**MAT 17A - DISCUSSION #2**

**GEOMETRIC AND ALLOMETRIC SCALING IN BIOLOGY**

**PROBLEM 1. A DANGER OF BEING SMALL: GETTING WET.**

“A man coming out of a bath carries with him a film of water about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed.”


How do you think Haldane came up with these conclusions? Did he go out and weigh humans, mice, and flies before and after dipping them in water? Probably not. In fact, these statements are probably not that precise. The main point of Haldane's statement is that as you get smaller the more dangerous getting wet becomes. Let's build an idealized mathematical model to see why this is.

(i) Let $R$ denote the ratio between the mass of the water film clinging to an animal's body after getting wet ($M_w$) and the dry animal's body mass ($M_B$). Find an expression (i.e., a mathematical model) that approximates $R$ as a function of $M_B$. Sketch a graph of $R$ as a function of $M_B$.

* Why is $R$ as a function of $M_B$ something we'd like to know?

* Construct your model by making simplifying assumptions, including:
  - assume that all animals are approximately the same shape, and model the body of an animal as a simple geometrical shape (i.e., a sphere).
  - assume that the mass of the water film on a wet animal is proportional to the animal's body surface area, and use Haldane’s estimate of the thickness of the water film ($l=0.05cm$).
  - assume that the density of an animal is the same as the density of water. Recall that density is mass per volume.

(ii) Compute $R$ for a human (60 kg), a cat (5 kg), a rat (0.25 kg), mouse (0.02 kg), a shrew (0.004 kg), a bee (0.0001 kg), a housefly (0.00002 kg), and a mosquito (0.0000025 kg). Plot the data points corresponding to these animals on your graph $R$ vs $M_B$ from (i).

(iii) What can you conclude from your mathematical analysis? Do your calculations agree with Haldane’s assertions? Do you see any issues with your/Haldane’s conclusions that getting wet poses serious dangers to small animals?

(iv) Consider the assumptions that you made in coming up with your mathematical model. Are they reasonable assumptions? How could you make your model more precise? Do you think your basic conclusions would change significantly if you constructed a more precise (but probably more complicated) model.
PROBLEM 2. Weight and wingspan of birds - "Why did dodos go extinct?" and "What should the wingspan of a flying human be?"

Ornithologists have measured and cataloged the wingspans and weights of many different species of birds. The table below shows the wingspan for a bird of weight.

(a) Use R Studio to make a scatter plot of the data. Make a semi-log plot, and a log-log plot. Are there any outliers in the data? Is there any justification for removing them from the set?

(b) Find an exponential function model and a power function model for the data (with outliers removed).

(c) Graph the models from b). Which fit appears better?

(d) The dodo is a bird that has been extinct since the late 17th century.* It weighed about 45 pounds and had a wingspan of about 20 inches. Why couldn't the dodo fly? For what reason(s) might the dodo have gone extinct? What data backs up your hypothesis?

(e) Based on your model, what wingspan would we need to be flying-humans?

[ see R commands on last page. ]

<table>
<thead>
<tr>
<th>BIRD</th>
<th>average body weight, W (lb)</th>
<th>average wingspan, L (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey vulture</td>
<td>4.40</td>
<td>69</td>
</tr>
<tr>
<td>Bald eagle</td>
<td>6.82</td>
<td>84</td>
</tr>
<tr>
<td>Great horned owl</td>
<td>3.08</td>
<td>44</td>
</tr>
<tr>
<td>Cooper's hawk</td>
<td>1.03</td>
<td>28</td>
</tr>
<tr>
<td>Sandhill crane</td>
<td>9.02</td>
<td>79</td>
</tr>
<tr>
<td>Atlantic penguin</td>
<td>0.95</td>
<td>24</td>
</tr>
<tr>
<td>King penguin</td>
<td>29.0</td>
<td>28</td>
</tr>
<tr>
<td>California condor</td>
<td>17.8</td>
<td>109</td>
</tr>
<tr>
<td>Common loon</td>
<td>7.04</td>
<td>48</td>
</tr>
<tr>
<td>Yellow warbler</td>
<td>0.022</td>
<td>8</td>
</tr>
<tr>
<td>Emu</td>
<td>138</td>
<td>69</td>
</tr>
<tr>
<td>Common grackle</td>
<td>0.20</td>
<td>16</td>
</tr>
<tr>
<td>Wood stork</td>
<td>5.06</td>
<td>63</td>
</tr>
<tr>
<td>Mallard</td>
<td>2.42</td>
<td>35</td>
</tr>
<tr>
<td>Dodo*</td>
<td>45</td>
<td>20</td>
</tr>
</tbody>
</table>
R CODE FOR PROBLEM 2
Type the uncommented lines of R-code (i.e., the lines without "#" at the beginning) in the console window.

# A. INPUT DATA AND PLOT

# data is listed in this order:
# (Turkey vulture, Bald eagle, Great horned owl, Cooper’s hawk, Sandhill crane, Atlantic puffin, King penguin,
# California condor, Common loon, Yellow warbler, Emu, Common grackle, Wood stork, Mallard, Dodo*)

# corresponding Weights of birds
w=c(4.4,6.82,3.08,1.03,9.02,0.95,29,17.8,7.04,0.022,138,0.2,5.06,2.42,45)

# corresponding wingspan of birds
s=c(69,84,44,28,79,24,28,109,48,8,69,16,63,35,20)

#plot data
plot(w,s,xlab="weight (lbs)",ylab="wingspan (in)")

# B. TAKE LOG OF DATA (BOTH w and s) AND RE- PLOT

# take log_10 of weight and wingspan data
wlog=log10(w)
slog=log10(s)

#plot log-log data on a new graph
plot(wlog,slog,xlab="log(weight)",ylab="log(wingspan)")

# What linear function y = a x + b would be a good fit to the log(wingspan) vs log(weight) data
# except for outliers (penguin, emu and dodo), where x = log(weight) and y = log(wingspan)?

# C. CREATE A LINEAR FUNCTION WITH SLOPE "a" and VERTICAL INTERCEPT "b" AND PLOT THE
# CORRESPONDING LINE ON A FIGURE THAT ALREADY EXISTS.

# change parameters "a" and "b" so that the line will fit your data
a = 2
b = -1

# create a sequence of numbers "x" from "xmin" to "xmax" with steps of "xstep".
xmin = -2
xmax = 2.5
xstep = 0.1
x=c(from = xmin, to = xmax, by = xstep)

# create a corresponding sequence of numbers "y=ax+b" for each value in the sequence "x"
y=a*x+b

# plot the curve going through points (x,y) on top of the graph that you already have plotted
lines(x,y,"l")