 = 0.30015\text{ m/s}$. Perform these calculations for your measurements from part (a), and write your answers in the following Table 2.

Frame	1-2	2-3	3-4	4-5	5-6
time (sec)	1/23				
Δt (sec)	1/23				
Δy (m)	0.01305				
v (m/s)	0.30015				

Table 2: Fill out the last four columns of this table to complete problem 1b.

- (c) Plot your measurements and fit them with a straight line, using R. Here's how:

```
t=seq(from = 1/23, to = 5/23, by = 1/23)
v=c(0.30015,data2,data3,data4,data5)
G=lm(v~t)
plot(t,v)
lines(t,fitted(G))
```

Note that you will have to modify the second line of this code, so that v stores your data for v from Table 2. To get the equation for the best-fit line, type in “G” at the prompt in R (or R-studio). If the linear fit is of the form $y = mx + b$, then the coefficient called “intercept” is b and the other coefficient is the slope. What are these values?

- (d) You may recall from high school physics that the velocity of an object pulled by gravity is

$$v(t) = v_0 + gt \quad (1)$$

where v_0 is the initial speed and g is the gravitational constant. This equation says that your data, which estimate $v(t)$, should be fit by a straight line with intercept v_0 and slope g . Based on your answer to part (c), what is g ?

- (e) How do you think you could make a more accurate measurement of the penny's speed, and hence of g ?
- (f) Suppose the position data you measured corresponds to the red dots in this figure, and suppose that if you measured the position at more points in time, the data would be on the black curve. To estimate the velocity at time point t_3 , we used that

$$v(t_3) \approx (y_3 - y_2)/(t_3 - t_2).$$

Explain what would happen to your velocity estimate at time point t_3 if you had data at more time points. Would it be higher or lower than the original estimate? Would it be more accurate?

