MAT 17A - DISCUSSION #5

Problem 1: Minutes of Daylight
The number of minutes of daylight in Davis, CA in 2018 can be approximately described by the function

$$L(t) = C + A\sin\left(\frac{2\pi t}{365}\right) + B\cos\left(\frac{2\pi t}{365}\right), \tag{1}$$

where $C = 731.83$, $A = 30.91$, $B = -154.38$, and t is time in days beyond January first. That is, $t = 0$ represents January first.

(a) Make a plot of this function in R/RStudio using the following commands:

```
C= 731.83
A=30.91
B=-154.38
t=0:364
L=C + A*sin(2*pi*t/365) + B*cos(2*pi*t/365)
plot(t,L,type="l",xaxt="n",xlab="day of year",ylab="daylight time (minutes)")
axis(side=1,at=seq(0,365,30))
```

Note that the `axis` command is used to control the location of the tick marks on the horizontal axis. This puts tics between 0 and 365 every 30 days. You can adjust this if you want different tick marks.

(b) Based on the graph, on what day of the year is the amount of daylight increasing the fastest? On what day of the year is the amount of daylight decreasing fastest? From the graph you can read of the number of the day. Convert this to a date.

(c) Compute the derivative of $L(t)$, and make a plot of $L'(t)$ vs. t. Use this graph to estimate the date on which the amount of daylight is increasing fastest and decreasing fastest. How much is the amount of daylight changing at these two dates?

(d) Based on the graph of $L'(t)$, for approximately what values of t is $L'(t) = 0$? What are these dates? What's the significance of these dates?

(e) Tuesday, November 6th is the 310th day of this year. In equation (1), this corresponds to $t = 309$. Compute the equation of the tangent line to the graph of $L(t)$ at $t = 309$. The tangent line is a good approximation to $L(t)$ near $t = 309$. Make a plot of $L(t)$ and this tangent line on the same graph. Recall that the `lines` command will add a graph to an existing plot.

Problem 2: Differential Equations

Suppose that a population grows at 3% per year. Earlier in the quarter, we discussed how to model the population with a discrete time equation

$$N_{k+1} = (1.03)N_k,$$

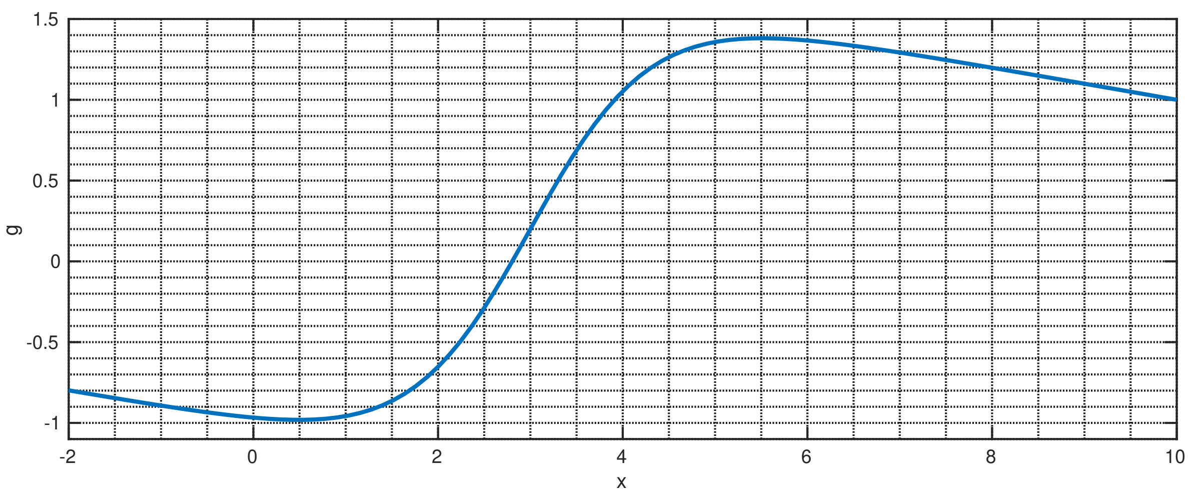
where N_k is the population at year k. This type of discrete time model describes how the population grows from year to year, but it does not give information about how the population grows over the course of the year. We could use a continuous time model and describe the growth by the differential equation

$$\frac{dP}{dt} = rP,$$

where $P(t)$ is the population as a function time t (in years), and r is a constant. For a population growing at 3% per year, what should the value of r be?

Problem 3: Estimating Derivatives from a Graph

Below is a graph of the function $g(x)$. Use the graph to answer the questions below.



(a) At what values of x is $g'(x) = 0$?

(b) Estimate g' at a few values other values of x and use the information to make a sketch of the graph of $g'(x)$.

(c) On what intervals is $g''(x) > 0$, and on what intervals is $g''(x) < 0$.

(d) Make a sketch of the graph of $g''(x)$. Your sketch should get the qualitative features of the graph (i.e. get the right shape), but there is no need for it to be quantitatively accurate.